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SUBJECT: ENERGY DISSIPATION IN SQUARE LOOP FERROMAGNETIC MATERIALS WITH SPECIFIC APPLICATION TO SWITCH CORES

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Date: May 12, 1953

Abstract: A determination is made of the energy dissipation in a square loop ferromagnetic material due to domain wall motion. These energy losses are separated into three terms, representing the relaxation loss, the eddy current loss, and the hysteresis loss. The order of magnitude of the energy loss per cycle is found for an F-262 ferrite and a 140 wrap 4-79 molybdenum permalloy core. From the values thus determined, the power loss of these cores is calculated for a frequency of 100 kilocycles, assuming a semi-infinite step function input. The calculated order of magnitude is in agreement with the experimental results for a ferrite; no data is available for the metallic cores. The results indicate that external cooling will be needed for switch core matrices operated at this frequency. A number of reasons are given to show that metallic cores should be preferable to ferrites as switch cores.

A high speed multiposition switch can be made using ferromagnetic cores,[1, 2] and a number of these switch-core matrices are operating successfully at the present time. However, J. Mitchell has noted that when these cores are operated at frequencies in excess of 5 kilocycles they heat up and, with increasing frequency, soon lose their ability to operate as a switching device. This is not surprising since the successful operation of a ferromagnetic toroid is limited by the shape of the hysteresis loop (see reference 1) and this loop is a function of the temperature.

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[1] Olsen, K. H., "A Magnetic-Matrix Switch and its Incorporation into a Coincident Current Memory," Digital Computer Laboratory Report R-211 (1952).

[2] Katz, A. and Gudits, E. A., "Swtich Core Analysis, I" Digital Computer Laboratory, Engineering Note E-500 (1952).

The heating problem is a serious one since the switch cores, when used in a computer, will operate at a frequency of the order of 100 kilocycles. It is therefore important that the factors which contribute to the temperature rise be understood. This note investigates the energy dissipated by a core on reversing its magnetization and, on the basis of the results obtained, determines the power loss at 100 kilocycles.

Energy Losses

In the calculations which follow the hysteresis loop is assumed to be square and the input signal is assumed to be a sequence of semi-infinite step functions. This corresponds to a cyclic input of the form shown in figure 1.

Upon reversing the magnetization of a ferromagnetic material the equation of motion of a domain wall is given approximately by the equation[3]

$$\beta\vec{v} = 2(\vec{H}-\vec{H_0}) \cdot \vec{I_s} \qquad (1)$$

wherein $\underline{\beta}$ is the damping factor, $\underline{\vec{v}}$ the velocity of the domain wall, $\underline{\vec{H}}$ the applied field, $\vec{H_0}$ the threshold field, which is closely related to the coercive field, and $\vec{I_s}$ is the saturation magnetization. The damping factor $\underline{\beta}$ is the sum of two factors

$$\beta = \beta_e + \beta_r \qquad (2)$$

where $\underline{\beta_e}$ is the damping factor due to eddy current effects and $\underline{\beta_r}$ is the damping factor due to relaxation effects. Equation 1 can therefore be rewritten as

$$\beta_e\vec{v} + \beta_r\vec{v} + 2\vec{H_0}\cdot\vec{I_s} = 2\vec{H}\cdot\vec{I_s} \qquad (3)$$

In equation 3

$2\vec{H}\cdot\vec{I_s}$ is the energy/unit volume supplied by external field $\vec{H}$.
$\beta_e\vec{v}$ is the energy/unit volume dissipated as eddy current losses.
$\beta_r\vec{v}$ is the energy/unit volume dissipated as relaxation losses.
$2\vec{H_0}\cdot\vec{I_s}$ is the energy/unit volume dissipated as hysteresis losses.

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[3] Goodenough, J. B. and Menyuk, N., "Nucleation of Domains of Reverse Magnetization and Switching Characteristics of Magnetic Materials," Digital Computer Laboratory, Engineering Note E-532 (1953).

