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SUBJECT: MAGNETIC-CORE MEMORY MATRIX ANALYSIS (EFFECT OF DRIVER IMPEDANCE)

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Abstract: The following analysis of a magnetic-core memory shows the extent to which the quiescent internal impedance of the row and column drivers affects the selection ratio.

Consider a square N by N matrix of identical magnetic cores (Fig. 1a). Having selected a row and a column, the matrix can be redrawn (Fig. 1b) as it is seen looking between the selected row and the selected column. One core, the selected core, links the selected row and column. From the selected row one core links each of the (N-1) unused columns and from the selected column one core links each of the (N-1) unused rows. In addition, each of the (N-1) unused rows is linked to each of the (N-1) unused columns (dashed lines).

Whatever the currents in the unused rows, they will, by symmetry, be identical. We can therefore connect them in series for this analysis (Fig. 1c). Similerly, we can connect the unused columns in series.

The final step (Fig. 1d) is to slide together cores which link the same pair of wires giving a composite reduction of the matrix into four cores; core A is the selected core, cores B are the cores lying along the selected row and column but not at the intersection, and C represents all other cores in the matrix. If core A has a cross-sectiohal area A, cores B each have area (N-1)A and core C has area $(N-1)^2A$.

$$1 + 2(N-1) + (N-1)^2 = N^2$$

let:

H_A = magnetic field acting on core A
H_B = magnetic field acting on cores B
H_C = magnetic field acting on core C
i_1 = current in selected row and column
i_2 = current induced in unused rows and columns
ℓ = mean magnetic path (circumference) of all cores
Single-turn coupling throughout

From Fig. 1d,

$$H_A = \frac{2i_1}{\ell}$$

$$H_B = \frac{i_1 - i_2}{\ell}$$

$$H_C = \frac{2i_2}{\ell}$$

We will now consider two driving condition extremes; the first is one in which the quiescent internal impedance of the row and column drivers is infinite and the second is one in which the quiescent internal impedance of the row and column drivers is zero. We will define the selection ratio as

$$H_A : H_B \text{ if } H_B > H_C$$

$$H_A : H_C \text{ if } H_B < H_C$$

Infinite Quiescent Internal Impedance Row and Column Drivers

In this condition, no induced currents are allowed to flow in the unused rows and columns.

Therefore: $i_2 = 0$

$$H_A = \frac{2i_1}{\ell}$$

$$H_B = \frac{i_1}{\ell}$$

$$H_C = 0$$

In this driving condition, the selection ratio, $H_A:H_B$, is 2:1 regardless of matrix size.

Zero Quiescent Internal Impedance Row and Column Drivers

The current induced in the unused rows and columns, i_2, is due to a voltage produced by flux changes in cores B. This current is limited only by a voltage of opposite polarity produced by flux changes in core C. Since the loop is assumed to have zero impedance, these voltages, and therefore the changes in flux, must be equal.

$$\frac{d\phi_B}{dt} = \frac{d\phi_C}{dt}$$

but:

$$\frac{d\phi_B}{dt} = \mu_B (N-1)A \frac{dH_B}{dt}$$

$$\frac{d\phi_C}{dt} = \mu_C (N-1)^2 A \frac{dH_C}{dt}$$

Assuming $\mu_B = \mu_C$, and equating these two, we have:

$$\frac{dH_B}{dt} = (N-1) \frac{dH_C}{dt}$$

Assume that H_B and H_C both start from zero. At any time, T, then:

$$H_B = (N-1) H_C$$

We see that for any size matrix, $H_B > H_C$, and the selection ratio therefore is $H_A : H_B$

From:

$$H_B = (N-1) H_C$$

$$H_B = \frac{i_1 - i_2}{\ell}$$

$$H_C = \frac{2i_2}{\ell}$$

We can write:

$$\frac{i_1 - i_2}{\ell} = (N-1)\left(\frac{2i_2}{\ell}\right)$$

$$i_1 = (2N-1) i_2$$

And:

$$H_B = \left(\frac{2N-2}{2N-1}\right) \frac{i_1}{\ell}$$

$$H_A = \frac{2i_1}{\ell}$$

And the selection ratio, $H_A : H_B$, equals $\frac{2N-1}{N-1} : 1$

We see that the selection ratio in this driving condition is a function of the matrix size. Tabulated below is $\frac{2N-1}{N-1} : 1$ for several values of N:

N	Selection Ratio $\frac{2N-1}{N-1}:1$
2	3.00:1
3	2.50:1
4	2.33:1
5	2.25:1
6	2.20:1
7	2.16:1
8	2.14:1
16	2.06:1

Summary

For drivers with infinite quiescent internal impedance, the selection ratio is always 2:1. For drivers with zero quiescent internal impedance, the selection ratio is always better than 2:1 and it depends upon the matrix size. It can be as high as 3:1 for a 2 by 2 matrix but it drops off rapidly as the matrix size increases. For a 16 by 16 matrix, the selection ratio varies but 3% as the quiescent internal impedance of the drivers is varied over the entire range from zero to infinity.

