Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, LIV

To: Group 63 Staff

From: A.L. Loeb, J.B. Goodenough, N. Menyuk, P.K. Baltzer

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An invited paper was presented at the 1953 Spring meeting of The American Physical Society by C. L. Hogan. This paper is discussed in this report.

The electrical resistivity of ferrites is 10^6 to 10^{13} times as great as that of iron. Thus the eddy current effect is tremendously reduced by the use of ferrites. Since eddy current effects delay any magnetic switching mechanism, the use of ferrites is highly desirable.

The permeability of ferrites varies from 100 to 400. The permeability displays a resonance effect as the frequency is varied and this was believed to occur when the frequency of the applied field was equal to the precession frequencies of the electrons. However, the behavior of the ferrite ferroxcube III was not compatible with this explanation and led Brockman to predict, and later to find, a high dielectric constant within the material. The dielectric constant was found to vary from 4000 to 0.5×10^6 through the frequency range considered, and exhibited resonance.

As a result of this high dielectric behavior, displacement currents as well as ohmic currents will flow in a ferrite core. This displacement current will then give rise to additional eddy current effects.

Consider a cylindrical ferrite specimen to which an alternating field $\vec{H}_a$ is applied, as shown in figure 124.

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* Hogan, C.L., "Electromagnetic Phenomona in Ferrites."

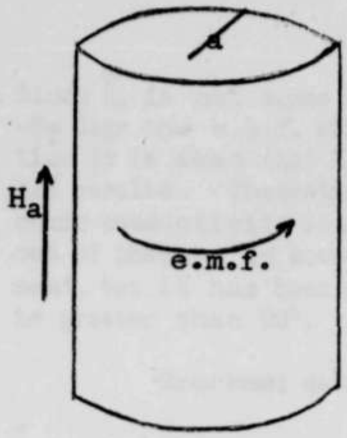


FIGURE 124

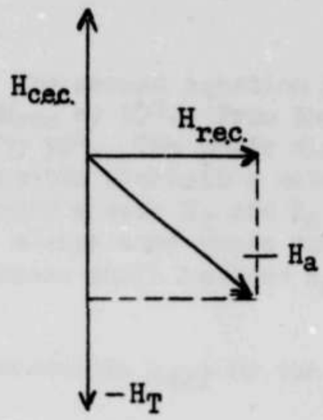


FIGURE 125

This alternating field will induce an electromotive force given by

$$\text{e.m.f.} = -\int_s \frac{\partial \vec{B_T}}{\partial t} \cdot d\vec{S}$$

where $\vec{B_T} = \mu\vec{H_T}$ is the total flux density. The total field $\vec{H_T}$ is composed of contributions from the applied field $\vec{H_a}$ and the field resulting from the eddy currents. Since there are two types of eddy currents induced in the material, an ohmic and a displacement current, the field resulting from the eddy currents is composed of an ohmic (resistance) eddy current contribution $H_{r.e.c.}$ and a displacement (capacitative) eddy current contribution $H_{c.e.c.}$. Thus

$$\vec{H_T} = \vec{H_a} + \vec{H}_{r.e.c.} + \vec{H}_{c.e.c.} \qquad \text{LIV-1}$$

The phase relationship between these components follows from the two Maxwell equations (M.K.S. units) when the electric and magnetic fields are expressible in the form $\vec{H} = He^{iwt}$.

$$\nabla x\vec{H_T} = \nabla x\vec{H}_{r.e.c.} + \nabla x\vec{H}_{c.e.c.} + \nabla x\vec{H_a} = \sigma\vec{E} + \frac{\partial\vec{D}}{\partial t} + 0$$

$$\text{since } \nabla x\vec{H_a} = 0 \qquad \text{LIV-2}$$

$$\nabla x\vec{E} = -\frac{\partial\vec{B_T}}{\partial t}$$

Since $\vec{H}_a$ is not space dependent, $\nabla \times \vec{H}_a = 0$. The second equation indicates that $-H_T$ lags the e.m.f. which is in phase with H_{rec} by 90^o. From the first equation it is seen that H_{cec} leads the e.m.f. by 90^o. The phase diagram of figure 125 results. Theoretically it should be possible to obtain a material with zero ohmic conductivity in which $H_{cec} > H_T$. In such a case H_T and H_a would be 180^o out of phase. In actual practice there are always some ohmic eddy currents present, but it has been possible to obtain a phase shift between H_a and H_T which is greater than 90^o.