Eddy Current Loss

The value of β_e, as given in reference 3, page 26, is

$$\beta_e = \frac{64\pi^2 I_s^2}{\rho_e c^2 \langle\cos\theta\rangle} \langle r\rangle \ln\frac{R_m}{\langle r\rangle} \tag{4}$$

where R_m is half the thickness of the ferromagnetic material, ρ_e is the resistivity of the material, $\langle r\rangle$ is the effective value of the radius of the base of a cone of reverse magnetization as seen on a plane through the material, and $\langle\cos\theta\rangle$ is the effective value of the cosine of the angle between the normal to this plane and the direction of magnetization within the domain (see figure 2). The wall velocity v and $\frac{d\langle r\rangle}{dt}$ are related by

$$v = \frac{1}{\langle\cos\theta\rangle}\frac{d\langle r\rangle}{dt} ,$$

therefore,

$$\beta_e v = \frac{\beta_e}{\langle\cos\theta\rangle}\frac{d\langle r\rangle}{dt} = \text{Eddy current loss} = \frac{64\pi^2 I_s^2}{\rho_e c^2\langle\cos^2\theta\rangle}\langle r\rangle\frac{d\langle r\rangle}{dt}\ln\frac{R_m}{\langle r\rangle} \tag{5}$$

The switching time τ is the time required for the core to completely reverse its magnetization direction, and in this time the domain wall is assumed to travel a distance ρ. On integrating equation 5 over the proper limits one finds

$$\text{Eddy current loss} = \begin{cases} \dfrac{16\pi^2 I_s^2 R_m^2}{\tau\rho_e c^2\langle\cos^2\theta\rangle} & \text{for } R_m < \rho\langle\cos\theta_2\rangle \quad (6a) \\[2ex] \dfrac{16\pi^2 I_s^2\rho^2}{\tau\rho_e c^2}\left\{2\ln\dfrac{R_m}{\rho\langle\cos\theta\rangle} + 1\right\} & \text{for } R_m > \rho\langle\cos\theta_2\rangle \quad (6b) \end{cases}$$

The eddy current loss in ferrites is negligible because of the high resistivity of these materials. In matallic cores this is no longer the case. For thin ribbon metallic cores of the dimensions used in this laboratory (1/8 mil and 1/4 mil) $R_m < \rho\langle\cos\theta_2\rangle$ and equation 6a is applicable.

Relaxation Loss

The value of β_r given in reference 3, page 25, is

$$\beta_r = \frac{2 I_s^2 \Lambda}{(\Lambda^2 + I_s^2\gamma^2)}\sqrt{\frac{K}{A}} \tag{7}$$

where Λ is the relaxation frequency, γ the magneto-mechanical ratio, K the anesotropy constant, and A the exchange constant. Therefore,

$$\text{Relaxation Loss} = \frac{2I_s^2\Lambda}{(\Lambda^2+I_s^2\gamma^2)\langle\cos\rangle}\sqrt{\frac{K}{A}}\,\frac{d\langle r\rangle}{dt} \qquad (8)$$

and on integrating one finds

$$\text{Relaxation Loss} = \frac{2I_s^2\Lambda\rho}{\tau(\Lambda^2+I_s^2\gamma^2)}\sqrt{\frac{K}{A}} \qquad (9)$$

Hysteresis Loss

The hysteresis loss, as given earlier, is $2H_o \cdot I_s$ or $2H_oI_s\langle\cos\rangle$. H_o is the value of the magnetic field at which domain wall motion ceases. It is obtained experimentally as described in reference 3, page 28. This value is based on the assumption of a square hysteresis loop.

Total Loss

The total energy loss per unit volume in the course of a single magnetization reversal is thus

$$\frac{E}{V} = \frac{16\pi^2I_s^2R_m^2}{\tau\rho_s c^2\langle\cos^2\theta\rangle} + \frac{2I_s^2\Lambda\rho}{\tau(\Lambda^2+I_s^2\gamma^2)}\sqrt{\frac{K}{A}} + 2H_oI_s\langle\cos\rangle \qquad (10)$$

where equation 6a has been used for the eddy current loss. Since a full cycle represents two reversals, the energy loss per cycle will be twice the value given above.

Comparison with Experiment

Ferrite Core

At present, the General Ceramic ferrite core MF-1312B is being used in switch core matrices. Many of the factors which appear in equation 10 are unknown for this material. However, reasonable values can be obtained from a knowledge of these values for similar materials, and the order of magnitude obtained can then be checked with experimental results. In ferrites the eddy current effect is neglected.

