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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, Vol. 2, I
           Ferromagnetic Anisotropy

To: Group 63 Staff

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Introduction

This memorandum covers the first two meetings of the 1953-1954 seminar on magnetism. It represents a survey of the literature on ferromagnetic anisotropy and is intended to serve as an introduction to the subject. It does not entail an intensive study of any particular aspect of anisotropy; this will be done at later meetings.

It was felt that this survey could best be accomplished by splitting the subject into four broad subdivisions: (1) definition of the anisotropy constants, (2) important applications of anisotropy, (3) theoretical explanations of anisotropy, (4) methods of measuring the anisotropy constants.

I. Definition of Anisotropy Constant

It has been observed that in single crystals of ferromagnetic materials, including those with cubic symmetry, different field strengths are required to saturate the material in different crystallographic directions. This is shown for cobalt, iron and nickel in Figure 1.

Field strength H →
in oersteds
(a) Cobalt

(b) Iron

(c) Nickel

Figure 1
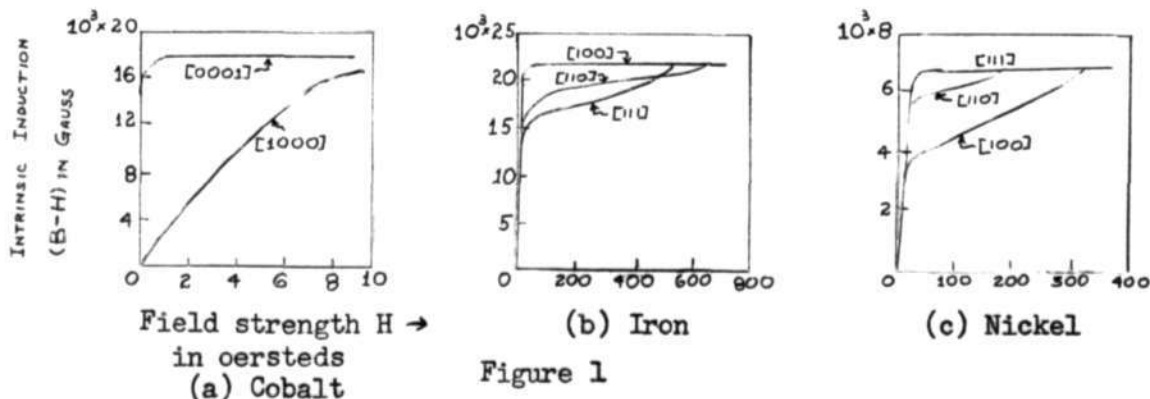
In obtaining quantitative expressions to describe this phenomenon it has been found convenient to use anisotropy constants K_1, K_2, etc., to describe the free energy density of anisotropy. In cubic crystals, the

anisotropy energy density is expressed in the form

$$E_k = K_o + K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2) + K_2 (\alpha_1^2 \alpha_2^2 \alpha_3^2) \quad (1)$$

where α_1, α_2, α_3 are the direction cosines of the angle the magnetization vector makes with the crystallographic axes.

In the expression K_o is the energy required to magnetize the material to saturation along one of the crystal axes.

The crystalline energy density can be determined by measuring the energy needed to saturate a crystal in that direction. The energy density needed to magnetize the crystal in the [hkl] direction will be given by

$$A_{hkl} = \int_0^{I_s} H dI \quad (2)$$

Since forces other than anisotropy will be involved in A_{hkl}, (e.g. magnetostriction) the work done in any given direction will not be due to anisotropic forces alone. However, if the other forces are assumed isotropic, the difference in the energy required to magnetize the material in two different directions will give a measure of the difference in anisotropy energy between the two directions.

