Report 3644

EXPERIMENTAL VERIFICATION OF THE SHEAR DEFORMATION EFFECT ON THE MOMENT DISTRIBUTION CALCULATION USED IN THE DESIGN OF SHIP COMPONENTS

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20034

EXPERIMENTAL VERIFICATION OF THE SHEAR DEFORMATION EFFECT ON THE MOMENT DISTRIBUTION CALCULATION USED IN THE DESIGN OF SHIP COMPONENTS

by

E.A. Zwenig

Distribution limited to U.S. Government agencies; Test & Eval. Info.; 7 Apr 1971. Other requests for this document must be referred to NSRDC, Code 700.

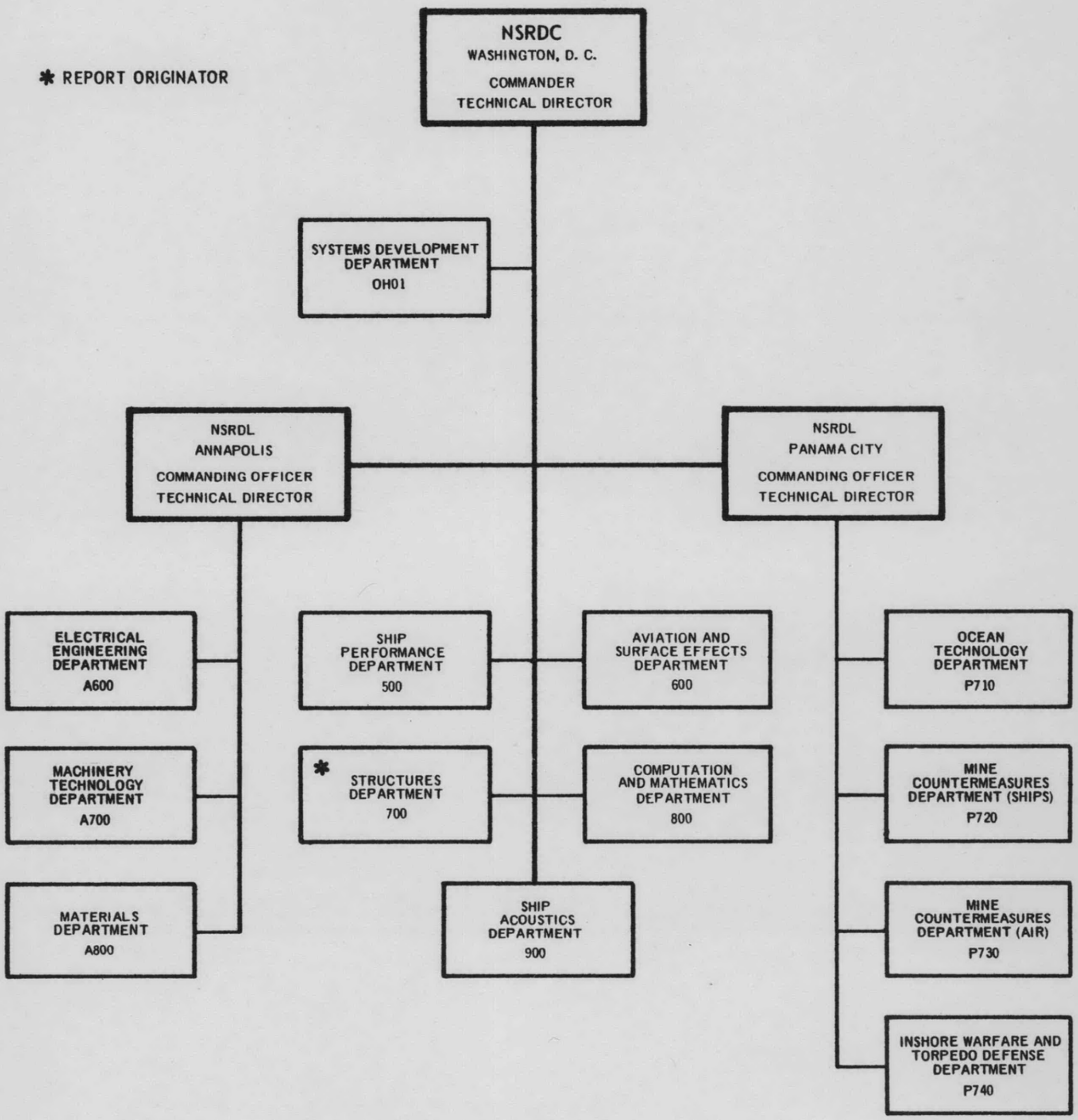
STRUCTURAL MECHANICS DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

May 1971 Report 3644

The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory (now Naval Ship R & D Laboratory) at Annapolis, Maryland. The Mine Defense Laboratory (now Naval Ship R & D Laboratory) Panama City, Florida became part of the Center in November 1967.

Naval Ship Research and Development Center
Washington, D. C. 20034

MAJOR NSRDC ORGANIZATIONAL COMPONENTS



DEPARTMENT OF THE NAVY
NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
WASHINGTON, D. C. 20034

EXPERIMENTAL VERIFICATION OF THE SHEAR DEFORMATION EFFECT ON THE MOMENT DISTRIBUTION CALCULATION USED IN THE DESIGN OF SHIP COMPONENTS

by

E.A. Zwenig

May 1971

Report 3644

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

A,B,C••••	Locations on beam
A_s	Area considered effective in shear
A_sG	Shearing rigidity of beam
a,b,c	Distances on beam
C.O.	Carryover factor
D	Distribution factor
dA	Differential area
dx,	Differential length in x-direction
∂x	Partial differential of x
E	Young's modulus of elasticity in flexure
EI	Flexural rigidity of beam
FEM	Fixed end moment
G	Modulus of elasticity in shear
h	Beam depth
I	Moment of inertia (a measure of the beam capacity to resist bending, $\int Y^2 \cdot dA$)
j	Dimensionless ratio $3EI/k_sA_sGL^2$ (see Appendix A)
K	A stiffness factor (see Moment Distribution)
k_s	Numerical value given to the effectiveness of the shear area to resist shear loading, this value is not yet free from controversy (see Appendix C); for the purposes of this report $k_s \doteq 1.0$
L	Length
M	Moment
P	External force or load
Q	Static moment $(Y \cdot dA)$
R_A, R_B, R_C	Reaction at Points A, B, and C
t	Thickness
U	Total strain energy - the internal work of a beam; subscript b denotes bending energy and v denotes shear energy
V	Shear force, subscripted V_E refers to Shear Energy
x,y,z	Major axes
δ,Δ	Deflection
ε	Strain
θ	Slope, rotation

ABSTRACT

Beam tests were conducted to determine the degree to which shear distortions affect moment distribution results as used in a practical design. Through simple experimentation it was found that, in selected cases, the shear distortion was important enough to suggest changes in design procedures. Appendixes supplement the theory and problems relative to the test report.

ADMINISTRATIVE INFORMATION

Following a conference of personnel from the Naval Ship Engineering Center (NAVSEC) and the Naval Ship Research and Development Center (NSRDC) on 19 January 1968, the work described herein was planned as NSRDC Proposal 7100, 761:LAB:d11, Serial 760-205 dated 5 February 1968. The program was carried out with NAVSEC-administered OM and N funds.

INTRODUCTION

Moment distribution is one of the methods used to analyze ship structures such as the bents that support the flight deck of modern aircraft carriers. The usual solution considers the energy due to bending but ignores that due to shear distortion. In most cases this solution gave results that were sufficiently accurate for design purposes.

However, as aircraft carrier decks became larger and were subjected to heavier aircraft loads, the size of the supporting bents increased to the point where shear distortion could no longer be ignored if accurate stress distributions were needed. Moment distribution solutions which consider shear distortion are available in the literature, but these have not been verified experimentally.

The objective of this test program was to clearly demonstrate the effect of the shear distortion on the carryover and distribution factors as well as to verify that the fixed-end moment does exist in the elastic region and can be shown in a simple experiment. Since basic structural analysis principles were involved, the tests were developed at the most elementary level, using a wide-flanged beam to verify the shear distortion effects. These experiments were the first to utilize the new NSRDC Free-Form Test Facility.[1]

The results of the test and evaluation program conducted in fiscal year 1969 verified the modified distribution. Details of the formula derivation are given in Appendix A together with analytical illustrations and calculations. This is followed by an error analysis (Appendix B) and a brief survey of the theoretical approach (Appendix C) to provide a fuller understanding of the effects of shear distortion.

TEST PROGRAM AND PROCEDURE

As already stated, the experimental program was designed to demonstrate principles only. Therefore the tests were done on a continuous two-span beam; see Figure 1. This test beam is a 6-in. steel, wide-flange section weighing 25 lb/ft (6 WF 25). The beam was tested in the as-rolled condition. This beam was chosen because of its relatively thick web; this feature was desirable in order to sustain the large shear loads which had to be produced during the test program if the actual carrier bent load simulation was to be realized.

Prior to testing, the beam was instrumented with strain gages as shown in Figure 2. These gages were all on the flanges of the beam and were oriented so as to measure longitudinal bending.

The beam was then set up as shown in Figure 3. Hydraulic jacks were used to support it at the midspan and to load it at the two ends. The load at each of these points was determined by the load cell shown on top of each jack. These three points (P_1, P_2, and B in Figure 1) did not move longitudinally during tests. Also shown in Figures 1 and 3 are two intermediate reactions. These are stands which hold load cells. The position of these stands changed for each test and is listed in Table 1. A further description of the program may be found in Appendix B.

[1]References are listed on page 33.