Brockman defined an effective permeability μ_{eff} by the equation:

$$\vec{B}_T = \mu_{eff} \vec{H}_a \qquad \text{LIV-3}$$

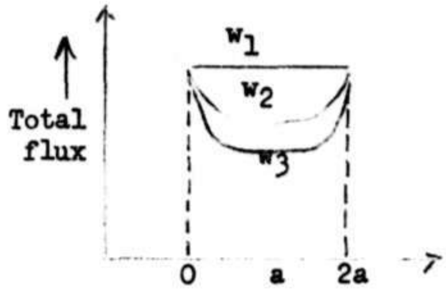
He then found an exact solution for μ_{eff} by imposing the boundary conditions $\vec{H}_a = \vec{H}_T$ at the surface of the cylinder. $\vec{H}_T$ then decreases as one goes into the material. If $\underline{a}$ is the radius of the cylinder he obtained for an average across the speciman ,

$$\mu_{eff} = \frac{2\mu J_1 \,(a\omega\sqrt{\mu\varepsilon})}{a w \sqrt{\mu\varepsilon}\, J_o(a\omega\sqrt{\mu\varepsilon})}\,, \qquad \text{LIV-4}$$

when J_o and J_1 are Bessel functions of the zeroth and first orders, respectively. Thus μ_{eff} will have a resonance at those values of $\underline{a}$ for which

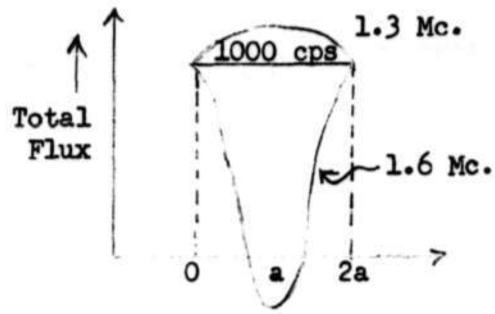
$$J_o(aw\sqrt{\mu\varepsilon}) = 0.$$

Brockman found a variation of the flux density through the cylinder at different frequencies as shown in figures 126 and 127. These curves can best be understood from a consideration of the phase diagram. If H_a is fixed, the phase relations will be as shown in figure 128. For metals H_{cec} is negligibly small. The field induced by eddy currents increases as one goes toward the axis of the cylindrical specimen. If H_{rec} increases, $(H_T - H_{cec}) \approx H_T$ decreases as the axis of the cylinder is approached. Because most of the eddy current is induced near the surface, the largest rate of change in H_T will be near the surface of the specimen. In ferroxcube III, however, H_{cec} is large. If $H_T > H_{cec}$, H_a and H_T will be less than 90^o out of phase and the component of H_T parallel to H_a will be positive. If H_{cec} increases more rapidly than the phase component of $(H_T - H_{cec})$ which is parallel to H_a decreases with increasing H_{rec}, as is the case in ferroxcube III at 1.3mc, H_T will increase toward the axis of the cylindrical specimen. As $H_{rec} \rightarrow H_a$, the phase component of $(H_T - H_{cec})$ parallel to H_a decreases more rapidly than H_{cec} increases so that H_T decreases. $H_{rec} = H_a$ when $H_T = H_{cec}$. If $H_{cec} > H_T$, there is more



Distance through Cylinder

FIGURE 126
Metal Cylinder (ω_1 ω_2 ω_3)



Distance through Cylinder

FIGURE 127
Ferroxcube III Cylinder

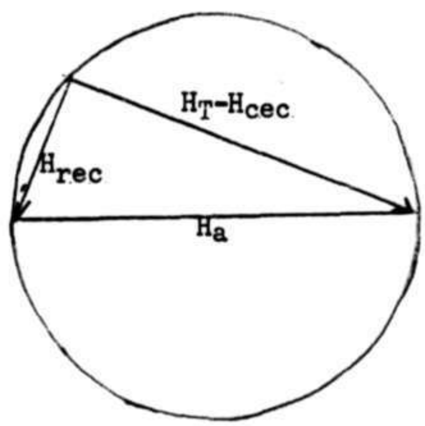


FIGURE 128

than a 90° phase shift between H_T and H_a so that H_T is negative when H_a is positive. This can result in a negative total flux at the axis of the cylinder compared to the flux at the surface of the cylinder. This latter case occurs in ferroxcube III at the resonance frequency of 1.6 Mc.

Hogan, at this meeting, added nothing new physically. He presented, however, a simple approach with the hope of obtaining a better physical intuition for the mechanisms involved in Brockman's mathematics. His phase diagrams help one to obtain an understanding of the curves of figures 126 and 127. From the phase relationships it is a natural step to go to an equivalent circuit for an explanation of the resonance phenomena observed in μ and $\mathcal{E}$. Since an equivalent circuit deals with average values, the flux density is assumed constant throughout. This is the simplifying assumption which introduces error into his analysis.

