Digital Computer Laboratory
Massachusetts Institute of Technology
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SUBJECT: <u>GROUP 63 SEMINAR ON MAGNETISM, APPENDIX I</u>

To: Group 63

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Eddy currents are set up in conducting media when there is a changing magnetic field, and are so directed as to counteract a change in the field producing them.

To show this, consider a varying field $\vec{B}$.

From Maxwell's equation

$$\frac{\partial \vec{B}}{\partial t} = - c(\vec{\nabla} \times \vec{E})$$

Hence a time variation of $\vec{B}$ induces an electric field and current will flow since

$$\vec{E} = \frac{\vec{i}}{\gamma}$$

Further, since $\vec{\nabla} \times \vec{H} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t} + \frac{4\pi \vec{i}}{c}$

and the first term on the right hand side of the equation is negligible for a conducting material,

$$\vec{\nabla} \times \vec{H} \approx \frac{4\pi \vec{i}}{c} \qquad (1 - 1)$$

We see that the magnetic field has its curl in the direction of current flow. If we assume μ = constant, this means that B will diminish at increasing depths within the material. We will show this for the special configuration shown in Figure I.

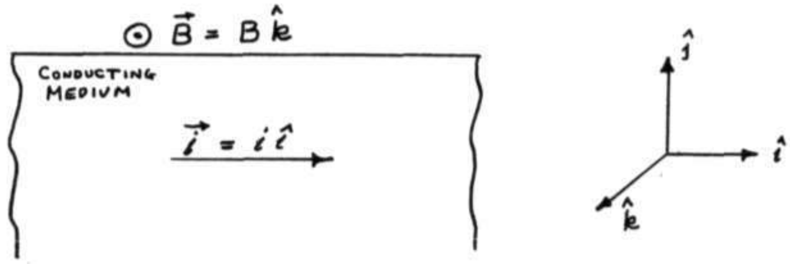

Figure I

By definition; and defining $\hat{k}$ parallel to $\vec{H}$

$$\vec{\nabla} \times H = \left[\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right] \times H \hat{k}$$

$$= \hat{i} \frac{\partial H}{\partial y} - \hat{j} \frac{\partial H}{\partial x}$$

And from equation 1 - 1

$$\vec{\nabla} \times \vec{H} = \frac{4\pi \vec{i}}{c} = \frac{4\pi i}{c} \hat{i}$$

Therefore,

$$\frac{\partial H}{\partial x} = 0$$

$$\frac{\partial H}{\partial y} = \frac{4\pi i}{c}$$

Since $\vec{\nabla} \times \vec{H}$ is in the same direction as $\vec{i}$, $\frac{\partial H}{\partial y}$ will have the same sign convention as $\vec{i}$, and the magnetic field will be smaller as we penetrate further into the material. For a continually changing field, the magnetic field strength in the interior may only be a small fraction of the surface field strength. The variation of $\vec{H}$ with depth is shown pictorially in Figure II.

If the magnetic field stops varying with time ($\frac{\partial \vec{B}}{\partial t} \rightarrow 0$), then the induced electric field and current will quickly go to zero, and and $\vec{\nabla} \times \vec{H} \rightarrow 0$.

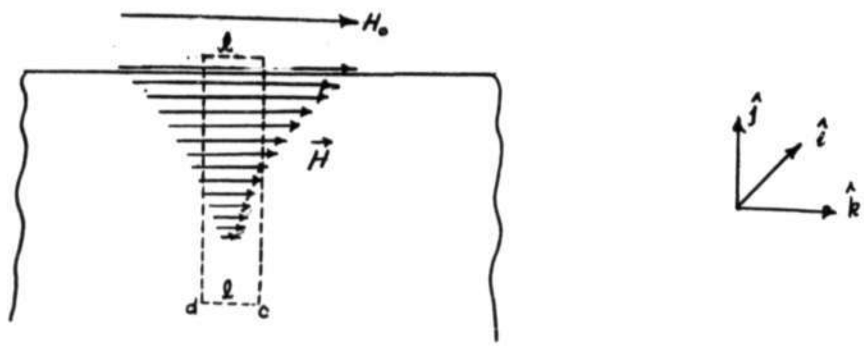

Figure II

In addition to reducing the magnetic field within the material, eddy currents give rise to a considerable power loss. It is, therefore, desirable to decrease its magnitude as much as possible. With the help of Figure II we see how this can be done.

We know

$$\int_s (\vec{\nabla} \times \vec{H}) \cdot ds = \oint \vec{H} \cdot \vec{dl} = \int \frac{4\pi i}{c} \, ds = \frac{4\pi I}{c}$$

where I = current

From Figure II, let us consider the line integral of $\vec{H} \cdot \vec{dl}$ taken around the path abcda. Then for line ab we have the contribution Hl. Since bc and ad are perpendicular to H, they contribute nothing. If cd is sufficiently far inside the material $H_{cd} \approx 0$, and therefore

$$\oint_{abcda} \vec{H} \cdot \vec{dl} = Hl = \frac{4\pi I}{c}$$

Therefore, reducing l would cause a proportional reduction in current. This is accomplished by laminating the material, with laminations electrically insulated from each other.

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