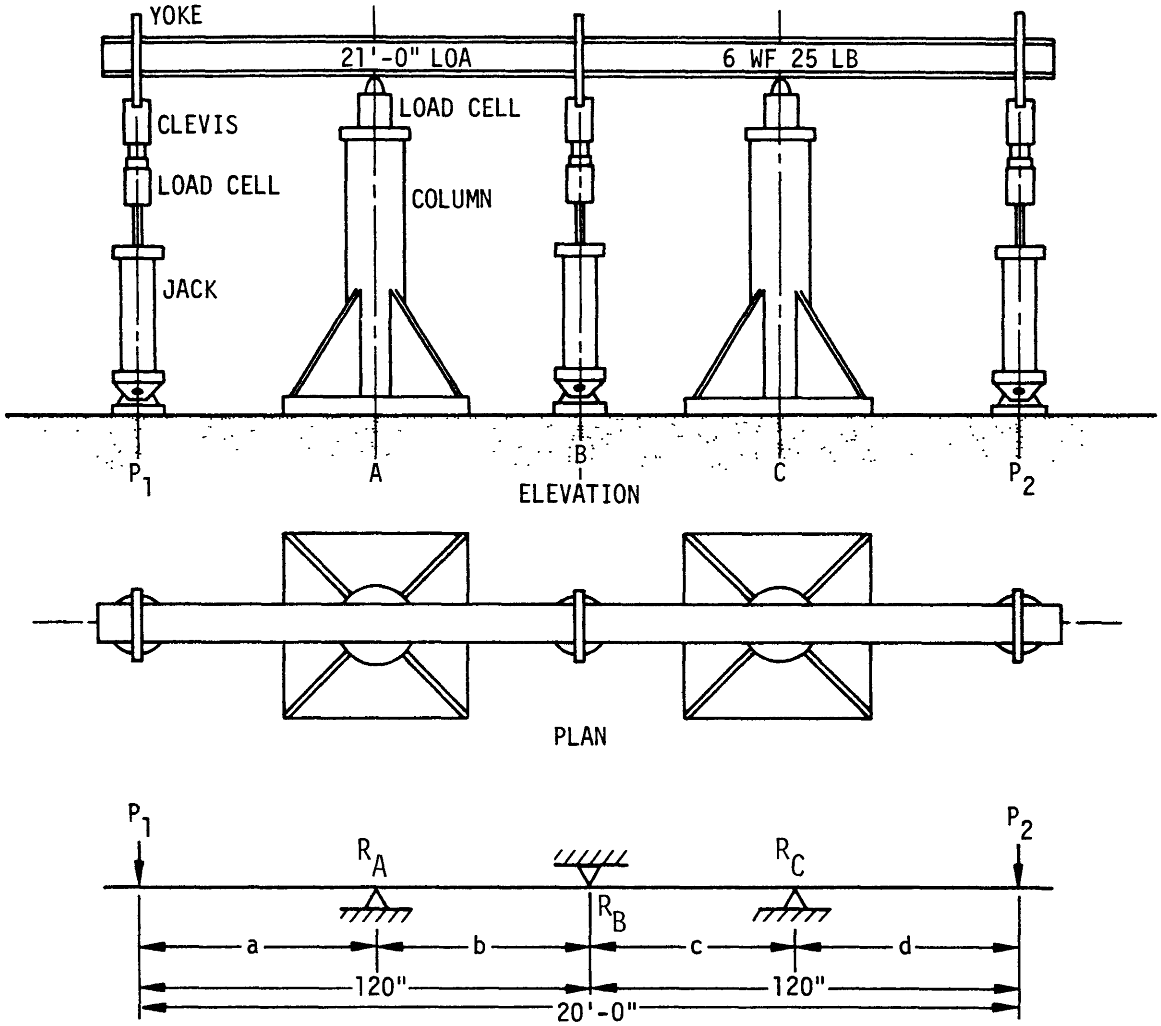


Figure 1 - Test Beam Arrangement

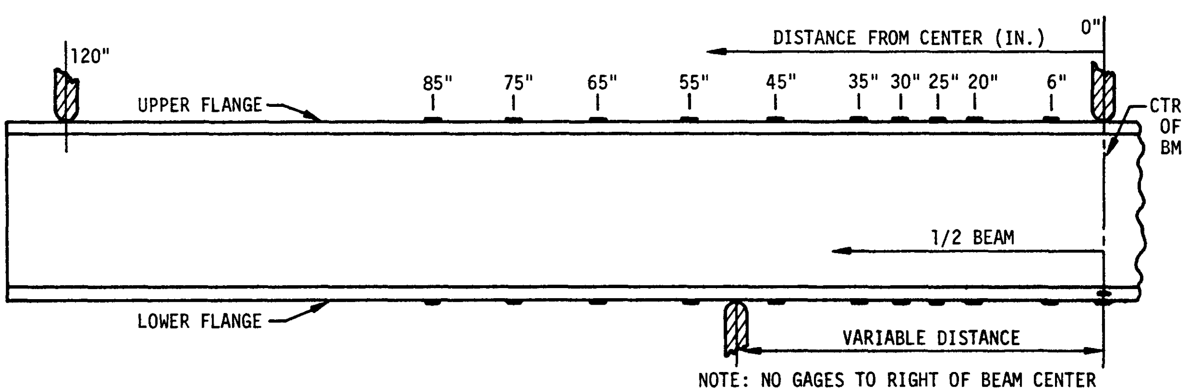


Figure 2 - Strain-Gage Locations

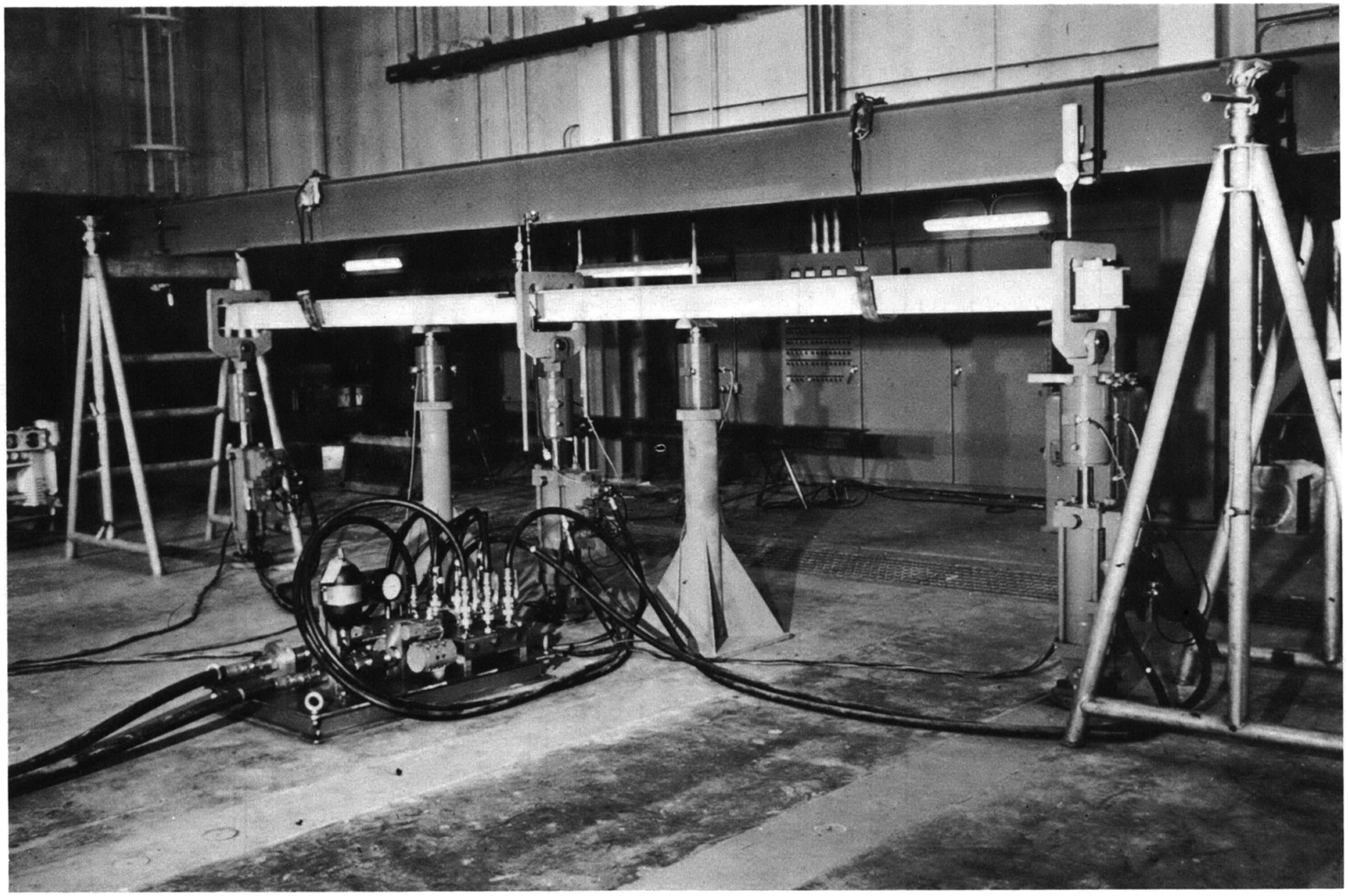


Figure 3 - Test Setup

TABLE 1

Loads and Reactions

(Experimental error in total load was ±3 percent).

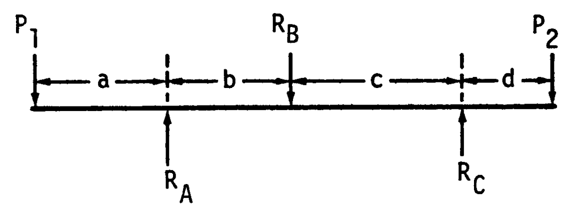
Test Number	Distance in.				Load kip		Reaction kip											
							R_A				R_B				R_C			
	a	b	c	d	P_1	P_2	S_s	S	S_e	%D	S_s	S	S_e	%D	S_s	S	S_e	%D
1	90	30	50	70	2	2	9.600	10.583	9.0	9	11.360	12.933	10.9	12	5.760	6.350	5.8	9
2	90	30	50	70	2	4	10.583	12.040	9.5	12	15.728	18.064	15.3	13	11.148	12.024	11.4	7
3	90	30	50	70	4	2	18.176	19.708	17.0	8	18.282	20.733	17.5	12	6.106	7.025	6.0	13
4	70	50	70	50	2	2	5.768	5.967	5.6	3	5.888	6.229	5.8	5	4.120	4.262	4.2	3
5	70	50	70	50	2	4	6.240	6.550	6.1	5	8.125	8.656	8.1	6	7.885	8.106	7.9	3
6	70	50	70	50	4	2	11.040	11.350	10.8	3	9.500	10.029	9.0	5	4.460	4.679	4.2	5
7	80	40	40	80	2	2	7.320	8.000	6.6	9	10.640	12.000	9.5	11	7.320	8.000	6.6	9
8	80	40	40	80	2	4	7.980	9.000	7.7	11	15.960	18.000	15.5	11	13.980	15.000	13.5	7
9	100	20	20	100	2	2	11.920	17.000	11.3	30	19.840	30.000	19.8	34	11.920	17.000	11.3	30

S_s - Calculated with shear

S - Calculated without shear

S_e - Experimental

$$\%D - \frac{S-S_s}{S} \times 100$$



One test restriction must be mentioned. To prevent the introduction of additional moment, it was specified that the three reaction points must not move in any direction with respect to each during any test. This was accomplished (1) by mounting a linear potentiometer at point B (see Figure 4) and using its output to keep point B from vertical displacement and (2) by making the supports rigid enough to prevent movement in the other directions.

The test procedure was as follows. Loads P_1 and P_2 were applied, causing a slight upward bow in the beam as shown in Figure 5a. The deflection signal in the potentiometer caused the jack at B to reach to bring points A, B, and C into vertical alignment. The reactions at each support as well as the loads were determined from the load cells. Longitudinal strains from each of the strain gages shown in Figure 2 were recorded. The load was then increased to the next increment and the procedure repeated.

The loads listed in Table 1 for each test were applied in four equal increments. The load magnitudes shown in the table were chosen to produce strains high enough to give accurate results without yielding the test beam. Four load increments were chosen to ensure linearity.

TEST RESULTS AND DISCUSSION

The loads and reactions as determined from the load cells are listed in Table 1 for the nine tests together with calculated values with and without the shear distortion effect and the difference between these two analytical values. The moment distribution techniques including and excluding the shear distortion effect can be found in Reference 2 and Appendix A. Finally as a check on experimental accuracy, moments throughout the spans were calculated from the measured loads and test geometries. These moments were compared to those obtained by converting measured strains to stress (stress = strain x elastic modulus) and then calculating the moment (moment = stress ÷ section factor).