The ferrite MF-1312B has $I_s \approx 155$ gauss and $H_o \approx 0.5$ oersteds. The distance $\rho \sim 5\text{x}10^{-3}$ cm, $\langle\cos\theta\rangle \sim 1$, $\gamma = 2\text{x}10^{7}$ (gauss-sec)$^{-1}$. The relaxation frequency $\Lambda \sim 10^{-8}$ sec^{-1} and $\sqrt{\frac{K}{A}} \sim 10^5$ cm^{-1}. The switching time τ is taken as $1.1\text{x}10^{-6}$ seconds, which is the experimental value. Substituting these values,

$$\text{Relaxation loss} = \frac{4I_s^2\Lambda\rho}{\tau(\Lambda^2+I_s^2\gamma^2)}\sqrt{\frac{K}{A}} \approx 450 \text{ ergs/cm}^3\text{/cycle}$$

$$\text{Hysteresis loss} = 4H_0I_s\langle\cos\rangle \approx 300 \text{ ergs/cm}^3\text{/cycle.}$$

The volume of these F-262 cores is 0.145 cm^3. The relaxation loss/core/cycle is therefore $\approx$62 ergs and the hysteresis loss/core/cycle is 44 ergs.

The experimental value of the hysteresis energy loss, as obtained from a measurement of the static hysteresis loop area, was found to be 75 ergs. The discrepancy between the calculated and experimental results can arise from our assumption of a perfectly square hysteresis loop. Actually, the loop is only approximately square, and increases in size with increasing input. Also, the experimental value of H_0 was taken from a single core, and it was not one of the cores used in obtaining the energy loss data. Since H_0 can vary by as much as 20% between supposedly similar cores, this leads to a large uncertainty in the calculated result.

The total energy loss per core per cycle has been found experimentally to be 162 ergs as compared with our calculated value of approximately 106 ergs. Thus the calculated relaxation loss of 62 ergs compares quite well with the experimental value of 87 ergs.

<u>Metallic Core</u>

A 1/4 mil 140 wrap 4-79 molybdenum permalloy core has recently been obtained by A. Katz for possible use as a switch core. The parameter constants, as given on page 29 of reference 3 are: $I_s \sim 700$ gauss, $\rho_e = 6\text{x}10^{-17}$esu-cm, $\sqrt{\frac{K}{A}} \approx 10^5 \text{cm}^{-1}$, $\Lambda \approx 2\text{x}10^8 \text{sec}^{-1}$, $\rho \approx 5\text{x}10^{-3}$, $\langle\cos\theta\rangle \approx 1$, and $\gamma = 2\text{x}10^7 (\text{gauss-sec})^{-1}$. Since this is 1/4 mil material, $R_m = 3.2\text{x}10^{-4}$ cm, and the switching time τ is again taken as $1.1\text{x}10^{-6}$sec. The switching time can be chosen arbitrarily as it is dependent upon the amplitude of the applied field. This value was chosen in order to obtain results which can be compared meaningfully with the results obtained for the ferrite MF-1312B. H_0 is 0.14 oersteds.

$$\text{Eddy Current loss} = \frac{32\pi^2 I_s^2 R_m^2}{\tau\rho_e c^2\langle\cos^2\theta\rangle} \approx 36 \text{ ergs/cm}^2\text{/cycle}$$

$$\text{Relaxation loss} = \frac{4I_s^2\Lambda\rho}{\tau(\Lambda^2+I_s^2\gamma^2)}\sqrt{\frac{K}{A}} \approx 900 \text{ ergs/cm}^3\text{/cycle}$$

$$\text{Hysteresis loss} = 4H_0 I_s \langle\cos\theta\rangle \approx 392 \text{ ergs/cm}^3\text{/cycle}$$

The total volume of this core is 0.13 cm^3. The calculated energy loss per core per cycle is therefore approximately 170 ergs.

No experimental values of energy losses are available as yet for comparison purposes.

Conclusions

The magnitude of the energy losses discussed above leads to a large power dissipation at high frequencies. At 100,000 cycles/second the calculated power loss for the MF-1326B and the metallic-ribbon core is about 1.1 and 1.7 watts per core respectively. Furthermore, in view of the parameters involved, it is highly unlikely that these figures will be decreased in order of magnitude. An external cooling system will therefore be needed in conjunction with a switch core matrix operating at high frequencies. Within this limitation, let us compare the relative merits of the metallic and ferrite cores as switch-matrix components.