Thus if the two directions chosen are the [110] direction in which $\alpha_1 = 1/\sqrt{2}$, $\alpha_2 = 1/\sqrt{2}$, $\alpha_3 = 0$ and the [100] direction in which $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$, substitution into (1) yields

$$A_{110} - A_{100} = E_{110} - E_{100} = K_o + K_1 (\tfrac{1}{4}) - K_o = \frac{K_1}{4}$$

Similarly, taking the difference in energy between the [111] direction in which $\alpha_1 = \alpha_2 = \alpha_3 = 1/\sqrt{3}$ and the [100] direction.

$$A_{111} - A_{100} = E_{111} - E_{100} = \frac{K_1}{3} + \frac{K_2}{27}$$

From these equations one finds

$$\left.\begin{array}{l} K_o = A_{100} \\ K_1 = 4(A_{110} - A_{100}) \\ K_2 = 27(A_{111} - A_{100}) - 36(A_{110} - A_{100}) \end{array}\right\} \quad (3)$$

Table I, based on Equation (3), shows the relation between the ease of magnetization in the different directions and the values of K_1 and K_2.[1]

Table I

K_1	+	+	+	-	-	-
K_2	$+\infty$ TO $-9K_1/4$	$-9K_1/4$ TO $-9K_1$	$-9K_1$ TO $-\infty$	$-\infty$ TO $9|K_1|/4$	$9|K_1|/4$ TO $9|K_1|$	$9|K_1|$ TO $+\infty$
Easiest direction	[100]	[100]	[111]	[111]	[110]	[110]
Intermediate direction	[110]	[111]	[100]	[110]	[111]	[100]
Hardest direction	[111]	[110]	[110]	[100]	[100]	[111]

II. Important Applications of Anisotropy

1. Domain Structure

The domain wall energy per unit area $\sigma_w = \sqrt{K_1 A}$ in cubic crystals,[2] where A is the exchange parameter and K_1 the anisotropy constant. The relative values of K_1 and A determine the width of the domain wall. Since the formation of new domains involves the creation of Bloch walls at the boundary, the energy of these walls is a major factor in the determination of the size and shape of the domains which is energetically most favorable.

In addition, the type of anisotropy exhibited by a material can strongly effect its domain structure. In cubic crystals, with three axes of easy magnetization, the formation of domains of closure bounded by $90°$ walls is common. However, in uniaxial crystals with large anisotropy constants such as cobalt, the closure domains involve high anisotropy energies. Therefore one does not find domains of closure in cobalt.

2. Square Loop Materials

A condition for square loop materials has been found to be[3]

$$\sigma_w = \sqrt{K_1 A} > C M_s^2 (\cos \theta_1 - \cos \theta_2)^2 \qquad (4)$$

where C is a constant dependent upon geometric considerations of the microscopic structure of the sample, M_s is the saturation magnetization, and θ_1 and θ_2 are a measure of the grain orientation of the sample.

3. Approach to Saturation

The approach to saturation is dependent upon the anisotropy of the material. For a polycrystalline material in a high field, assuming a random orientation of the directions of easy magnetization from grain to grain, the approach has been found to be[4]

$$\frac{I}{I_s} = 1 - \frac{8K_1^2}{105 I_s^2 H^2}$$

4. Resonance Phenomena

In resonance absorption experiments carried out at microwave frequencies, the resonance frequency in classical theory is given by the equation[5]

$$\omega_o = \gamma H_{eff} \tag{5}$$

Except for the use of H_{eff} instead of applied magnetic field H_z, this is equivalent to the Larmor frequency. H_{eff} is used in Equation (5) to take into account the demagnetization energy and the anisotropy energy. In this equation it is convenient to think of the anisotropy energy in terms of an equivalent magnetic field $\underline{H}^e$ which is defined such that the torque exerted on the sample by this fictitious field is equal to the torque exerted by the anisotropy energy.

$\underline{H}^e$ is usually taken parallel to the x or y axis, and is then expressed in terms of an effective demagnetizing factor N^e such that

$$H_x^e = - N_x^e M_x$$

$$H_y^e = - N_y^e M_y$$

Then it can be shown[5] that in an ellipsoidal specimen, where the demagnetizing factors in the x, y, and z directions are given by $-N_x M_x$, $-N_y M_y$, $-N_z M_z$ respectively, the effective field H_{eff} of Equation (5) is given by

$$H_{eff} = \left\{ [H_z + (N_y + N_y^e - N_z)] \times [H_z + (N_x + N_x^e - N_z) M_z] \right\}^{1/2} \tag{6}$$

The values of N_x^e and N_y^e are dependent upon the anisotropy constant, the type of anisotropy (e.g. uniaxial or cubic), and the direction of the applied field. Thus Kittel[6,7] obtains the following results:

a) Uniaxial Crystal; H_z along axis

$$N_x^e = N_y^e = \frac{2K_1'}{M_z^2}$$

b) Cubic Crystal; xy plane, containing both static and rf field (100)

$$N_x^e = \frac{2K_1}{M_z^2} \cos 4\Theta$$

$$N_y^e = (\frac{3}{2} + \frac{1}{2} \cos 4\Theta) \frac{K_1}{M_z^2}$$

where Θ is angle between the magnetization and a [100] axis.