He first rewrites Maxwell's equations to define a magnetic current $\vec{I}_m$ in analogy with the customary electric current $\vec{I}_T$. Thus he defines

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \equiv \vec{J}_T$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \equiv \vec{J}_m$$

$$\vec{I}_m = \int_s \vec{J}_m \cdot d\vec{S} = -\frac{\partial \Phi_T}{\partial t} \qquad \text{LIV-5}$$

Similarly a magnetic impedance is defined by analogy with the electric impedance by the relations

$$\text{e.m.f.} = ZI_T$$

$$\text{m.m.f.} = Z_m I_m \qquad \text{LIV-6}$$

An equivalent circuit is then set up wherein the phase diagram is used for the assignment of equivalent electrical components for the respective m.m.f.'s. This is shown in figure 129 wherein the components exhibit the values assigned by Hogan. A is the cross-sectional area of the cylinder and σ its conductivity.

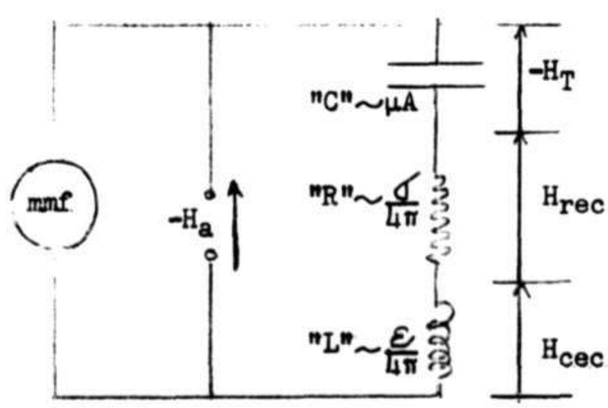


FIGURE 129

To calculate these components, one proceeds as follows. Electrically the voltage across a capacitor is $V_c = \frac{1}{C}\int Idt$. In the magnetic analogy,

$$-H_T = \frac{1}{\text{"C"}}\int I_m dt = -\frac{\phi_T}{\text{"C"}} = -\frac{\mu A}{\text{"C"}}H_T.$$

Therefore "C" = μA. Also from equation LIV-2 there follows

$$\nabla x \vec{H}_{rec} = \sigma \vec{E} \qquad \text{LIV-7}$$

$$\nabla x \vec{H}_{cec} = \frac{\partial \vec{D}}{\partial t} \qquad \text{LIV-8}$$

Whereas, if everything is expressed in cartesian coordinates, there follows from equation LIV-5

$$\nabla x \vec{I}_m = A \nabla x \vec{J}_m = A \nabla x \nabla x \vec{E} = A\left[\nabla\nabla\cdot\vec{E} - \nabla^2\vec{E}\right]$$

But $\nabla\cdot\vec{E} = 4\pi\rho/\varepsilon = 0$ and $\nabla^2\vec{E} = \sigma\mu\frac{\partial\vec{E}}{\partial t} + \varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} \cong -\varepsilon\mu\omega^2\vec{E}$ for the ferrites at

high frequencies wherein $\varepsilon\omega/\sigma$ is small. It is assumed here that the fundamental frequency component predominates. Therefore

$$\nabla x \vec{I}_m = \varepsilon\mu\omega^2 A\vec{E}. \qquad \text{LIV-9}$$

Electrically the voltages across a resistance and an inductance are, respectively, $V_R = IR$ and $V_L = L\frac{dI}{dt}$. In the magnetic analogy, therefore,

$$\vec{H}_{rec} = \vec{I}_m \text{"R"}$$

$$\vec{H}_{cec} = \text{"L"}\frac{dI_m}{dt}$$

If the curl of each equation is taken, there follows immediately from equations LIV-7, 8, and 9,

$$\sigma \vec{E} = "R" \varepsilon \mu \omega^2 A \vec{E}$$

$$\varepsilon \frac{d\vec{E}}{dt} = "L" \varepsilon \mu \omega^2 A \frac{d\vec{E}}{dt}$$

The condition for resonance in an LC circuit is $\omega L = 1/\omega C$. Since this condition is satisfied by the values of "L" and "C" above, the frequency may be taken as the resonance frequency ω_o and "R" = σ/Ω_o, "L" = ε/Ω_o when $\Omega_o = \varepsilon\mu\omega_o{}^2 A$. This agrees with Hogan's parameters only if $\Omega_o = 4\pi$. According to Brockman's solution, equation LIV-4, the resonance frequency ω_o occurs when $J_o(a\omega\sqrt{\mu\varepsilon}) = 0$. Here $\underline{a}$ is taken as the radius of the cylinder of cross-sectional area $A = \pi a^2$. The first root of $J_o(a)$ is $x = 2.4 \approx 2$. Therefore the resonance frequency is given approximately by

$$\omega_o^2 = 4/\mu\varepsilon a^2 = 4\pi/\mu\varepsilon A \qquad \text{LIV-10}$$

which is the necessary relation for $\Omega_o = 4\pi$.

