Wiener

p. 26. $2_\times$ & 2_+ are not Dfs. There is need of a notation for "the". What is alleged does not enable you to put "$0 =$ etc. Df". It was a discussion on this very point between Schröder & Peano in 1900 at Paris that first led me to think Peano superior.

p. 27. [Bottom] Ref.ce in Principia is to Huntington, not to Whitehead.

p. 29. "Class" in Schröder is indefinable; in Princ. it is defined. The contradictions give ground for regarding it as not a possible indefinable.

p. 30. "=" is not primitive in Princ. cf. ∗13·01.

p. 37. The notion of "elementary" props is necessitated by the "Liar" & kindred paradoxes. How does Schröder's system avoid these?

p. 39. $2_\times$ & 2_+ subject to same defect as on p. 26. What is meant by the "value" of a false prop? This notion of "value" will have to be a prim. idea, won't it?

pp. 48–9. Schröder's remarks might be rendered thus: When i is used to stand for an individual, it is assumed to satisfy the hyp "$(\alpha): \exists ! \, i \cap \alpha . \equiv . \sim \exists ! \, i - \alpha$"; thus we may put Indiv $= \hat{i}\{(\alpha): \exists ! \, i \cap \alpha . \equiv . \sim \exists ! \, i - \alpha\}$ Df Then Indiv will be the same as what I call 1.

p. 67. In my notation, $R = \dot{s}`\{\dot{2} \cap Rl`R\}$

p. 77. For $\breve{R}$, refer also to Prin. p. 33, where Schröder is mentioned.

p. 87. I do not think a relation ought to be regarded as a class of couples — I merely suggest this as a view wh. may help the reader to begin with.

p. 85. I don't see how $R/\dot{V}$ can give anything interesting, because it $= R$.

p. 101. You ought not to put assertion-signs before a hyp, like "$z = \iota`x$". Assertion signs only go before true props — "$z = \iota`x$" is not a prop, let alone a true one.

p. 126. Have you any evidence that Schröder knew there was a difference between Peter & the class whose only member is Peter?

p. 127. It is a poor defence of Schröder to say he can't discuss many of the most important forms of props that occur in logic; e.g. Apostles ε Cls.

p. 128. What you say doesn't seem to meet the point. Whether Schröder says $\alpha = 2$ or "α is a couple" doesn't matter. The point is that $\underline{\alpha}$ is a member of a certain class of classes (couples), & that a logic wh. refuses to mention this fact is very inadequate. His way of meeting the difficulty, according to you, is to change the subject.

p. 135. "Perfectly justifiable limitation" – only if you are prepared to omit many of the most important topics.

p. 138. The passage you quote shows that Schröder once felt the need of distinguishing ε & C, which makes it all the worse that he didn't see its importance. I don't believe he knew that $\underline{x}$ & $\iota' x$ are different, & I think this made him hit up against insoluble difficulties in trying to distinguish ε & C.

pp. 139–141. The confusion as to Peano's meaning could never have been made by a man who had thought the matter out as clearly as you suppose.

You deal only with the more conventional parts of <u>Principia Mathematica</u>. I shd. rest its claims mainly upon 3 Dfs, *14.01, *20.01, & *30.01. The rest is mainly working out these three. Can Schröder's methods express these?

On p. 3 You fall into the "fallacy of initial predication":

Red may be immediately given, & green, & also the diffce betw. red & green. It may be that animals see red & see green, & behave & feel differently towards the two, but never see "the difference between red & green" — but there is no logical difficulty in adding this experience to those of red & green.

pp. 4-5. What you say about relational structure of colour. Blind man's experience is quite true. But it is part of a more general fact: communication between people, so far as it is accurate, is in respect of universals, while in regard to particulars identity & converse are not strictly possible.

pp 5-6. How do you know that what can't be communicated is not more "essential" (whatever that means) than what can?

p. 6. Your "(2)" seems to me sheer mistake, depending on assuming that what is immediately given can't also be given in a context.