Two important cases were examined to determine what differences did exist between the calculations in Table 1 and the strain results. These cases were the nonsymmetrical Test 3 and the symmetrical Test 9. The results may be seen in Figures 6 and 7, respectively. Since the solution is inherent in the gaged portion of the beam, only that half of the beam, i.e., from station at the centerline to P_1 is shown.

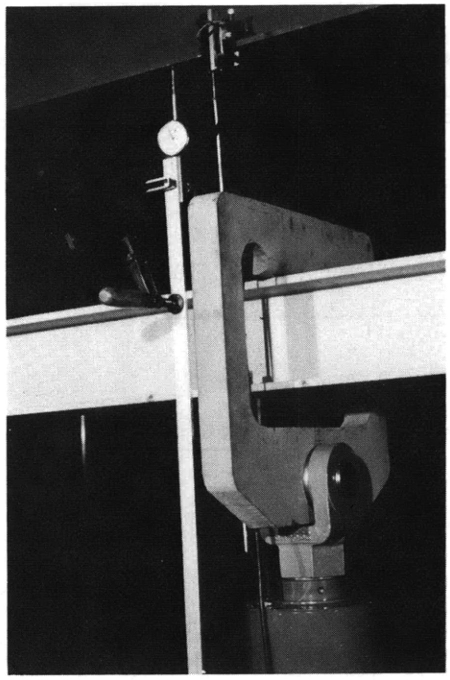


Figure 4 - Load Application and
Linear Potentiometer

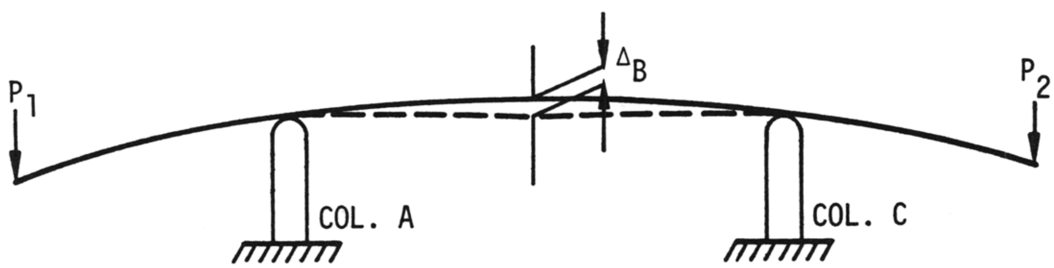


Figure 5a

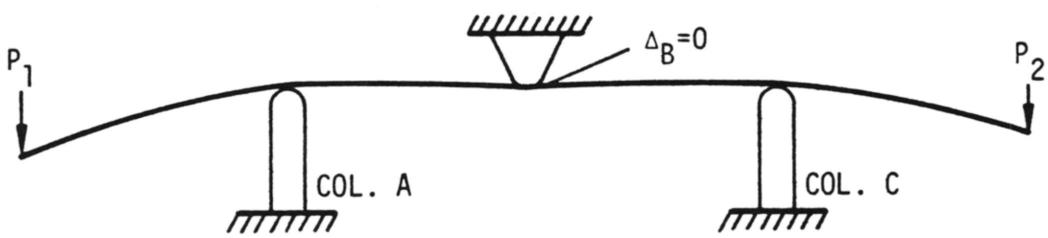


Figure 5b

Figure 5 - Loading Method

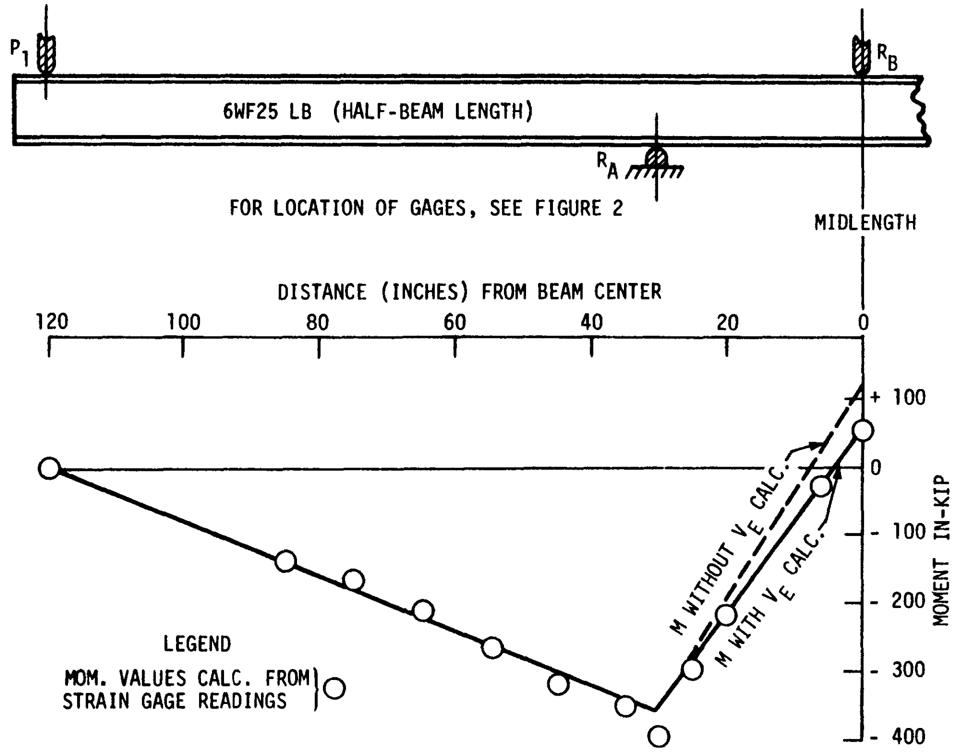


Figure 6 - Comparison of Analytical and Empirical Values for Test 3

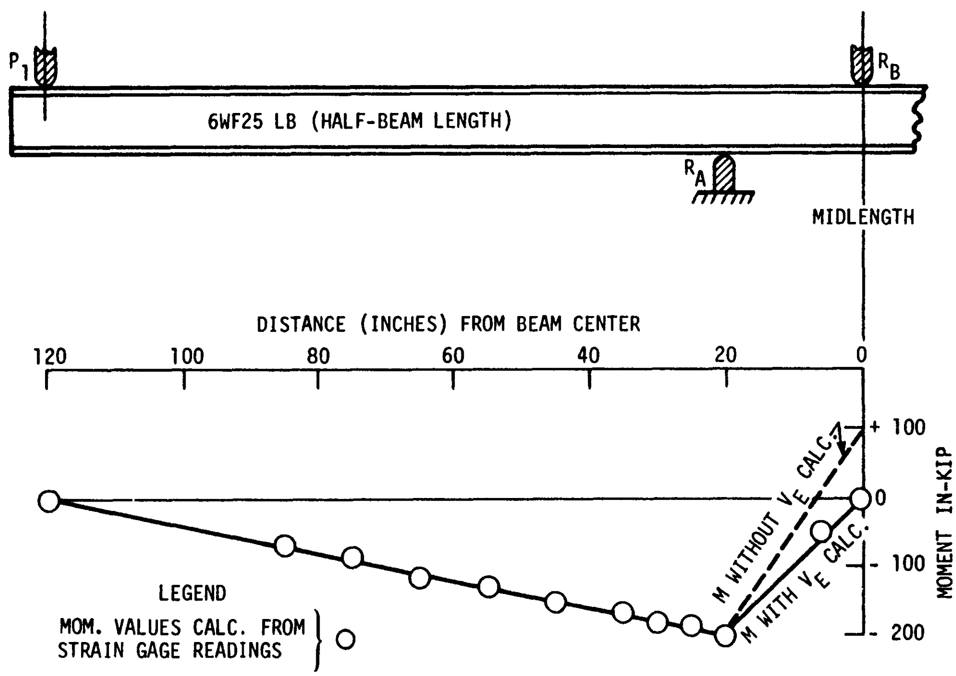


Figure 7 - Comparison of Analytical and Empirical Values for Test 9

As can be seen from Table 1, the experimental results that included the shear distortion effect were closer in every case to the calculations than were the results that excluded that effect. This held true for spans where the difference ranged from 0 to 30 percent of the results without shear distortion. It is also interesting to note that no difference occurred at the spans for which no difference was predicted.

Table 1 shows that in almost every instance the experimental values were less than the analytical values even when shear distortions were included. The experiment was not prepared for this result and did not have instrumentation to record the possible physical phenomenon. A tentative explanation lies in the fact that the estimate of web area to resist the loading, did not include the fillet and a portion of the flange area. This may account for the slightly lower experimental value in most of the cases. Discussion of the approximations in shear stress distributions may be found in Appendix C.

To check experimental results for a specified moment, the case of a 20-in. span length (Test 9), as shown in Figure 7, was examined in more detail. This is a case where the span length was chosen so that the carryover factor would be zero (see Appendix A or Reference 2 for the derivation). To do this, carryover expression 0.5-j was set equal to 0. Since $j = (3\ EI/L^2A_sG)$, then j must equal 0.5 or $0.5 = 3\ EI/L^2A_sG$. Assuming E/G = 2.6 for steel and knowing that the beam is a 6WF25 the required span is $(2 \times 3 \times 2.6 \times 53.5/2.0384)^{1/2}$ or 20.2 in.

Thus, at a distance of approximately 20 in. from the support, one may expect zero moment for Test 9. It is seen from Figure 7 that when shear distortion effects were included (solid line) the moment was zero for the case, including shear distortion effects; for the conventional moment distribution analysis (dotted line), it was 100 in.-kip, significantly different from the experimental results.

CONCLUSION

The results of these tests verify the effect of shear distortion on moment distribution as predicted by theory. It is concluded on the basis of the test results that failure to consider this distortion in the design calculations of short deep beams (span-to-depth ratios of approximately 9:1 or less) results in calculated moments which are different from the actual value by as much as 15 percent. Thus more efficient designs are possible if the more complex moment distribution equations are used.

APPENDIX A

DERIVATION OF EQUATIONS AND SAMPLE CALCULATIONS

The objective of the test program was to verify the shear distortion effects in moment distribution analysis. Moment distribution, a standardized technique used in redundant structural analysis, was introduced primarily by Hardy Cross.[3] Since the shear distortion effect is not ordinarily incorporated in this technique, a detailed derivation was developed to help provide an understanding and logical linkage between the analysis and the experiment.

Deflections due to shearing distortion alter some of the factors used in moment distribution. These factors are functions of the dimensions and geometry of the members as well as of such material properties as the modulii of elasticity and rigidity. Thus, these factors affect the manner in which the distributions of fixed-end moments are made in analysis.

The following derivation accounts for the alteration in the stiffness or restraint of the members. It also includes the relationship between the moment at the fixed end to the moment that produces a rotation (angular displacement) of the beam at the rotating end, termed the carryover factor.

Appendix A first gives the derivations necessary to supplement the design expressions given in Reference 2 and then applies the technique in an example and provides the attendant moment and shear diagrams. The example contrasts the results obtained when the calculations include and exclude the effects of shear distortion. Figures 6 and 7 illustrate the experimental verification of the effect of shear distortion.

The derivation is based on the method of least work (Castigliano's Theorem)[2,3] which may be expressed in the following manner. If an equilibrated force system acts on a structure to produce an internal strain energy U, the partial derivative of U with respect to any force of the loading system gives the displacement in the direction of that force.