It is now possible for Hogan to calculate a μ_{eff} from the equivalent circuit since

$$\frac{-\vec{H}_a}{Z_m} = \vec{I}_m = -A \frac{d\vec{B}_T}{dt}$$

and $$\vec{B}_T = \mu_{eff}\vec{H}_a = \left| \frac{1}{i\omega_o\, iZ_m} \right| \vec{H}_a.$$

Since $$"Z"_m = \left\{ \frac{\sigma}{4\pi} + i\omega_o \left(\frac{\varepsilon}{4\pi} - \frac{1}{\omega_o^2 \mu A} \right) \right\} \approx \sigma/4\pi,$$

$$\mu_{eff} = \frac{2\sqrt{\mu\varepsilon}}{a\sigma} \qquad \text{LIV-11}$$

Finally the equivalent circuit analysis was extended to calculate the switching time in a ferrite core. This analysis does not concern itself with the actual mechanism by which a core is switched, ie. the creation and motion of domain walls. If, a memory core is switched by a square wave input H_a, the time dependence of the voltage out from the core can be calculated by determining the time dependence of $I_m = \partial\phi_T/\partial t$. The **Laplace** transform for the differential equation of the equivalent circuit is

$$"L"pg(p) + "R"g(p) + \frac{g(p)}{p"C"} = H_a$$

$$g(p) = \frac{H_a}{"L"} \frac{p}{(p+\alpha)^2 - \omega_d^2}, \quad \alpha = \frac{"R"}{2"L"}, \quad \omega_d^2 = \left(\frac{"R"}{2L}\right)^2 - \frac{1}{"L""C"} .$$

Therefore

$$I_m(t) = \frac{H_a}{"L"} e^{-\alpha t} \frac{\sinh \omega_d t}{\omega_d}$$

where the case of critical damping has been taken. for small values of t this becomes

$$I_m(t) \simeq \frac{H_a}{"L"} e^{-\alpha t} \cdot t$$

which has an extremum when $t = 1/\alpha = 2"L"/"R" = 2\,\varepsilon/\delta$. Although the components "R" and "L" are frequency dependent, nevertheless $2\varepsilon/\delta$ is not explicitly so. This, however, is not sufficient justification for applying a square wave pulse to an equivalent circuit which requires linear components and a single frequency component. The parameters of the equivalent circuit are all frequency dependent. However, if the values of ε and δ at resonance for ferroxcube III are used, the maximum in the voltage output of a switching memory core occurs at $t = 2\varepsilon/\delta = 1.75\mu sec$. The curves for this are shown in figure 130.

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Approved David R. Brown

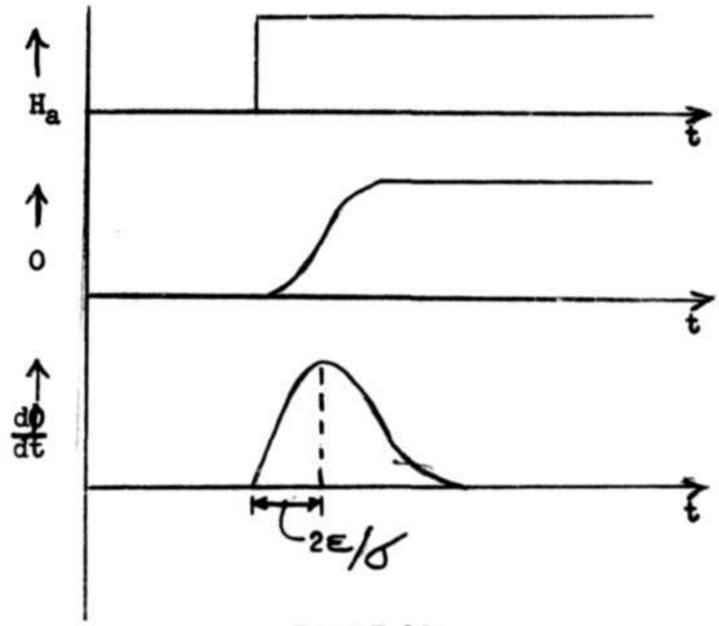

FIGURE 130