Beckford[8] has found the corresponding equations for the (110) plane to be

$$N_x^e = (2 - \sin \Theta - 3 \sin^2 2\Theta) \frac{K_1}{M_z^2}$$

$$N_y^e = 2(1 - 2 \sin^2 \Theta - \frac{3}{8} \sin^2 2\Theta) \frac{K_1}{M_z^2}$$

5. Domain Wall Velocity

At small magnetic field values, magnetization reversal is primarily due to domain wall motion. In various applications (e.g. magnetic cores for digital computer memory arrays) it is desirable to have this motion as rapid as possible. The velocity of the domain wall in materials wherein eddy current effects can be ignored is inversely proportional to the square root of the anisotropy constant. The equation of wall velocity $\underline{v}$ in a cubic crystal is[9]

$$v = \frac{2(\gamma^2 M_s^2 + \Lambda^2)}{\Lambda M_s} \quad (H-H_o) \sqrt{\frac{A}{K_1}}$$

where γ is the magnetomechanical ratio, M_s the saturation magnetization, Λ the relaxation frequency, H the applied field, H_o the threshold field value, A the exchange parameter, and K_1 the anisotropy constant

6. Coercive Force

A number of factors enter into the coercive force of a ferromagnetic material. Upon the assumption of cylindrical domains within the material, one of these factors will be dependent upon the anisotropy constant through the relationship[10]

$$H_w(\sigma_w) \sim \frac{10^3 \sigma_w}{M_s} \sim 10^3 \frac{\sqrt{K_1 A}}{M_s}$$

where $H_w(\sigma_w)$ is the contribution to the coercive force due to the surface tension of the Bloch walls opposing domain growth.

III. Theoretical Explanations of Anisotropy

One of the first theoretical explanations of anisotropy was introduced by Mahajani.[11,12] He assumed the elementary magnets to be electron orbits. Due to their magnetic interaction, these magnets have a mutual potential energy which is dependent on their orientation. This leads to an anisotropy. However, this theory leads to the objection that the magnetic moment is due to electron spin and not orbital motion. Furthermore, his results led to an anisotropy effect which was much too small.

A number of authors then introduced the concept of spin-orbit coupling to explain the anisotropy effect. Bloch and Gentile[13] dealt with a one atom model in which anisotropy arose from the interplay between the spin-orbit interaction and the quantization of a single atom in a crystalline field.

Powell and Fowler[14,15] introduced an anisotropic Weiss field through the concept of a crystalline field which lies in a number of directions bearing simple relations with the crystal structure. This field is associated with spin-orbit coupling.

Bozorth and McKeehan[16,17] pointed out that if the dipole associated with each lattice point forms a small magnet of finite length, a negative value of K_1 results. That is, the easy direction of magnetization is along the [111] axis, as in nickel. On the other hand, if the dipoles form a disk equivalent to a circular current loop, a positive value of K_1 results. On the basis of this model an anisotropy of the proper order of magnitude can be obtained from magnetic interaction alone. The introduction of electrostatic interaction is then un-necessary.

The above articles are not discussed in any detail since Van Vleck,[18] in 1937, published a more quantitative article on magnetic anisotropy theory which included a critique of the aforementioned anisotropy models.

The exchange energy which is responsible for ferromagnetism cannot lead to anisotropy, for the exchange forces are equivalent to a coupling energy $-2J\vec{S}_i \cdot \vec{S}_j$ where J is the exchange energy. Since $\vec{S}_i \cdot \vec{S}_j$ is a scalar product, it is invariant under rotation.

It is therefore necessary to look for some other form of interaction, and the first one considered in this article is dipole-dipole coupling. Van Vleck points out that parallel dipoles cannot give anisotropy in cubic crystals since the mutual potential energy of parallel dipoles transforms like a quadratic form under axis rotation. However, cubic anisotropy cannot be expressed by a form of lower than fourth order.