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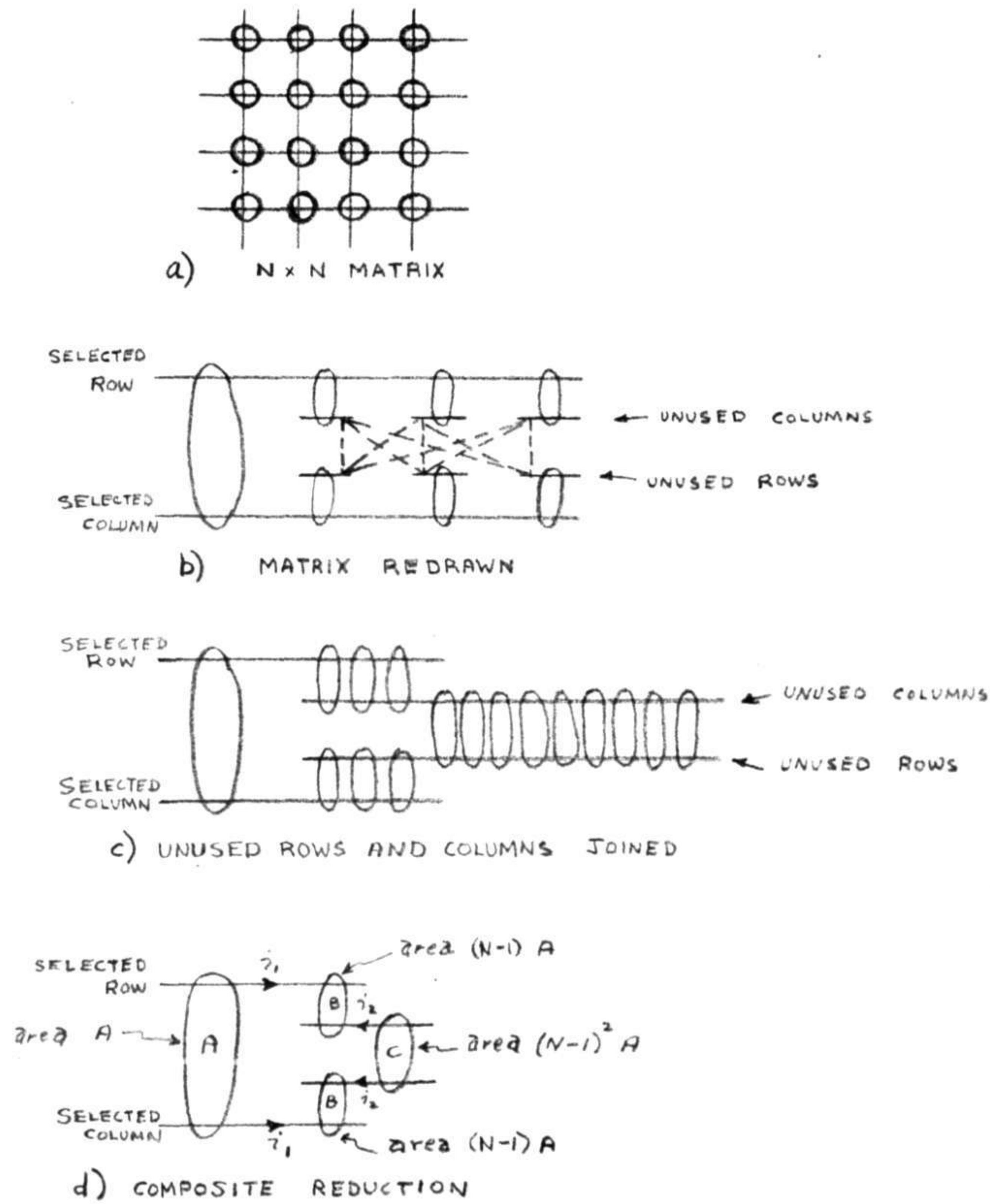

FIG. 1. MAGNETIC MATRIX REDUCTION