1. Hysteresis loop - The hysteresis loop of the MF-1326B ferrite core is less square than that of the 140 wrap 1/4 mil 4-79 molybdenum permalloy core, as shown in figure 3. The pulse characteristics of the metallic core should therefore be preferable to that of the ferrite and should reduce the need for any "tailoring" of the input signal with its resultant complication of the electronic circuit.

2. Heating - As shown above, the metallic core dissipates 1.7 watts at 100kc. as compared to 1.1 watts for the ferrite core. However, in view of the assumptions made in these calculations, the difference is not necessarily significant. Furthermore, since the saturation magnetization is almost five times as great in the metallic core as in the ferrite, the output per metallic core should be considerably greater than the output per ferrite core. At present, seven ferrite cores are needed for each switch-matrix component to obtain the desired output. Since fewer metallic cores will be needed, the heat generated per component will probably be lower using metallic cores.

3. Surface Area-Volume Ratio - For efficient cooling it is desirable to have as large a surface area to volume ratio as possible. This ratio is rather poor in both the F-262 size ferrite core now in use and the 140 wrap metallic core which has been received. This ratio can be increased in the metallic core by widening the core and decreasing the number of wraps, maintaining the same

volume. It would probably be more difficult to effect a large improvement in this ratio with a ferrite.

4. Temperature Sensitivity - Since molybdenum permalloy has a higher Curie temperature (~460°C.) than the ferrites (~300°C.), the metallic core hysteresis loops should be somewhat less sensitive to small temperature changes in the vicinity of room temperature than the ferrites.

Applied Field Value - The 1/4 mil molybdenum permalloy core has a smaller value of H_o than the MF-1326B ferrite. In addition, the switching coefficient S_w, where $S_w = (H-H_o)\tau$, is also smaller for the metallic core. The figures are:

$S_w = 6.4 \times 10^{-7}$ oersted-second for 1/4 mil 4-79 molybdenum permalloy, and

$S_w = 10.2 \times 10^{-7}$ oersted-second for MF-1312B.

A metallic switch-core matrix designed to operate at the same speed as the ferrite switch therefore requires a smaller magnetic field. Thus, when using metallic cores, a smaller current amplitude will be required of the electronic equipment used in conjunction with the matrix. This may permit a simplification of the circuitry.

Recommendation

The above considerations all indicate that the metallic switch core matrix is preferable to a ferritic switch core matrix. Future experimentation along this direction should therefore be emphasized. In particular, wider cores should be used to increase the surface area.

Acknowledgment

The writer wishes to thank A. Katz for bringing the problem to his attention and for supplying all the experimental information pertaining to switch cores. Useful discussions pertaining to this problem were held with J. B. Goodenough, P. K. Baltzer and A. Katz.

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NM/djd
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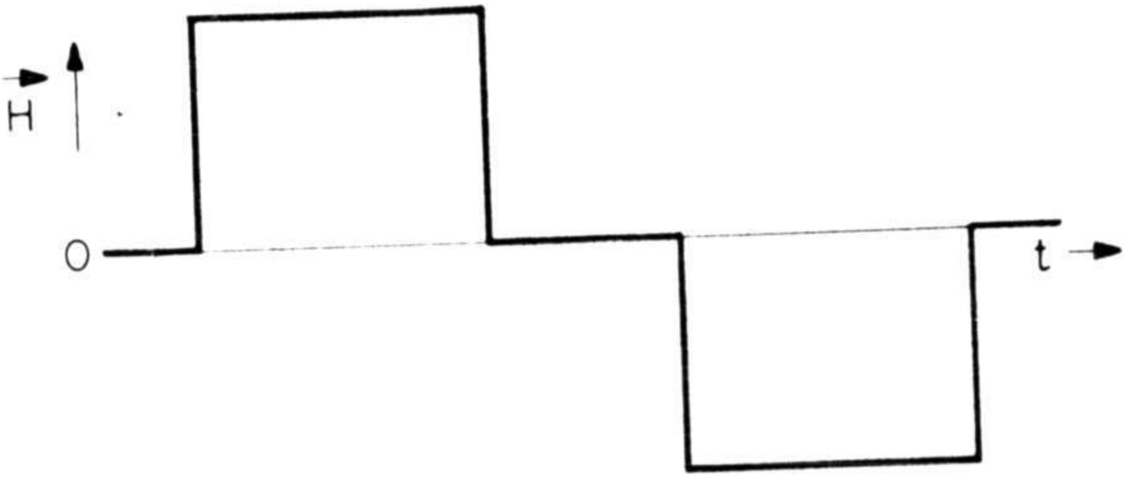
A-55036
A-55037