When the dipoles are not perfectly parallel, anisotropy can result. The determination of this effect involves a second order perturbation calculation.

Van Vleck next considers quadrupole-quadrupole interaction and shows that spins of 1/2 cannot generate quadrupole coupling. Since the nickel atom has a saturation magnetization of 0.6 Bohr magnetons at absolute zero, which presumably arises from the fact that 60 percent of the nickels ions have a d^9 magnetic configuration and 40 percent a d^{10} non-magnetic configuration, its atomic spin is almost certainly 1/2. Its anisotropy must therefore be due to dipole-dipole coupling. Iron, on the other hand, has two electrons less than nickel and so may have spins greater than 1/2. It may therefore have interactions of quadrupole-quadrupole and lower order terms.

Thus far no attempt has been made to consider the physical origin of the anisotropic interactions. The most likely explanation arises from the fact that the mutual electrostatic energy of two atoms is dependent upon the alignment of their orbital angular momentum as well as the spin. The orbital coupling is dependent upon the orientation of the orbital angular momentum with respect to the axis joining the two atoms as well as their orientation relative to each others. Thus the energy of orbital coupling is of the form $f(\vec{L}_i \cdot \vec{L}_j, \vec{L}_{zi}, \vec{L}_{zj})$, where z is the axis joining the two atoms. The exchange interaction is not dependent on the axis direction and is of the form $F(\vec{S}_i \cdot \vec{S}_j)$.

Since orbital angular momentum is strongly quenched in ferromagnetic materials, the anisotropy of the orbital moment might seem unimportant. However, the spin-orbit coupling is not negligible so the energy will depend upon the spin orientation relative to the crystal axes.

Simply stated, the orbital angular momentum is in a position of minimum energy with respect to the spin direction and the crystalline axes when the spin vectors are in the direction of easiest magnetization. Rotation of the spin vector direction will tend to produce changes in both these effects. The orbital-axis coupling will change the relative positions of the spin and orbital angular momentum while the spin-orbit coupling will cause a rotation of the orbital momentum with respect to the axes. These effects result in an energy configuration which is a function of the magnetization direction. They give rise to the second and third terms of the Hamiltonian given in Equation (7).

The calculation of the anisotropy energy should follow from a Hamiltonian of the form

$$\mathcal{H} = 2 \sum_{j>i} J_{ij} \, \vec{S}_i \cdot \vec{S}_j + A \sum_i \vec{L}_i \cdot \vec{S}_i + \sum_{j>i} f_{ij} \, (\vec{L}_i \cdot \vec{L}_j, \vec{L}_i \cdot \vec{r}_{ij}, \vec{L}_j \cdot \vec{r}_{ij}) \quad (7)$$

where $\vec{S}_i$ and $\vec{S}_j$ are the spin angular momentum operators of the ith and j^{th} atoms respectively, $\vec{L}_i$ and $\vec{L}_j$ are the orbital angular momentum operators of the ith and j^{th} atoms respectively, $\vec{r}_{ij}$ is the radius vector connecting the ith and j^{th} atoms, J_{ij} is the exchange integral for the two atoms, A is the spin orbital coupling constant, and f_{ij} is some polynomial function of the indicated argument. The first term on the right-hand side of Equation (7) represents the Heisenberg exchange energy, the second term represents the spin-orbit coupling, and the third term represents the dependence of the orbital angular momentum coupling of two atoms on both their relative orientation and their orientation relative to the vector joining them.

A calculation based directly on Equation (9) would be prohibitively difficult. Van Vleck therefore based his calculation on the equation

$$\mathcal{H} = g\mu_B \, \chi I' \sum_i S_i + \sum_{j>i} C_{ij}\left[\vec{S}_i \cdot \vec{S}_j - 3\,(\vec{S}_i \cdot \vec{r}_{ij})(\vec{S}_j \cdot \vec{r}_{ij})/r_{ij}^2\right]$$

$$+ \sum_{j>i} \Upsilon_{ij}\,(\vec{S}_i \cdot \vec{r}_{ij})^2(\vec{S}_j \cdot \vec{r}_{ij})^2/r_{ij}^4 \tag{8}$$

Equation (8) is essentially a substitute for the more exact form given in Equation (7). The Heisenberg exchange coupling term has been replaced by an expression using the Weiss molecular field wherein χ is the constant of proportionality between the exchange energy and the Weiss field which is proportional to the magnetization intensity I. The additional terms of Equation (8) represent the constraints on the spin which would be found if the last two terms of Equation (8) could be treated exactly. The second term represents the dipole-dipole interaction and the third term represents the quadrupole-quadrupole interaction.