Castigliano developed this to analyze redundant structures. There is no relative motion at the supports (unyielding supports) in our application, hence

$$\frac{\partial U}{\partial P} = \Delta = 0$$

As shown in Figure 8, the work of the external force is transformed into the energy of strain in the beam (see page 68 of Reference 2), and since there is no axial loading,

$$U = \int_0^L \frac{\left(M_{AB} - V_A \cdot x\right)^2 \cdot dx}{2\ EI} + \int_0^L \frac{V_A{}^2\ dx}{2\ A_s\ G}$$

The explanation for the statement is that at some distance x from the support at A, the amount of internal work, expressed in terms of strain energy at section X-X, consists of two parts, the work in terms of the bending and the shear energies at that section.

In general, for a fixed beam with a couple M applied at the free end, the rotated angle is

$$\theta = \frac{ML}{EI}$$

and the displacement due to the work performed by M is

$$U = \frac{M\theta}{2}$$

which indicates the total external work from no displacement to the final position. Since the change in M per unit length may be expressed by $M - \frac{dM}{dx}(x)$, which is $M - V \cdot x$, that portion of contribution from flexure of Equation (1) is expressed by

$$U_{Bending} = \frac{\left(M_{AB} - V_A \cdot x\right)\left(M_{AB} - V_A \cdot x\right)}{2\ EI}\ dx$$

$$= \frac{\left(M_{AB} - V_A \cdot x\right)^2 \cdot dx}{2\ EI}$$

The shear contribution to deflection is

$$\Delta_{Shear} = \frac{\theta}{A_s\ G} \cdot \frac{dV}{dx}$$

and the work of the force from zero to V, displaced through the distance dx is

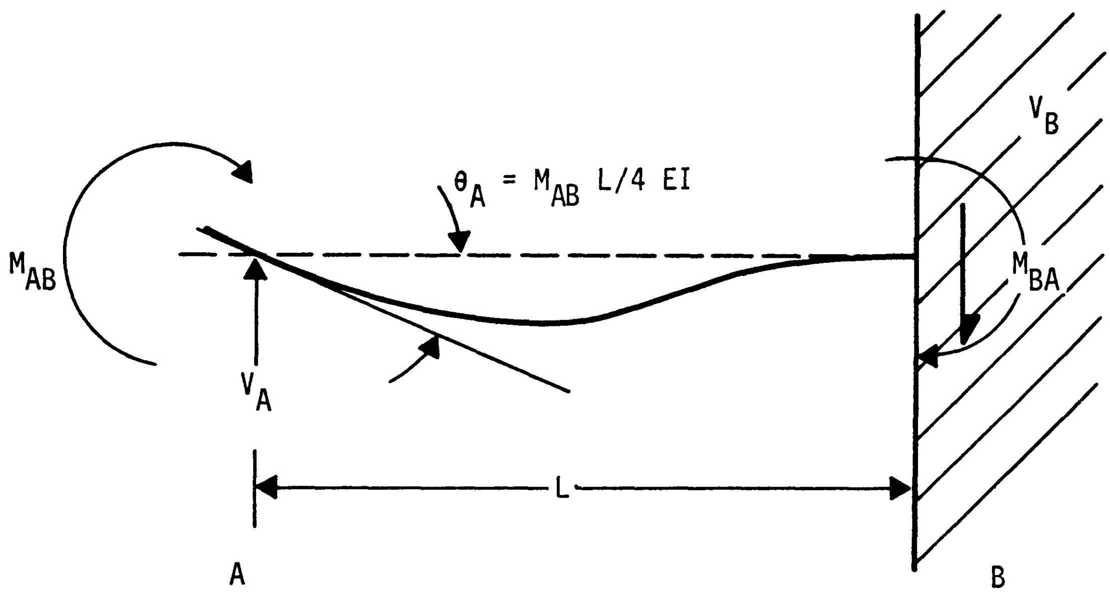


Figure 8 - Fixed-End Conditions

$$U_{Shear} = \frac{V^2 \, dx}{2 \, A_s \, G}$$

Combining the contributions from bending and shear, and applying the least work theorem

$$\Delta_A = \frac{\partial U}{\partial V_A} = \int_0^L \frac{\left(M_{AB} - V_A x\right) \left(0-x\right)^2}{2 \, EI} dx + \int_0^L \frac{V_A \, dx \cdot 2}{2 \, A_s \, G} = 0$$

$$= - \frac{M_{AB} \, x^2}{2 \, EI} + \frac{V_A \, x^3}{3 \, EI} + \frac{V_A \, x}{A_s \, G} \Bigg]_0^L = 0$$

Putting in the limits of integration and dividing by L gives

$$- \frac{M_{AB} \, L}{2 \, EI} + \frac{V_A \, L^2}{3 \, EI} + \frac{V_A}{A_s \, G} = 0$$

or

$$\frac{M_{AB} \, L}{2 \, EI} = V_A \left(\frac{L^2 \, A_s \, G + 3 \, EI}{3 \, EI \cdot A_s \, G} \right)$$

Dividing numerator and denominator by $L^2 \, A_s \, G$, gives

$$V_A = \frac{3 \, M_{AB}}{2L \left(1 + \frac{3 \, EI}{L^2 \, A_s \, G}\right)}$$

If $j = (3 \, EI/L^2 \, A_s \, G)$, then (see page 68 of Reference 2)

$$V_A = \frac{3 \, M_{AB}}{2L(1+j)} = \frac{M_{AB}}{L} \left(\frac{1.5}{1 + j} \right)$$

The relationship between the end moment and the rotation at A is

$$\theta_A = \frac{\partial U}{\partial M_{AB}} = \frac{\partial}{\partial M_{AB}} \left[\int_0^L \frac{\left(M_{AB} - V_A \cdot x\right)^2 dx}{2\ EI} + \frac{V_A{}^2\ dx}{2\ A_s\ G} \right]$$

$$= \int_0^L \frac{\left(M_{AB} - V_A \cdot x\right) dx \cdot 2}{2\ EI} + 0$$

$$= \frac{M_{AB} \cdot x - \dfrac{V_A \cdot x^2}{2}}{EI} \Bigg]_0^L = \frac{L\left(M_{AB} - 0.5\ V_A \cdot L\right)}{EI}$$

But

$$V_A = \frac{M_{AB}}{L} \left(\frac{1.5}{1+j}\right)$$

Then

$$\theta_A = \frac{L}{EI} \left[M_{AB} - 0.5 \frac{M_{AB}}{L} \cdot \left(\frac{1.5}{1+j}\right) L \right]$$

$$= \frac{M_{AB}}{EI} \cdot L \left(1 - \frac{0.75}{1+j}\right) = M_{AB} \left(\frac{L}{EI}\right) \left(\frac{0.25 + j}{1+j}\right)$$

and solving for M_{AB} gives

$$M_{AB} = \frac{EI}{L} \left(\frac{1+j}{0.25 + j}\right) \theta_A$$

In the terms of moment distribution, the stiffness factor K represents the equivalent force EI/L to produce a unit displacement, so in this case, based on Figure 8 and to correspond with the conventional presentation of the relative value of K, since E and I are constant, then

$$K = \frac{1}{4L}\left[\frac{1 + j}{1/4 + j}\right]$$

Referring to Figure 8 again, by statics

$$\Sigma M = 0 \ , \Sigma F = 0 \ , \ - V_A = V_B$$

$$M_{BA} = V_A \cdot L - M_{AB} = \frac{M_{AB}}{L}\left(\frac{1.5}{1+j}\right) L - M_{AB}$$

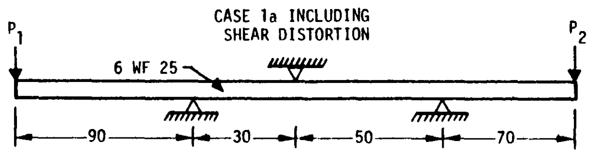
$$M_{BA} = M_{AB}\left(\frac{1.5}{1+j} - 1\right) = M_{AB}\left(\frac{0.5 - j}{1+j}\right)$$

The relationship between M_{BA} and M_{AB} is the carryover factor C.O. = (0.5-j)/(1+j) (see page 68 of Reference 2).

In terms of application, the meaning of the derivation is that the deformation due to shear distortion alters the stiffness, carryover factors and may affect the resulting distribution of moment. For the purposes of this report, experimental runs were made to physically illustrate the distributions.

Some of the cases are analyzed first by moment distribution (Figure 9) as modified by shear distortion and then by conventional moment distribution (Figure 10). The results are shown in the moment and shear diagrams of Figures 11.

A factor k_s modifies the area considered effective in shear and is dependent on the geometry of the cross-section. There is still some controversy regarding this factor, as explained in Appendix C and in references of that appendix. For purposes of computation, the factor k_s is approximately equal to unity for the wide flange beam.



E/G ≅ 2.6

I = 53.5"4

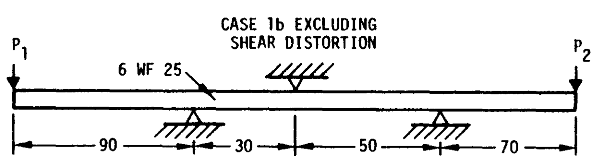
$A_s \cong k_s$ x WEB AREA ≅ 1.0 x 6.37 x 0.32 = 2.0384"2

NOTE: LOAD IN TERMS OF P

$j = \dfrac{3\ EI}{L^2 A_s G}$		0.22746	0.08188	
$K = \dfrac{1}{4L}\left[\dfrac{1+j}{1/4 + j}\right]$		0.02140	0.01627	
D = K/ΣK		0.56809	0.43190	
$C.O. = \dfrac{1/2 - j}{1+j}$		0.22203	0.38647	
	0 … 90 … 0	0 -90 … (7.0702) … -19.9827 … - 4.0165	… 0 … 70 27.0529 -3.0536	-70 … 0 0
Σ	… 90	-90 … -23.9992	+23.9993 … 70	-70 …
SIGN OF M	… (-)	… (+)	… (-)	
FREE BODY DIAG.	P			P