ASSUMED CORE SIGNAL INPUT

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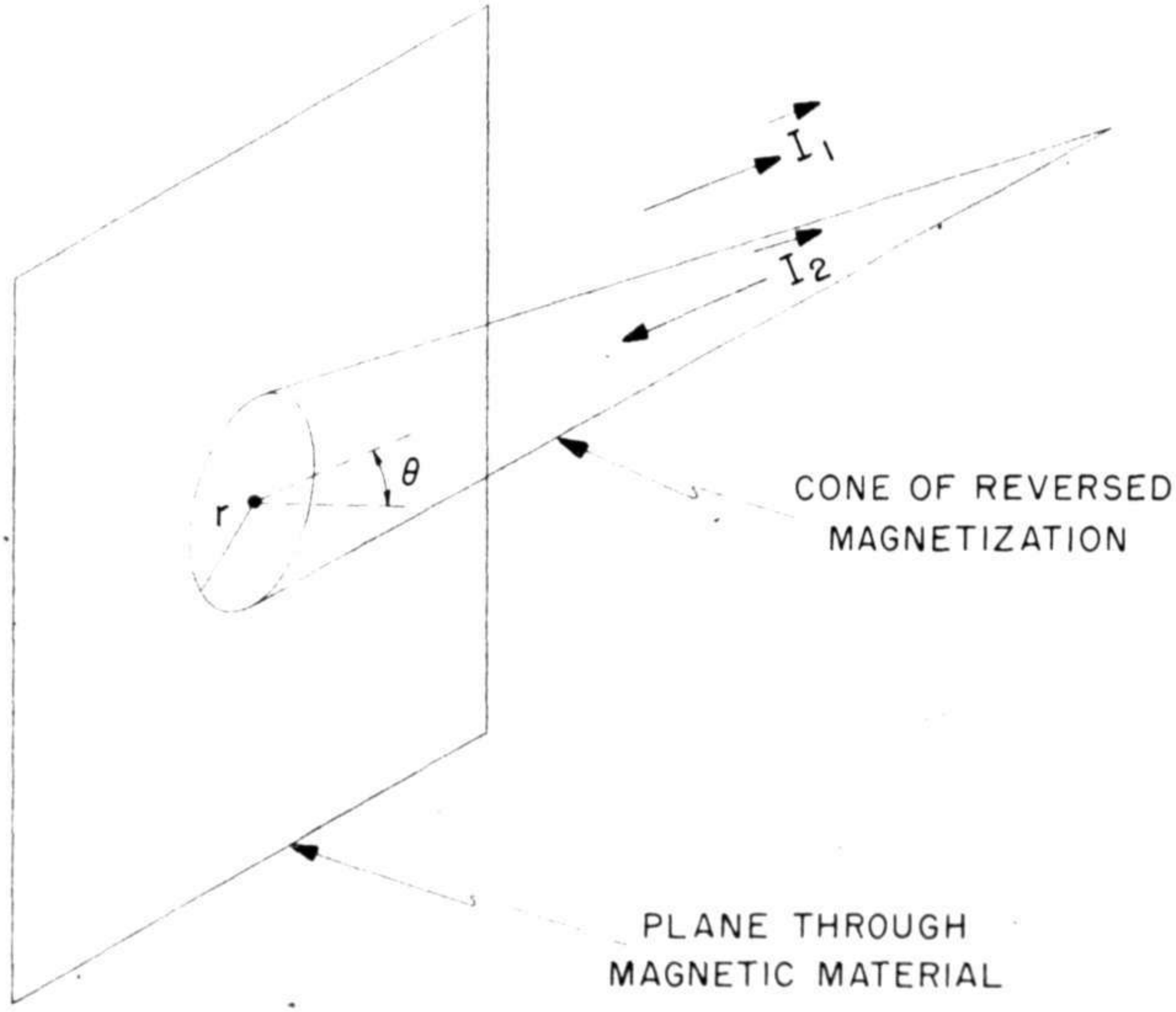
MH