The anisotropy constant K_1 can be obtained by regarding the C and Υ terms as pertubations carried to the second order in C or first order in Υ. Both C and r are taken to be functions of temperature in this model.

From calculations based on the above model Van Vleck obtained values of K_1 of the order of 10^5 ergs/cm^3. This compares favorably with experimental values for iron ($K_1 = 6 \times 10^5$) and nickel ($K_1 = -3 \times 10^4$).

The sign of K_1 cannot be determined if it is due to the quadrupole-quadrupole interaction of the third term in Equation (8). However, if K_1 is due only to dipole-dipole interaction as given in the second term of Equation (8), K_1 will be positive for a simple-cubic lattice and negative for a face-centered lattice. Thus nickel, which is face-centered and has no quadrupole-quadrupole interaction, is predicted to have a negative value of K_1. This is in accord with experiment.

The predicted temperature variation near the Curie point is only roughly correct. In this region the variation of the anisotropy constant with saturation magnetization is found to be of the order of I^{10}. The theory, however, yields a variation of the order of I^5 or I^6.

Van Vleck's article also contains a critique of various proposals previously set forth to explain anisotropy. The Bloch and Gentile model is based upon a single atom. Van Vleck maintains that the coupling of angular momentum vectors of different atoms is needed, and that the interaction of a single vector with the crystalline field cannot yield appreciable anisotropy. He then sets forth his reasons for this assertion.

The Powell and Fowler model involves the introduction of an anisotropic Weiss field which is somehow associated with spin-orbit coupling. However, while it may be considered a phenomenological approach to the problem it cannot be considered a real explanation of anisotropy.

The Bozorth and McKeehan model assumes a complete correlation between the spin and orbital alignment. In that case, from purely magnetic interaction, $K_1 \sim R^2 g^2 \mu^2 / r_{ij}^5$ where R is the atomic radius. Taking $R^2/r^2 \sim 1/10$, $r_{ij}^3 = 10^{-23}$, the value of K_1 is of the order of magnitude observed experimentally.[13] However, in Van Vleck's opinion correlation of spin and orbital alignment is weak and should lead to a value of K_1 of the order of 10^3 too small. This loss in magnitude is recovered on assuming an electrostatic interaction.

H. Brooks[19] treats ferromagnetic anisotropy from the standpoint of the collective electron model in which each electron is treated as belonging to the entire metal and moves in a self-consistent field determined by the potential of the ions and the "time exposure" field of the other electrons. Although this model is a poorer approximation for the bound d electrons than the Heisenberg model, the introduction of spin-orbit interaction leads directly to ferromagnetic anisotropy.

The calculations required to obtain quantitative results are quite involved. The anisotropy constant calculation is obtained by applying fourth order perturbation theory.

Using this model, Brooks obtained the correct order of magnitude of K_1 for nickel and iron with the proper sign in each case. However, this model could not be used to obtain the correct thermal variation of anisotropy.

IV. Measurement of Anisotropy Constants

An exhaustive discussion of the behavioral aspects of ferromagnetic anisotropy, including an extensive bibliography, is to be found in Chapter 12 of "Ferromagnetism" by Bozorth.[20] Rather than duplicate this entire work we will limit ourselves to that feature which is of most direct interest to us, namely methods of measurement of the anisotropy constants. The three methods discussed below are (1) the method of torque measurements, (2) the fitting of theoretical magnetization curves to experimental curves, and (3) the area measurement between magnetization curves.

1. Torque Measurements

The measurement of torque due to magnetic anisotropy using a torque magnetometer can be found in an article by Larasov[21] and others.[22,23] Figure 2, taken from page 557 of Bozorth's "Ferromagnetism" is based upon the instrument described in the Larasov article.