R_A R_B R_C

Figure 9 - Case-1 Including Shear Distortion Effect

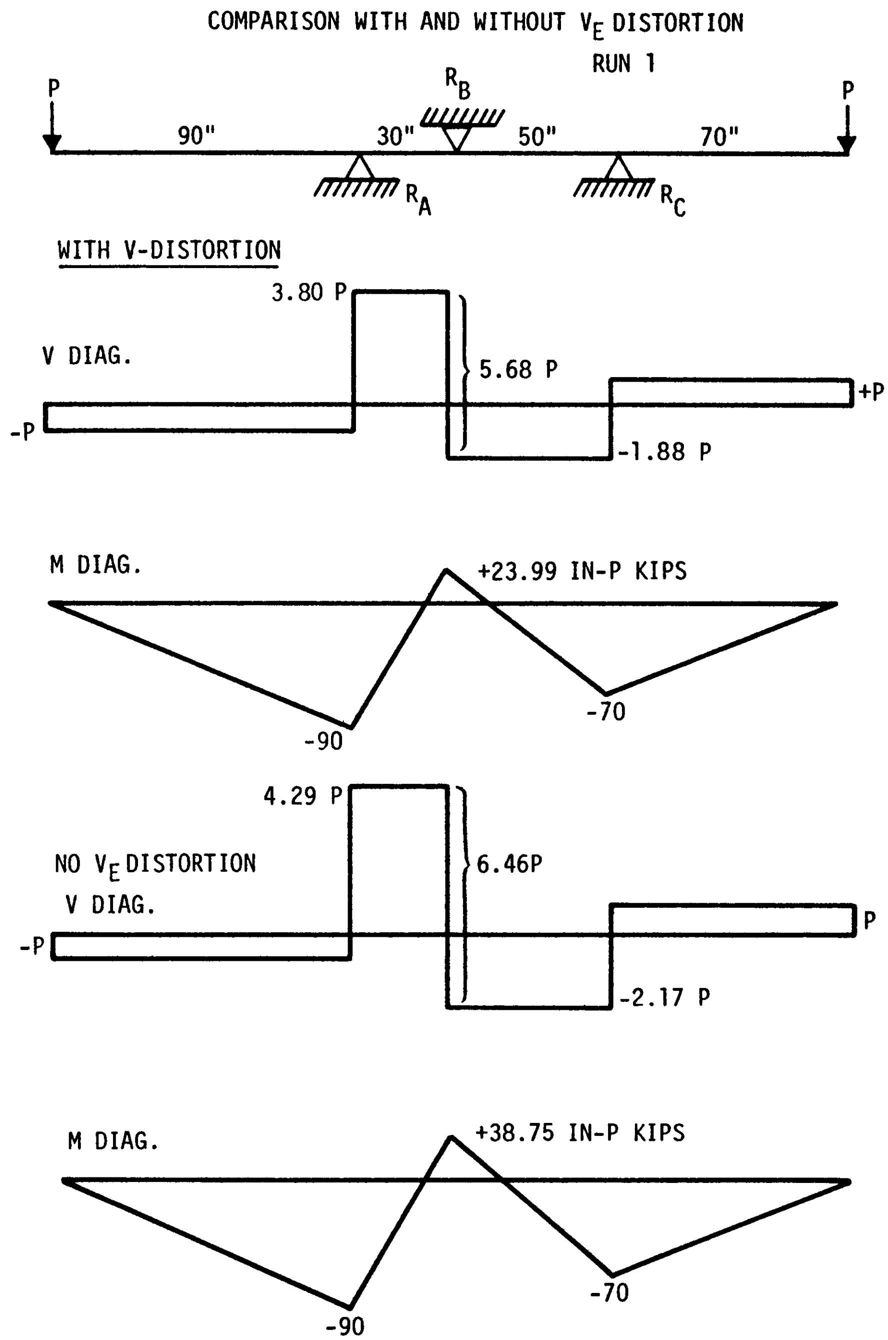


NOTE: LOAD IN TERMS OF P

K = 1/L		1/30	1/50	
D = K/ΣK	… 0	1 … 0.615	0.375 … 1	0
C.O. =		1/2	1/2	
	0 … 90 … 0	0 … 0 -90 … 0 … (-10) … -45.0 … + 6.25	0 … 0 0 … 70 35.0 +3.75	-70 … 0 0
Σ	… +90	-90 … -38.75	+38.75 … 70	-70 …
SIGN OF M	… (-)	… (+)	… (-)	
FREE BODY DIAG.	P			P

R_A R_B R_C

Figure 10 - Case-1 Excluding Shear Distortion Effect



RATIO $\frac{\text{INCL. } V_E}{\text{EXCL. } V_E}$	R_B RATIO	MOM RATIO
	$\frac{5.68}{6.46} = .879$	$\frac{23.99}{38.75} = .619$
DIFFERENCE IN %	~ 12	~ 38

Figure 11 - Comparison of Case-1 With and Without Distortion Effect

APPENDIX B

ERROR EVALUATION IN CALCULATIONS, TEST, AND THEORY

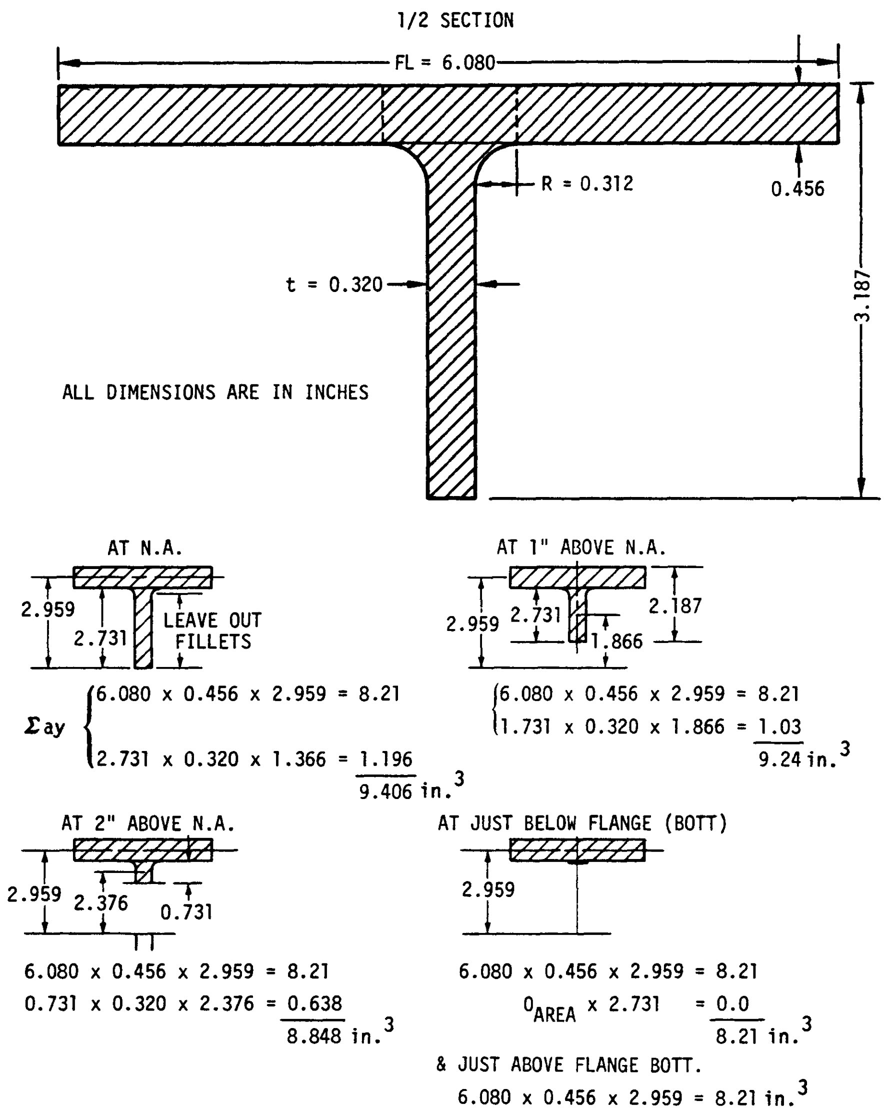
To determine the tests which had errors large enough to explain the vertical reactions at the supports were calculated and measured. These are listed in Table 2. Next, the individual errors and the average error were found. Finally, any error which was more than one standard deviation from this mean (circled values in Table 2) was examined. There were two such groups; the reaction at point a on the first three tests and all the reactions on Test 7.

The investigation of errors indicated that both sets of errors occurred in the running of the experiment. As already indicated, these were the first tests run in the new Free-Form Test Facility. The errors were not apparent at the time of testing and were actually part of the learning process for new equipment. In the first set (R_A alone), the error was found due to an incorrectly plumbed hydraulic jack. Subsequent runs were corrected by shims. The second set was due to an error in reading the Brush recorder (a line reading chart). The recorders have now become secondary readouts in later tests.

Another possible source of error might have occurred because of inconsistencies in the properties of the rolled I-beam. However, the variations were apparently such that the average results were close to that of the standard beam. Table 3 shows the relationship.

The error due to neglect of shear distortion will surpass most of the experimental or analytical errors, however. If the shear distortion analysis of Figure 15 (see Appendix C) is examined, it may be expected that the shear in the flanges will take about 6 to 7 percent of the total shear in the section (estimated from Figure 12). Actually, the effect may be well in excess of this amount for shorter, deeper beams. Further discussion and reference sources on shear distortion are given in Appendix C.

Shear in the flange is ordinarily ignored and thus the error due to neglecting it may be within the experimental error. The results of the experiment (Table 1) suggest that even with shear deformation effect included, the experimental reactions would generally tend to be less than predicted from the analysis. The explanation is that only the web area



$V_L = VQ/bI$

VALUE OF V AT VARIOUS LEVELS

$$V_{NA} = \frac{V_x\ 9.406}{0.320 \times 53.5} = 0.548\ V$$

$$V_{1''} = \frac{V_x\ 9.242}{0.32 \times 53.5} = 0.538\ V$$

$$V_{2''} = \frac{V_x\ 8.848}{0.32 \times 53.5} = 0.496\ V$$

$$V_{FL(-)} = \frac{V_x\ 8.21}{0.32 \times 53.5} = 0.478\ V$$

$$V_{FL(+)} = \frac{V_x\ 8.21}{6.080 \times 53.5} = 0.0249\ V$$

IGNORING THE FILLETS AND ASSUMING A STRAIGHT LINE VARIATION OF SHEAR STRESS FROM INNER FLANGE VALUE TO ZERO SHEAR AT THE OUTER FLANGE, THE PROPORTION TAKEN UP BY BOTH FLANGES IN SHEAR =

$$V_{FL} = 2[1/2 \times 0.0249 \times 6.080 \times 0.456] \times 100$$
$$= 6.82 \text{ PERCENT } V$$

Figure 12 - Shear Stress Distribution for the Test Beam

was included in the calculation, and it may be that part of the flange should be included, especially for short beam lengths (see Figure 13). This supposition seems to be borne out, as may be noted from Table 1.

In summary, the technique used in the experiment may be expected to give results with an approximate error of ±3 percent. This is with the jacks properly plumbed, using the digital voltmeter in place of the Brush recorder, and recording loads to the nearest 0.1 kip. Conventional analysis of shear stress distribution errors are of approximately the same magnitude as the experimental errors.

TABLE 2

Experimental versus Calculated Values

(Numbers with three decimals were calculated and those with one decimal were measured).