REGION OF REVERSE MAGNETIZATION

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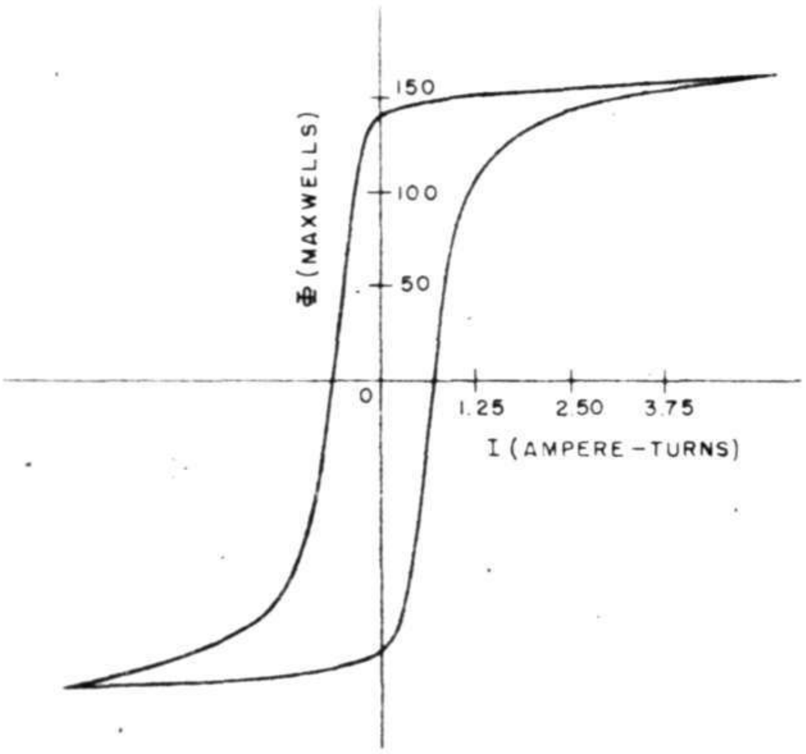
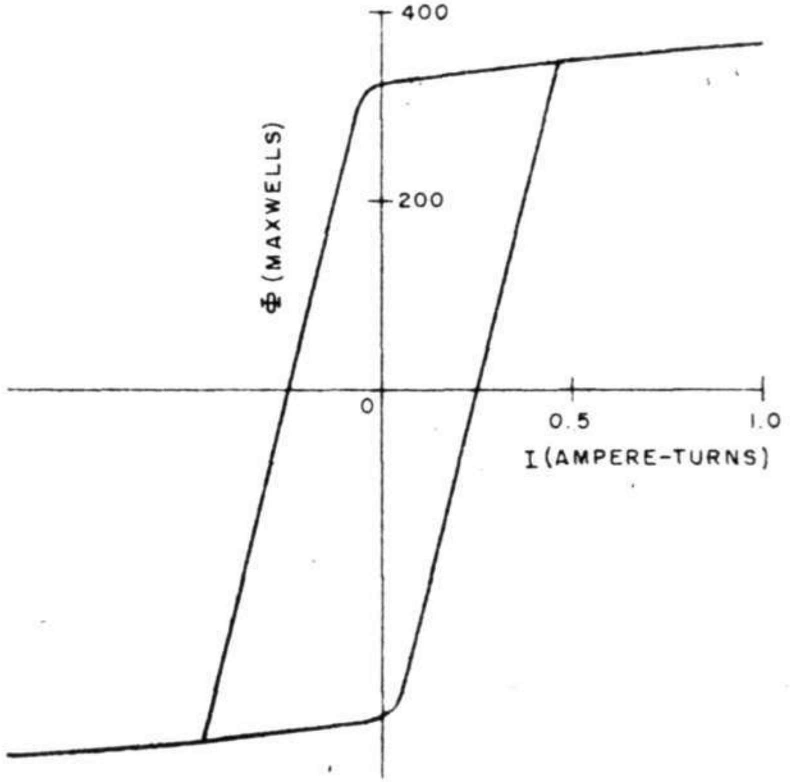
MN



(a) STATIC HYSTERESIS LOOP OF AN MF 1312 B FERRITE CORE.



(b) 60 CYCLE HYSTERESIS LOOP OF A 140 WRAP $\frac{1}{4}$ MIL 4-79 MO-PERMALLOY CORE.