As can be seen in Figure 2, the single crystal specimens are cut in the form of disks and are held by torsion fibers. These fibers are of phosphor bronze with a diameter of 0.032 inches. The upper wire is fastened rigidly to the torque meter frame while the lower wire is attached to a circular disk which is free to turn. A gear attachment used in the actual meter is not shown in this figure for, while permitting greater accuracy of measurement, it does not alter the operation of the mechanism as shown.
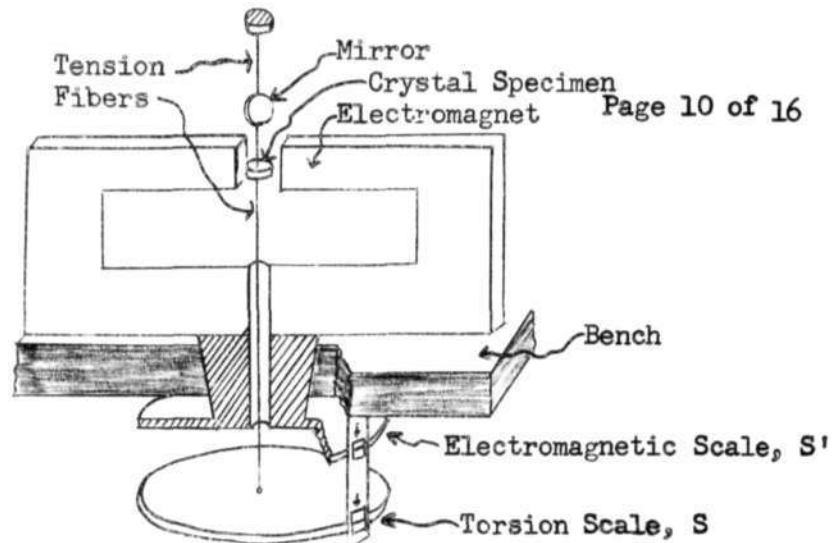
Figure 2 Torque Magnetometer

Upon application of a magnetic field, the magnetic specimen will tend to turn so as to place its direction of easy magnetization in the field direction. This torque is then balanced by turning the circular disk until the galvanometer mirror mounted on the upper torsion fiber has been returned to its original position. The difference in the torsion scale reading S before and after application of the applied field gives a measure of the torque.

The orientation of the crystal axes of the magnetic specimen with respect to the applied field can be varied by turning the electromagnet. The position of the electromagnet can be read on the scale S'.

A discussion of the relationship between the torque measurements and the anisotropy constants follows.

Extending earlier work of Akulov,[24] Heisenberg,[25] Gans,[26] and Czerlinsky[27] on magnetic anisotropy, Bozorth[1,28] presented a thorough study of torque effects in single crystals.

A disk cut from a single ferromagnetic crystal is generally subject to a torque because of the magnetic anisotropy of the crystal. The torque per unit volume of the crystal is the rate of change of energy density with angle:

$$L = - \frac{dE_k}{d\Theta} \qquad (9)$$

where, as before, $E_k = K_o + K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2)$.

Therefore

$$L = - K_1 \frac{d}{d\Theta} (\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) - K_2 \frac{d}{d\Theta} (\alpha_1^2\alpha_2^2\alpha_3^2). \qquad (10)$$

For the case of a cubic crystal magnetized parallel to the (100) plane, $\alpha_1 = 0$, $\alpha_2 = \cos\Theta$, and $\alpha_3 = \sin\Theta$. Then from Equation (10)

$$L = - K_1 \frac{d}{d\Theta} (\sin^2\Theta \cos^2\Theta)$$

$$L = -K_1 (2 \sin \Theta \cos^3 \Theta - 2 \sin^3 \Theta \cos \Theta)$$

$$= -2K_1 \sin \Theta \cos \Theta (\cos^2 \Theta - \sin^2 \Theta)$$

$$= -K_1 \sin 2\Theta \cos 2\Theta .$$

$$L_{100} = \tfrac{1}{2} K_1 \sin 4\Theta \qquad\qquad (11)$$

Similarly it can be shown that for a cubic crystal magnetized parallel to the (110) plane

$$L_{110} = -\frac{K_1}{8} (2 \sin 2\Theta + 3 \sin 4\Theta) - \frac{K_2}{64} (\sin 2\Theta + 4 \sin \Theta - 3 \sin 6\Theta) \quad (12)$$

The value of $\frac{dE_k}{d\Theta}$ for various planes, as originally given in reference (1), are shown in Table II.