Test No.	Differences in Percent			Comments
	R_A	R_B	R_C	
1	$\frac{9.600-9.0}{9.600} \times 100 = 6.3$	$\frac{11.360-10.9}{11.350} \times 100 = 4.0$	$\frac{5.760-5.8}{5.760} \times 100 = -0.7$	
2	$\frac{10.583-9.5}{10.583} \times 100 = 10.2$	$\frac{15.728-15.3}{15.728} \times 100 = 2.7$	$\frac{11.148-11.4}{11.148} \times 100 = -2.3$	Note R_A for Tests 1,2,3
3	$\frac{18.176-17}{18.176} \times 100 = 6.5$	$\frac{18.282-17.5}{18.282} \times 100 = 4.3$	$\frac{6.106-6.0}{6.106} \times 100 = 1.7$	
4	$\frac{5.768.5.5}{5.768} \times 100 = 2.7$	$\frac{5.888-5.8}{5.888} \times 100 = 1.5$	$\frac{4.120-4.2}{4.12} \times 100 = -1.9$	
5	$\frac{6.240-6.1}{6.240} \times 100 = 2.2$	$\frac{8.125-8.1}{8.125} \times 100 = 0.3$	$\frac{7.885-7.9}{7.885} \times 100 = -0.2$	
6	$\frac{11.040-10.8}{11.040} \times 100 = 2.2$	$\frac{9.5-9.0}{9.5} \times 100 = 5.2$	$\frac{4.460-4.2}{4.460} \times 100 = 5.8$	
7	$\frac{7.320-6.6}{7.320} \times 100 = 9.8$	$\frac{10.640-9.5}{10.640} \times 100 = 10.7$	$\frac{7.320-6.6}{7.320} \times 100 = 9.8$	Note Degree of Error for Run 7
8	$\frac{7.980-7.7}{7.980} \times 100 = 3.5$	$\frac{15.960-15.5}{15.960} \times 100 = 2.9$	$\frac{13.980-13.5}{13.980} \times 100 = 3.4$	
9	$\frac{11.920-11.3}{11.920} \times 100 = 5.2$	$\frac{19.840-19.8}{19.840} \times 100 = 0.1$	$\frac{11.920-11.3}{11.920} \times 100 = 5.2$	
	ESTIMATED Jack Error	(Neglecting Encircled Values)		
	3.2 $\left\{\begin{matrix}2.2\\ \text{to}\\ 5.2\end{matrix}\right.$	2.6 $\left\{\begin{matrix}0.1\\ \text{to}\\ 5.2\end{matrix}\right.$	1.2 $\left\{\begin{matrix}-2.3\\ \text{to}\\ 5.8\end{matrix}\right.$	Average Approx. 3 Percent ±

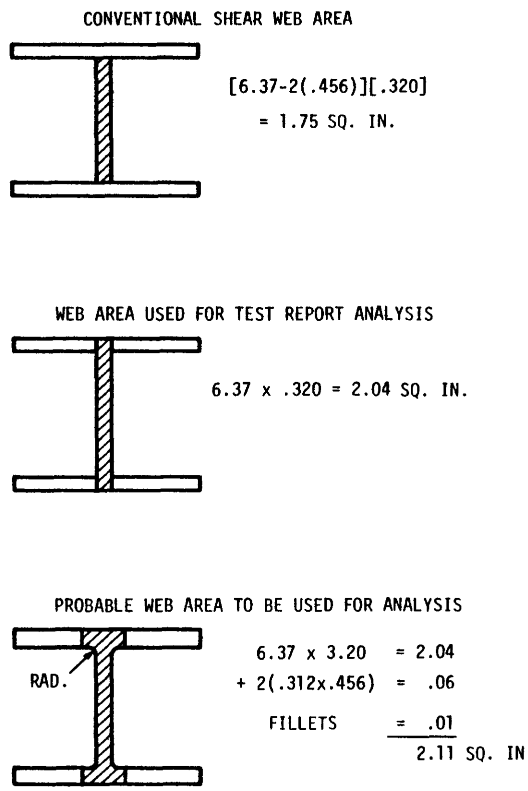


Figure 13 - Shear Area Consideration of a 6WF25 Beam

TABLE 3

Beam Comparison-Standard and Test*

Standard Bm. 6 In. Wide Flange 25 Lb		Test Beam Meas. Average Values		
depth	6.37	Avg. Diff. 0.0183"	6.383	6.373 S. End 6.390 Ctr. 3.387 Ctr. End
with flange	6.080	Avg. Diff. 0.010"	6.070	6.060 Top. Fl. 6.080 Bott. Fl.
Flange Thickness	0.456	Avg. Diff. 0.002"	0.458	0.472 Top West & 0.445 Top East Sides 0.455 Bott. West & 0.460 Bott. East Sides
Web Thickness	0.320	0	0.320	0.330 Ctr. 0.310 of 0.319 Bm.
Fillet Radius	0.312	0		11/32 Radii & 5/16 Gages

*Standard beam and averaged measured values are essentially the same.
All dimensions are in inches.

APPENDIX C

BRIEF SURVEY WITH COMMENTS ON SHEAR DISTORTION THEORY

Timoshenko was among those who included shear strain in beam deflection calculations. He showed that in some cases the inclusion of shear strains resulted in a decrease in deflection. He demonstrated that this was so even for several simple beam solutions.[4-6] Some publications[7,8] have criticized the various explanations given by Timoshenko on the grounds that they were obscure.

Since the basic concepts of the beam test are in line with the reasoning of Timoshenko, let us review various opinions on the shear strain concepts. A modification of the shear strain theory is presented here to help eliminate the obscurities and possibly to answer some of the criticisms and questions.

The shear effect has generally been explained by distinguishing between slope due to shear and slope due to bending. A fresh derivation of the Timoshenko beam theory was given recently[9] in direct answer to the criticisms of Reference 7. This led to a new value for the effective shear coefficient for the cross section of many beam shapes. It was a three-dimensional analysis, but although the results were multitermed, they did not differ greatly from those of previous authors (except for Timoshenko). Reference 9 is valuable for mathematical approaches in elastic theory, but it does not provide a physical interpertation to help eliminate the criticisms and alleged obscurities of the theory.

It is interesting to note that prior to Timoshenko, others developed similar theories e.g., Inglis[10] and Newlin and Trayer.[11] Later articles have also presented explanations to account for the effects of shear strain.[12-14] These studies generally have one feature in common, namely, they emphasize the fact that shear stresses are not uniformly distributed and thus that previously plane cross sections (in bending theory) become warped.

What appears to be required, however, is an explanation of the mechanism wherein this warping due to shear is translated into an effect which alters the distribution of the moment, that is, the bending of flexural stresses. This, then, is the modification to the plane bending theory. For the purposes of this test report, some of the ideas will be developed by illustration and commentary, utilizing available reference material for support. Recommendations for experiments and analysis are given as well.

Figure 14 (adapted from Reference 15) shows the kind of distortion expected from shear in a beam of rectangular section and is a good illustration of warping. Concepts developed to illustrate warping of the cross section are shown in Figure 15. These were adapted from sketches and illustrations in Reference 16.

With the basic concepts referenced and sketched, the listing of the criticisms are now in order. The main objection appears to be the "obscure" explanation given for shear effects. Although early pioneers in elasticity (St. Venant, Filon, etc) emphasized these effects, later scientists generally considered shear strain energy a minor item except in the case of a short deep beam with heavy loads. Since shear effects generally were considered minor with respect to bending, only the applications involving deleterious effects such as shear failures were developed in the analysis and testing. Hence any beneficial effects were neglected and the shear distortion effects remained obscure. As this test report indicates, further experiment and theory are still required.

The criticisms and questions, general or specific, are listed below:

1. If shear warping is accepted as a concept for free-ended elements, what are the consequences of restraining the free ends? If shear warping is then not allowed in the beam--for example at fixed ends and at symmetrical loading positions or at particularly restrained beams--how far away from the restraint does this effect last?

2. A specific criticism concerns this statement by Timoshenko (see Figure 149, page 170 of Reference 5):

"The elements of the cross sections at the centroids remain vertical." Reference 7 (page 181) asks why should shear warping leave the centroidal sections elements vertical?

3. In a similar vein, page 21 of Reference 8 indicates that Timoshenko was incorrect in assuming that shear strain can be computed from the maximum stress at the neutral axis of a beam, and that the stress distribution over the entire section cannot be ignored.