What you say at the end is a gross non sequitur due to ambiguity in the word "experience". If I see Jones running towards the station, & argue "evidently he is trying to catch the 10 o'clock train", I "experience" his running & my argument, but I evidently do not, in the same sense, "experience" the fact that he wants to catch the train. Thus among the props I think I know it is possible to distinguish some as primitive & some as deduced.

p. 26. It is true that postulate and definition are not clearly distinguished by S. However, there is no need for a notation for "the", as it is easy to prove that 0 & 1 are unique, since if $a \in 0$, whatever a, $0' \in 0$, and, similarly, $0 \in 0'$, so that $0 = 0'$.

p. 29. As Schröder sticks to one type, he isn't involved in the paradoxes. For Schröder a 'class of classes' is not a genuine class at all, or if it be a genuine class, it belongs to a quite different system of ~~inclusion~~ dependent on a different $\in$-relation ~~than~~ from the classes of which it is made up. See his theory of 'reine Mannigfaltigkeit'.

It is only when we stick to one type, in your system, therefore, that his 'Klasse' and your 'class' are strict synonyms.

p. 30. No, but ~~is~~ within Whitehead's postulates, it is primitive.

p. 37. As S's theory of propositions and ~~of propositions~~ is ~~derived from~~ a special case of his theory of classes, and as his universe of classes must be a "reine Mannigfaltigkeit", so must his universe of propositions. That is no proposition in a given universe should be dependent for its existence in that universe (not its validity) upon any other proposition.

pp. 48-9 Unless we follow out A. der L., I, §23, p. 482, Inhär. would have no meaning in Schröder's system.

p. 67, Yes, except that you involve expressions of several types, whereas S. does not.

p. 85. $\vdash : x(R|\dot{V})y . \equiv : \cancel{(\exists z) . \{xRz . z\dot{V}y} \; (\exists z) . xRz . z\dot{V}y : \equiv : (\exists z) . xRz.$

Now, $(\exists z) . xRz$ is not equivalent, in general, to xRy. I believe you are mistaken on this point.

p. 87. It seems to me that what is possible in mathematics is legitimate.

p. 126. "Peter" for Schröder means 'the class whose sole member is Peter. S. is simply not concerned with your Peter

p. 127. S. can and does discuss such propositions (see I, §23, p. 482), but to do so, he must confine the formulae which he uses to the "n-te abgeleitete Mannigfaltigkeit." See I, §9.

p. 128. It is unjust to Schröder to insist that his 'Klasse' should be precisely equivalent to your 'class'. Num. a = 2 is equivalent in meaning (or nearly so) to $\alpha \varepsilon 2$; whether it is expressed in the same set form seems to ~~be~~ beside the point. The question is not whether Schröder's concept of "Klasse" enables him to treat these problems, but whether he can treat of them in any consistent way. To appreciate S's work, it seems to me hardly fair to regard it as a sort of misstatement of the Principia; he must be approached by his own methods and through his own symbolism.

p. 135. But Schröder is able to make up the deficiency in other ways. The only things in the P. which cannot be expressed in his symbolism are the technicalities peculiar to the development of its particular notation.

P. 138. Quite true, — for S. x and ι'x are not distinguished — because he has, in general, nothing to do with x. ~~When~~ However, he can express, in his own symbolism, everything you say about x, by a statement about i or j — the equivalent of your ι'x. On the other hand, if one follows out the suggestion of I, §23, p. 482, and constructs a hierarchy of systems of classes, each system ~~bein~~ having as individuals the classes of the preceding ~~type~~ system, the ∈-relation, and the operations and relations derived therefrom, having a separate symbol for each type, no ambiguity will arise if we use but one symbol for x and ιx, for the type for which any formula is stated will be clear on the face of it from the symbols for the relations and operations involved!

As for *14.01, *20.01, *30.01, see pp. 20-21 of my thesis. Besides, although I have not worked this question out sufficiently, I have strong reason to believe that S's "fünfziffrige Rechnen" amounts to an anticipation of the very ideas involved in *14.01, *30.01.

p.26. It is true that postulate and definition are not clearly distinguished by S. However, there is no need for a notation for 'the', as it is easy to prove that 0 & 1 are unique, since, if $a \in 0$, whatever a, $0' \in 0$, and, similarly, $0 \in 0'$, so that $0' = 0$

p.29. As S. deals with only one type at a time, he isn't involved in the paradoxes. For S. a 'class of classes' belongs to an entirely different universe of discourse (the 'erste abgeleitete Mannigfaltigkeit') from that to which ~~his~~ the classes of which it is composed belong (the 'ursprüngliche Mannigfaltigkeit). See S's theory of 'reine Mannigfaltigkeit', vol I, §9. It is only, therefore, when one sticks to one of your types, that his 'Klasse' and your 'class' are strict synonyms.

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