From Equation (11) it can be seen that the slope of L_{100} vs. Θ at $\Theta = 0$ is

$$\left(\frac{dL_{100}}{d\Theta}\right)_{\Theta = 0} = -2K_1$$

and the maximum and minimum of the L_{100} vs. Θ curve occur at $\Theta = \frac{(2n-1)\pi}{8}$ where n is an integer, and have the value

$$(L_{100})_m = \pm K_1/2 .$$

Thus K_1 can be determined by measuring L in the (100) plane.

Both K_1 and K_2 are involved in torque measurements in the (110) plane. There the constants are determined by obtaining a best fit from the torque data. A typical torque curve in the (110) plane, obtained by Williams[29] using a crystal of silicon iron is shown in Figure 3.
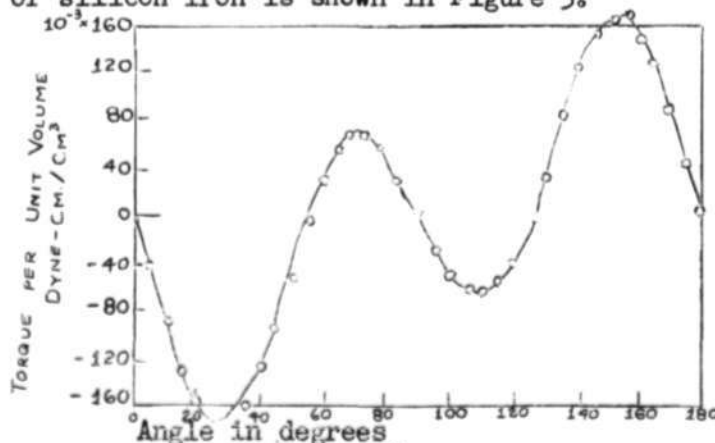

Figure 3
Torque curve of single xtal disk of silicon iron in (110) plane

It should be borne in mind that for finite fields the magnetization direction is undetermined, so the value of Θ is not known. However, as H is increased the magnetization direction approaches that of the applied field, and Θ is then known. Taking a measurement of torque for various field values and extrapolating the curve of torque vs. 1/H affords a simple means of determining the torque for an infinite field. This is shown in Figure 4 for 4 percent silicon iron.
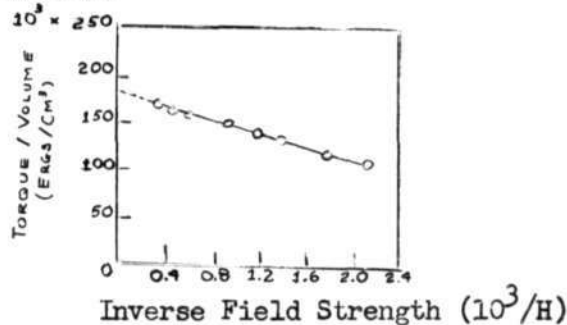

Inverse Field Strength $(10^3/H)$

Figure 4

2. Comparison of Theroetical and Experimental Magnetization Curves

A theoretical relationship between the magnetization curve of a magnetic material and the anisotropy constants is discussed in references previously cited,[24,27,1] and is given below.

If a single crystal with an easy direction of magnetization $[h\,k\,l]$ is subjected to a field $\vec{H}$ at some angle Θ_o from the $[h_o k_o l_o]$ direction, the magnetization $\vec{M}_s$ will lie somewhere between $[h_o k_o l_1]$ and the magnetic field direction, as shown in Figure 5. As H is increased, $\vec{M}_s$ will approach $\vec{H}$.
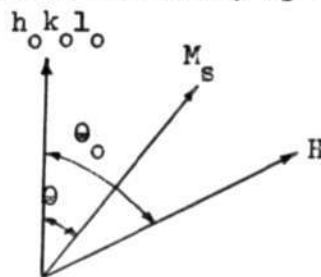