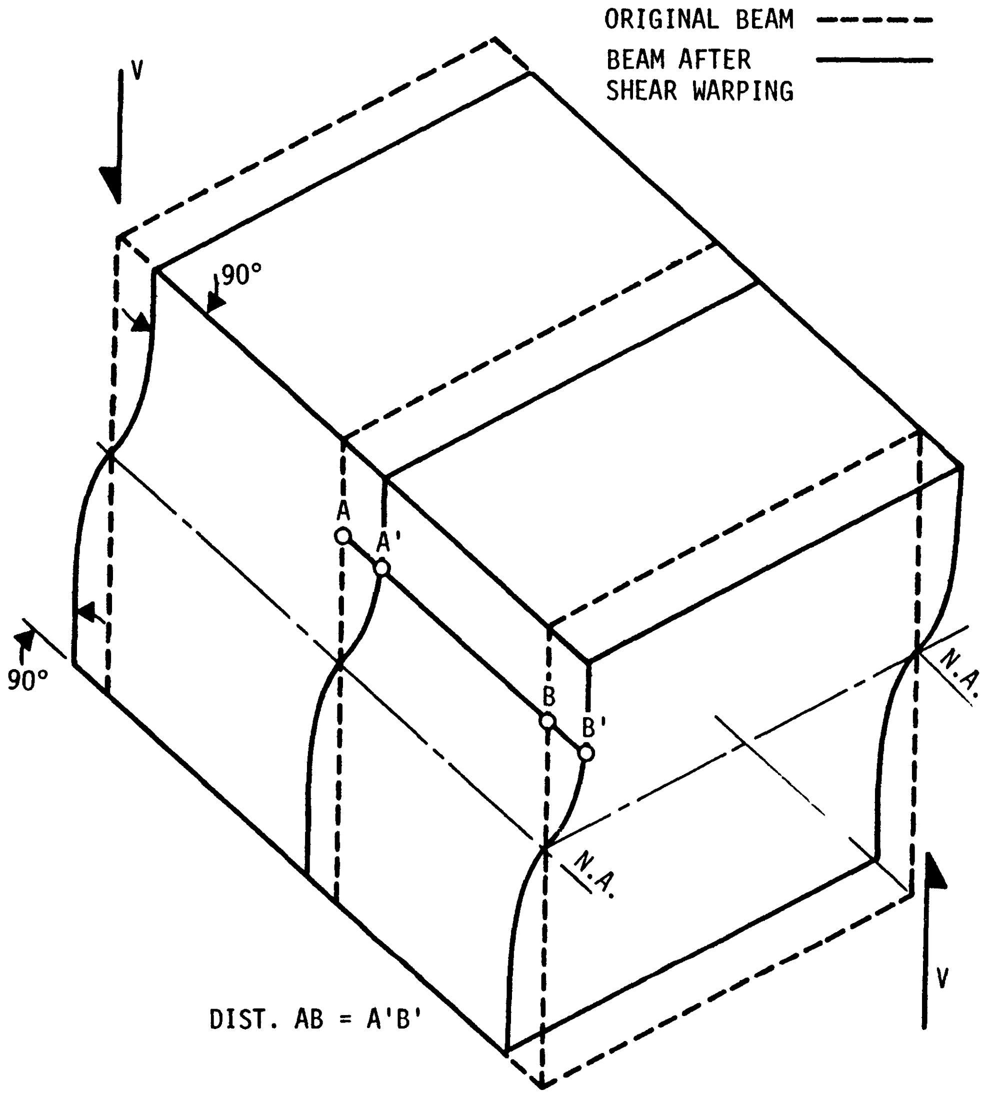


Figure 14 - Shear Warping Due to Constant Shear Force

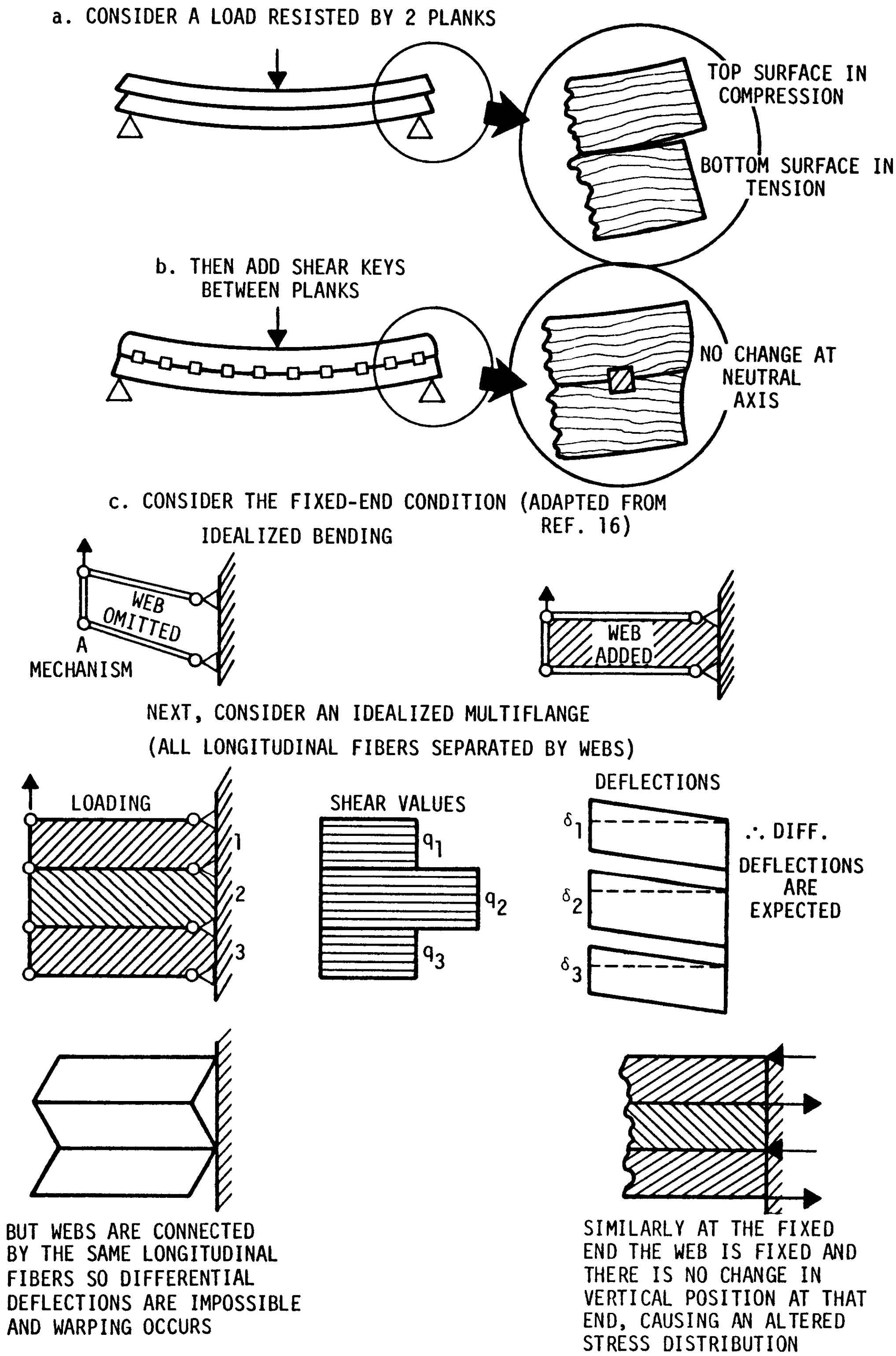


Figure 15 - Section Warping Due to Shear

4. Another general criticism is that although the slope due to bending at the fixed end equals zero, there will be a slope at that fixed end due to a constant shear. This is in contradiction to the Timoshenko "continuity of deformation" in shear as well as bending (illustrated on pages 173 and 174 of Reference 5 and in a corollary case given on pages 228-230 of Reference 4) involving shear on either side of a symmetrical case of loading.

This report makes no attempt to investigate all aspects of shear deflection theory; it merely seeks to verify the theory. However, it is properly within its province to provide a general approach and to review the concepts that are related to the experiments. Hence, by sketch and reference sources, a modified approach is given below to show that the experimental setup and analyses reconcile some of the differences of the criticisms just cited.

The primary point of contention among the several critics is whether or not there is a displacement or slope relationship dependent on the warping in shear. Criticisms 1 and 4 imply that there is not (see page 173 of Reference 7). Although no direct mathematical refutation has been found up to this point, the example taken from Timoshenko (page 174 of Reference 5) and the discussion by Shanley (page 379 of Reference 16) taken together indicate that when restraints occur and the cross sections are compelled to remain plane, the transverse shear will alter flexural stresses.

As far as Criticisms 2 and 3 are concerned, there is insufficient evidence to evaluate the true distribution of stresses at the beam sections. However, from the generalized view that has been taken for this report, one possible explanation is that instead of the Timoshenko expression of the elements at the centroid remain vertical, the answer might be that the slope at the section, (i.e., the "elastic line" of Reference 10) of the shear warped condition is presently unknown and probably may be expressed as a function of the boundary conditions, the geometry of the member, and thus is dependent upon the distribution of shear in the member. Some assumptions to solve this problem are advanced in a later section.

In view of the discussion of Inglis (Reference 10) shear stresses at the centerline (neutral axis) have patterns of nonuniformity which he terms overshooting, and his pictured distributions are different from conventional shear distributions.

Some of the experimental and analytical approaches to this problem are available in Reference 10. It is indicated that earlier photoelastic work (by Coker, Filon and others) may be used to set up present day experiments to more fully probe and compare with results of the past to determine shear along the centerline.* Although the past work was employed primarily for simple beams and plates, modern techniques certainly make it feasible to investigate indeterminate structures such as the configuration considered in this test report.

The actual stress distribution is still an unknown but for purposes of explanation, an assumed distribution is shown in Figure 16. Consideration was given to the abrupt change in section from web to flange although the stress concentration was relieved to an extent by the fillet. In addition, the shear stress in the flange area must be zero at both the inside and outside edges.

Finally, it is important to realize that this problem is still relevant in recent practice as shown by publications in several fields. For example, shear strain energy equations may be found in a design book in civil engineering,[17] in a report on submarine structure,[18] and in a report on surface ship structure.[19]

In view of the still present need, the following approaches are offered for explanation, analysis, and further experimentation. A tentative theory of the shear strain effect is illustrated in Figure 17. Combining the visual effect of continuity of deformation noted in the experiment with

---

*An historical comment is that the Inglis paper[10] was presented to the Royal Society under the sponsorship of Coker and that, in turn, J.N. Goodier, a pupil of Inglis, did his thesis in a closely related subject under the direction of Timoshenko. Many of these results may be found in Reference 6, with photoelastic verification in the Goodier thesis.

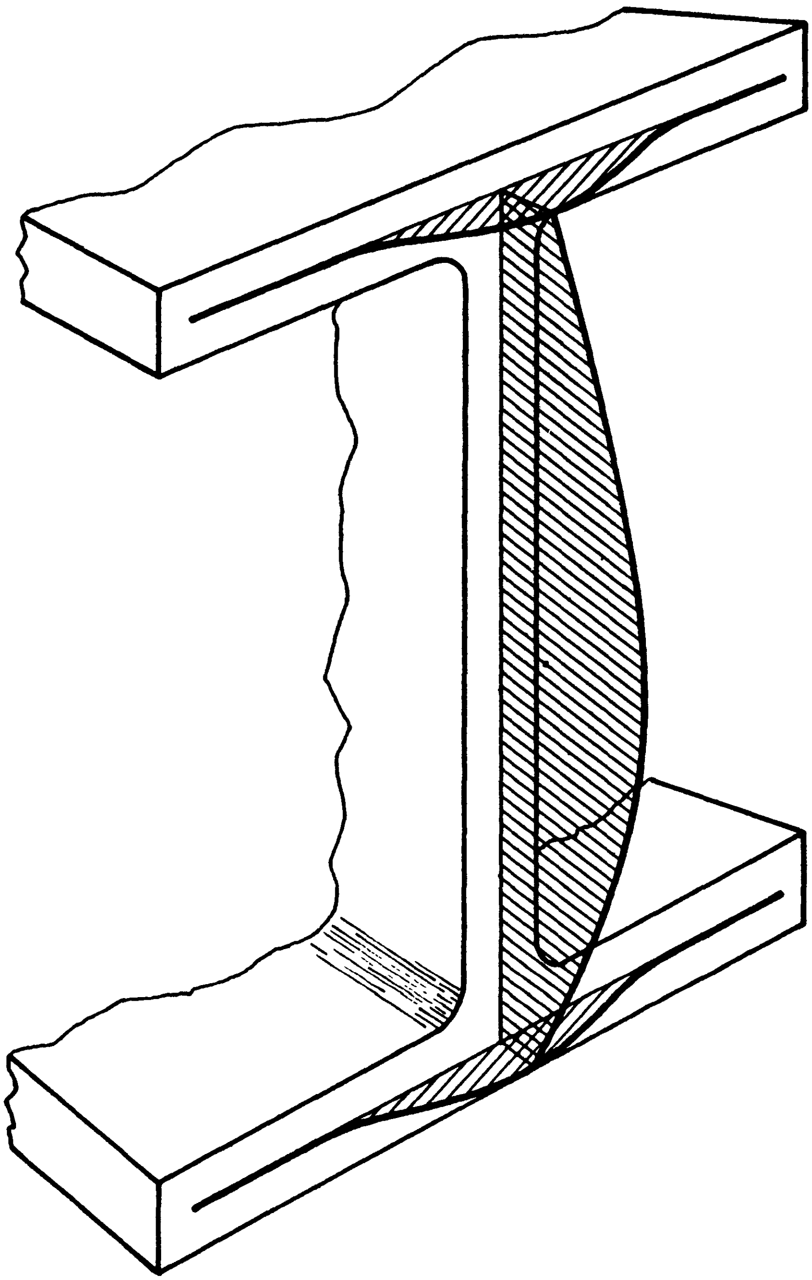


Figure 16 - Assumed Shear Stress Distribution
(Idealized in Planar Illustration)

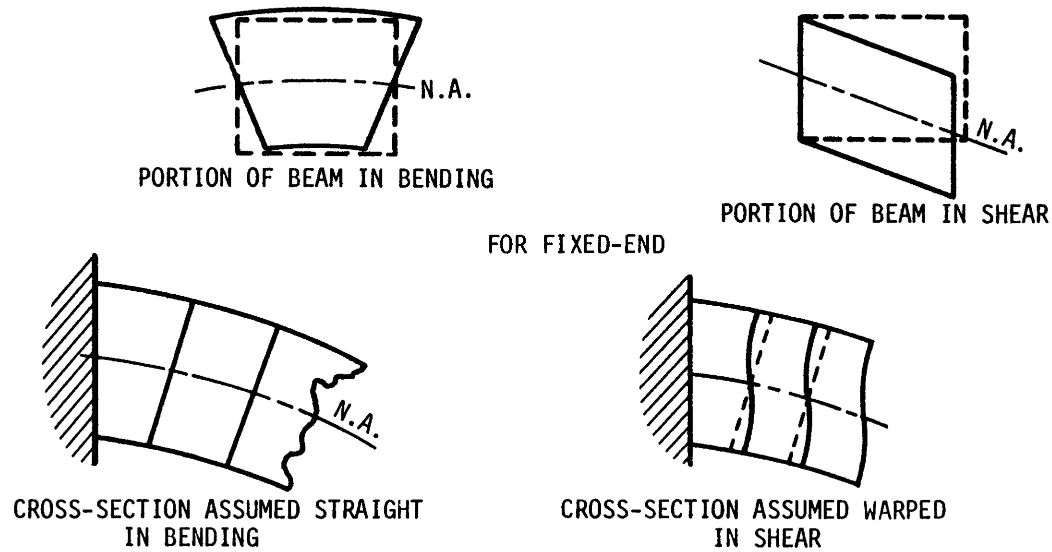


Figure 17a - Comparison of Deformations Due to Bending and Shear

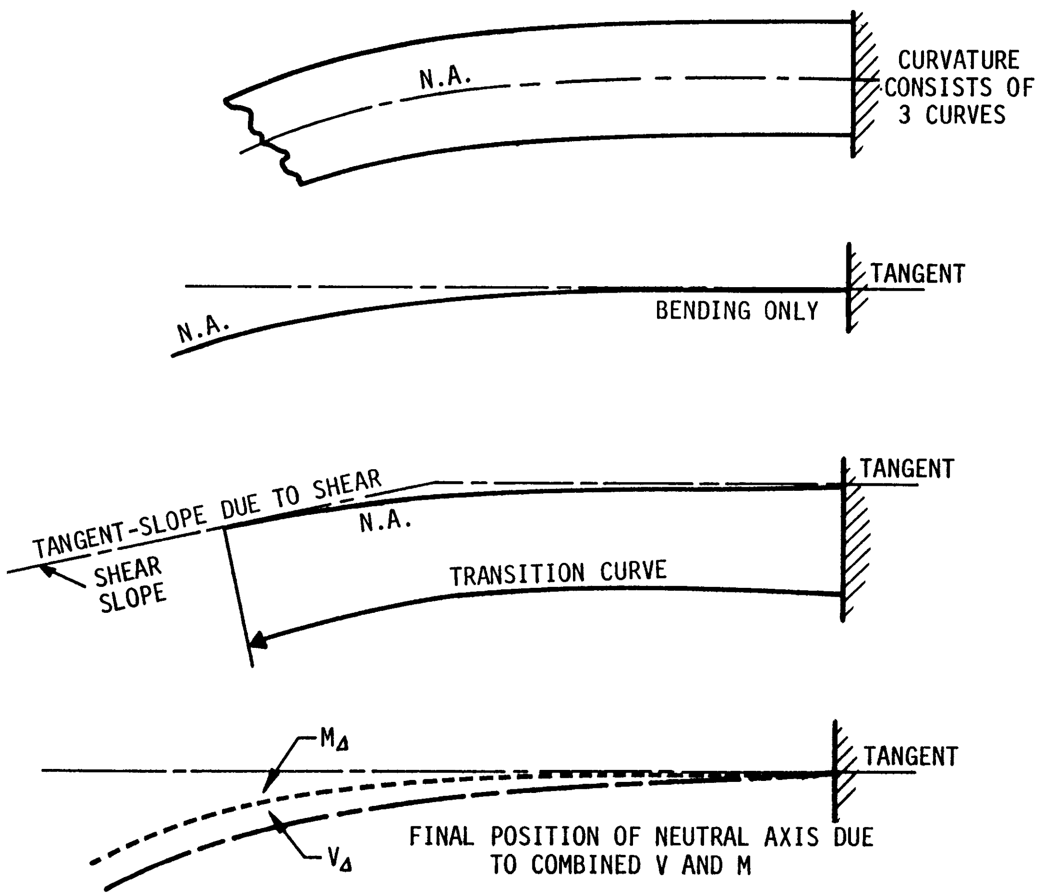


Figure 17b - Concept of Superposition for Shear and Bending

Figure 17 - Deformations Due to Shear and Bending

the concept of restraint in the vertical plane at the "fixed" condition, it is assumed that in its shearing distortion, the beam adjusts to two tangential lines. It is horizontal at the fixed end and thus establishes the first tangent. As a result of constant shear, there is a constant slope which establishes the second tangential line. Between these two slopes, the beam adjusts in a curve whose mathematical description would be determined by elastic theory derivation.

The method of investigation for analysis and experiment would be based on the following assumptions:

1. The true state of strain may be found by superposing on the state of strain due to bending only that due to shear distortion.
2. The true distribution of strain over the cross section of the beam at any station will be described by a function as yet undetermined.
3. The true restraint and the resulting true deflection or curvature will be described by a shear distortion deformation superposed upon the deformation due to bending effect.

It is obvious from this brief survey and comments that shear distortion energy is an area of study that has been overlooked. But it does have importance in selected cases. Hence a theoretical consolidation of concepts would be helpful. In addition, a further test program may be in order to verify the strain distortion theory more completely.

It is suggested that a more complete state-of-the-art study should be made in the hope of achieving a well-defined strain distortion theory. From this study, a test program should be developed to verify the theory and improve analytical techniques for future design.

To summarize, where the section is allowed to warp freely, shear distortion energy does not affect moment; however, where restraint of position occurs and free warping is prevented, then shear distortion energy alters the bending moment and thus the level of flexural stresses. A change in stress level results in a deflection change. The restraint is apparent in horizontal shear (see the plank analogy in Figure 14) and in the fixed-end analogy of vertical restraint (see Figure 17).

The indeterminate case is just such an application of vertical restraint at the supports. Hence, where the geometry is that of the short, deep beam for the portion under consideration, then shear distortion effects alter the distribution of moment at the supports. Further, the shear restraint in the flanges may account for a sizable portion of the internal work and thus enhance the shear distortion effect beyond the usual shear-in-web assumptions. This indicates new approaches to stress distribution problems.

It is suggested that further theoretical studies be accompanied by experimental work for more positive verification.

REFERENCES

1. Stavovy, A.B. et al., "A Free-Form Test Facility for Large-Scale Structural Models of Ship Section or Components," NSRDC Report 2979 (Mar 1969).

2. Maugh, L.C., "Statically Indeterminate Structures," John Wiley and Sons, New York (1946).

3. Cross, H. and Morgan, N., "Continuous Frames of Reinforced Concrete," John Wiley and Sons, New York (1932).

4. Timoshenko, J. and Young, D., "Elements of Strength of Materials," Fourth Edition, Van Nostrand, New York (1962).

5. Timoshenko, S., "Strength of Materials," Second Edition, Van Nostrand, New York (1940).

6. Timoshenko, S. and Goodier, J., "Theory of Elasticity," Second Edition, McGraw Hill, New York (1951).

7. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams, Including Methods of Solution and Application to Ships," David Taylor Model Basin Report 1317 (May 1961).

8. Mann, E., "Moment Distribution Including the Effect of Shear Strain," Thesis, Catholic University of America (Jun 1967).

9. Cowper, G.R., "The Shear Coefficient in Timoshenko's Beam Theory," J. Appl. Mech. (Jun 1966).

10. Inglis, C.E., "Stress Distributions in a Rectangular Plate Having Two Opposing Edges Sheared in Opposite Directions," Proc. Roy. Soc., Series A, pp. 598-610 (1923).

11. Newlin, J.A. and Trayer, G.W., "Deflection of Beams with Special Reference to Shear Deformations," NACA Report 180 (1924).

12. Southwell, R.V., "Theory of Elasticity," Second Edition, Oxford University Press, London (1941).

13. Sechler, E., "Elasticity in Engineering," John Wiley and Sons, New York (1952).

14. Biblis, E.J., "Shear Deflection of Wood Beams," Yale University School of Forestry Technical Report 32; see also, Forest Products Journal Vol. XV (Nov 1965).

15. Crandall, S.H. and Dahl, N.C. Ed., "An Introduction to the Mechanics of Solids," McGraw Hill, New York (1959).

16. Shanley, F., "Strength of Materials," McGraw Hill, New York (1957).

17. Lightfoot, E., "Moment Distribution," John Wiley and Sons, New York (1961).

18. Heller, S.R. and Palermo, P.M., "Methods of Elastic Analysis of Circular Bulkhead Stiffening Systems," David Taylor Model Basin Report 1336 (Nov 1959).

19. Ship Structures Committee, "Effect of Ship Stiffeners upon the Structural Response of a Cargo Ship to Impulsive Loads," SSC-186, Appendix G. (1969).

INITIAL DISTRIBUTION

Copies	
3	NAVMAT
	1 MAT 03L
	1 MAT 03P
	1 MAT 03T
8	NAVSHIPSYSCOM
	2 SHIPS 2052
	1 SHIPS 031
	1 SHIPS 034
	1 SHIPS 03423
	1 SHIPS PMS 377
	1 SHIPS PMS 3799
	1 SHIPS PMS 392
14	NAVSEC
	2 SEC 6034
	1 SEC 6101
	1 SEC 6102
	1 SEC 6110
	1 SEC 6113
	2 SEC 6120
	6 SEC 6128
1	ONR 459
1	NAVCIVENCLAB
1	USNA
1	NAVSHIPYD BNS
1	NAVSHIPYD CHASN
1	NAVSHIPYD HUNTERS PT
1	NAVSHIPYD L BEACH
1	NAVSHIPYD MARE ISLAND
1	NAVSHIPYD NORVA
1	NAVSHIPYD PEARL
1	NAVSHIPYD PHIL
1	NAVSHIPYD PTSMH
1	NAVSHIPYD BREM
1	SUPSHIP NWPT NEWS
1	SUPSHIP BATH
1	SUPSHIP GROTON
1	SUPSHIP SEATTLE
1	ABS

Copies	
1	MARAD
1	MIT
1	WEBB
1	STEVENS
1	Catholic U
1	USNPG School Monterey
2	DDC

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)* Naval Ship Research and Development Center Washington, D.C. 20034	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3 REPORT TITLE

EXPERIMENTAL VERIFICATION OF THE SHEAR DEFORMATION EFFECT ON THE MOMENT DISTRIBUTION CALCULATION USED IN THE DESIGN OF SHIP COMPONENTS

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Final

5 AUTHOR(S) *(First name, middle initial, last name)*

Eugene A. Zwenig

6. REPORT DATE May 1971	7a. TOTAL NO OF PAGES 39	7b. NO. OF REFS 19

8a. CONTRACT OR GRANT NO b. PROJECT NO. 0 & MN c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 3644 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Distribution limited to U.S. Government agencies; Test & Eval. Info.; 7 Apr 1971. Other requests for this document must be referred to NSRDC, Code 700.

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY NAVSEC

13 ABSTRACT

Beam tests were conducted to determine the degree to which shear distortions affect moment distribution results as used in a practical design. Through simple experimentation it was found that, in selected cases, the shear distortion was important enough to suggest changes in design procedures. Appendixes supplement the theory and problems relative to the test report.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

UNCLASSIFIED

Security Classification

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Moment Distribution						
Shear Energy						
Bent Design						
Structures						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

UNCLASSIFIED
Security Classification