Figure 5

The angle of deviation of M_s from $[h_o k_o l_o]$ is taken as Θ, and will lie in the position of minimum energy. This position can be found by minimizing the energy with respect to Θ.

$$E = E_k - HM_s \cos(\Theta_o - \Theta)$$

$$\frac{dE}{d\Theta} = \frac{dE_k}{d\Theta} - HM_s \sin(\Theta_o - \Theta) = 0$$

Therefore

$$H = \frac{dE_k/d\Theta}{M_s \sin(\Theta_o - \Theta)} \tag{12}$$

where $\frac{dE_k}{d\Theta}$ is just the torque expression discussed previously. The magnetization $M = M_s \cos(\Theta_o - \Theta)$.

A table of values[1] of $dE_k/d\Theta$ for various planes is given below:

Table II

Plane (hkl)	Easy Magnetization Direction $[h_o k_o l_o]$	$dE_k/d\Theta$
100	001	$K_1(\sin 4\Theta)/2$
100	011	$K_1(-\sin 4\Theta)/2$
110	001	$K_1(2\sin 2\Theta + 3\sin 4\Theta)/8 + K_2(\sin 2\Theta + 4\sin 4\Theta - 3\sin 6\Theta)/64$
110	1̄10	$K_1(-2\sin 2\Theta + 3\sin 4\Theta)/8 + K_2(-\sin 2\Theta + 4\sin 4\Theta + 3\sin 6\Theta)/64$
110	1̄10	$K_1(-2\sin 2\Theta - 7\sin 4\Theta)/7 + K_1(\cos 2\Theta - \cos 4\Theta)/3\sqrt{2}$ $+ K_2(-3\sin 2\Theta - 28\sin 4\Theta - 23\sin 6\Theta)/576$ $+ K_2(3\cos 2\Theta - 8\cos 4\Theta + 5\cos 6\Theta)/144\sqrt{2}$
111	1̄10	$K_2(\sin 6\Theta)/18$
111	1̄1̄2	$K_2(-\sin 6\Theta)/18$
211	011̄	$K_1(2\sin\Theta - 7\sin 4\Theta)/24 + K_2(-13\sin 2\Theta + 20\sin 4\Theta - 25\sin 6\Theta)/576$
211	1̄1̄1̄	$K_1(-2\sin 2\Theta - 7\sin 4\Theta)/24 + K_2(13\sin 2\Theta + 20\sin 4\Theta + 25\sin 6\Theta)/576$

Equation (12), along with the relationship $M = M_s \cos(\Theta_o - \Theta)$ and Table I can be used to evaluate K_1 and K_2 from the magnetization curve of a magnetic crystal if the direction and plane of the applied field are known. This is accomplished through the use of window frame crystals cut along various specified axes, including the direction of easy magnetization. For example, if the frames are cut along the [100], [110], and [111] directions in the (001), (001), and (110) planes respectively, the magnetization curve will be different for each case. Then K_1 and K_2 are adjusted to give the best fit between the theoretical curves obtained from Equation (12), the appropriate value of $dE_k/d\Theta$ in Table II, and the experimental data.

3. Area Measurement Between Magnetization Curves

A third method of measuring the anisotropy constants arises through the use of Equation (2). Since $A_{hkl} = \int_0^{I_1} H dI$ is a measure of the area to the

left of the magnetization curve, $A_{110} - A_{100} = \dfrac{K_1}{4}$ can be obtained from the area between the magnetization curves in these directions. Similarly, the difference in area between the magnetization curves in the A_{111} and A_{100} directions, $A_{111} - A_{100} = \dfrac{K_1}{3} + \dfrac{K_2}{27}$, enables one to determine K_2 as well.

Williams[29] measured the anisotropy constants of 3.85 percent silicon-iron using all three methods described above. His results are shown in Table III.

<div align="center">Table III</div>

Method	$K_1 (erg/cm^3)$	$K_2 (erg/cm^3)$
Area Measurement Between Magnetization Curves	272,000	150,000
Fitting Theoretical Magnetization Curves to the Experimental Curves	280,000	100,000
Torque Measurements	287,000	100,000

Signed _____
Norman Menyuk

Approved _____
David R. Brown

NM/jk

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