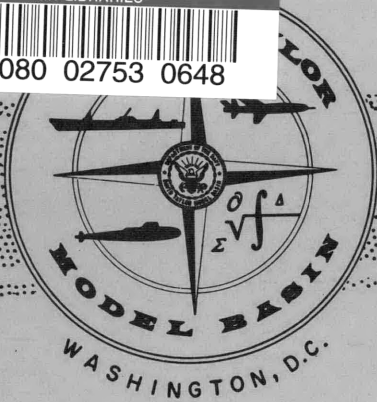


V393
.R46

MIT LIBRARIES



DEPARTMENT OF THE NAVY



HYDROMECHANICS



TESTS OF FABRICATED MULTILAYERED RING-STIFFENED CYLINDRICAL MODELS UNDER EXTERNAL HYDROSTATIC PRESSURE

AERODYNAMICS



by

Richard V. Raetz

STRUCTURAL MECHANICS



Distribution of this document is unlimited



APPLIED MATHEMATICS



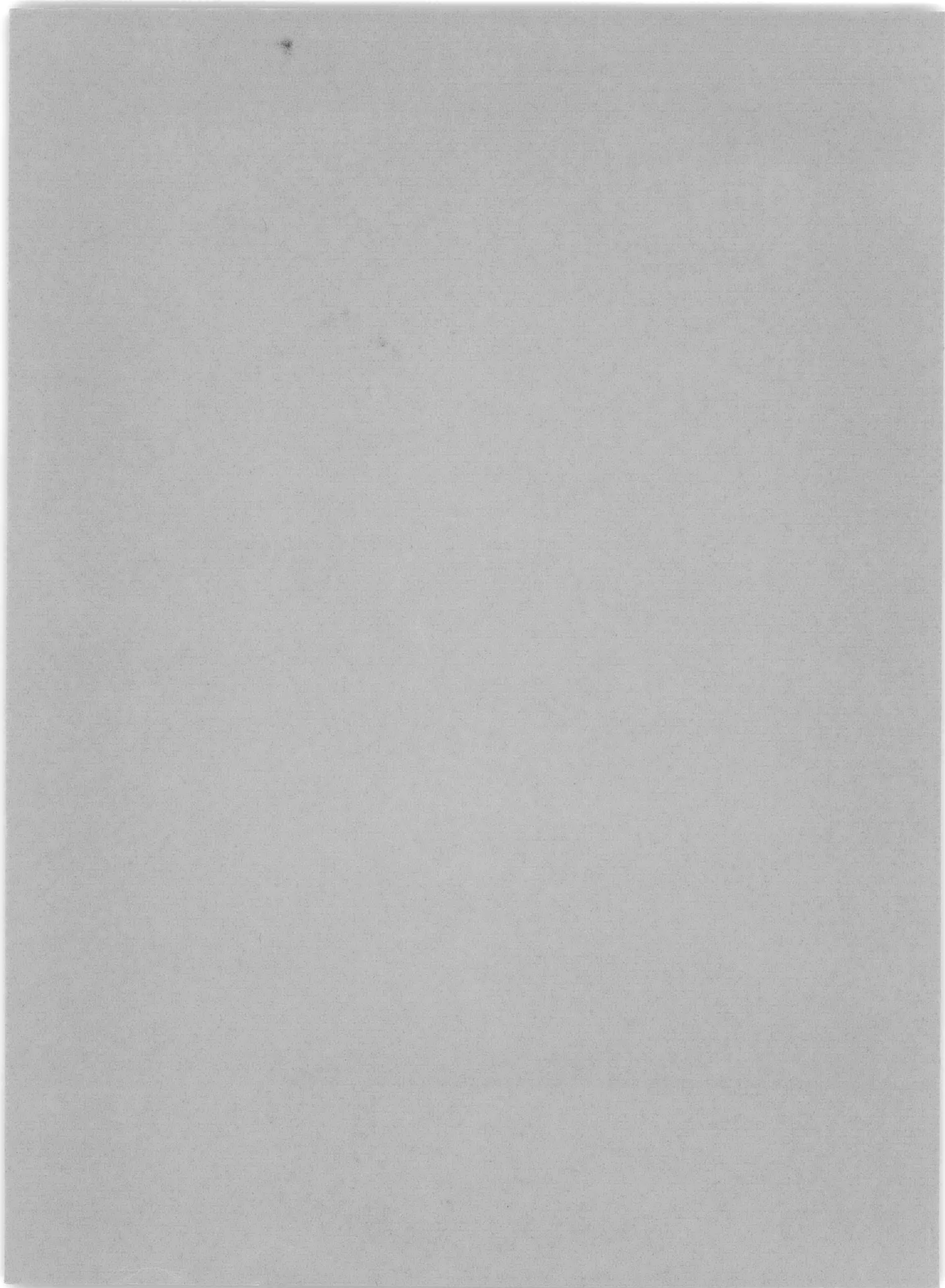
STRUCTURAL MECHANICS LABORATORY RESEARCH AND DEVELOPMENT REPORT

ACOUSTICS AND VIBRATION

April 1966

Report 2173

Report 2173



DAVID TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

TESTS OF FABRICATED MULTILAYERED RING-
STIFFENED CYLINDRICAL MODELS
UNDER EXTERNAL HYDROSTATIC PRESSURE

by

Richard V. Raetz

Distribution of this document is unlimited

April 1966

Report 2173
S-R011 01 01
Task 0401

TABLE OF CONTENTS

	Page
ABSTRACT	1
ADMINISTRATIVE INFORMATION	1
INTRODUCTION	1
DESIGN AND DESCRIPTION OF MODELS	2
INSTRUMENTATION AND TEST PROCEDURE	5
TEST RESULTS	5
INTERPRETATION OF RESULTS AND DISCUSSION	11
CONCLUSIONS	18
APPENDIX - THEORETICAL CALCULATIONS	20
REFERENCES	26

LIST OF FIGURES

	Page
Figure 1 - Dimensions and Design Details of Models ML-1, ML-2, and ML-3	3
Figure 2 - Dimensions and Design Details of Model ML-4	3
Figure 3 - Experimental Strain Sensitivities	6
Figure 4 - Models After Collapse	9
Figure 5 - Typical Pressure-Strain Diagrams	12
Figure 6 - Theoretical Stress and Strain Sensitivity Distribution for Typical Bays	15
Figure 7 - Nomenclature for Theoretical Calculations	21

LIST OF TABLES

	Page
Table 1 - Physical and Material Properties of Models	4
Table 2 - Experimental and Theoretical Collapse Pressures of Models	8

ABSTRACT

An exploratory investigation was carried out to study the strength of ring-stiffened multilayered cylindrical shells under external hydrostatic pressure. Four HY-100 steel models, having relatively large shell thickness-to-diameter ratios, were tested to collapse. The weight-to-displacement ratios for all models were approximately equal. The tests indicated that for the weight-to-displacement ratio studied, the strength of a single-layered ring-stiffened cylindrical model could be attained by a model with a shell consisting of two layers, each layer one-half the single-layered shell thickness, with a moderate increase in shell yield strength. It was also indicated that the strength of the single-layered model could not be attained in a model with a shell consisting of layers of one-fourth the single-layered shell thickness without a very large increase in the yield strength of both shell and frame material.

ADMINISTRATIVE INFORMATION

The work described in this report was conducted as part of the David Taylor Model Basin Fundamental Research Program, Subproject S-R011 01 01, Task 0401.

INTRODUCTION

In designing submarine hulls to operate at very great depths, we have a problem of obtaining and fabricating sufficiently thick shell material having the necessary physical properties to meet hull strength and weight requirements. One possible solution is to fabricate multilayer shells from thinner plating. Although the lamination of the shell in itself would seem almost certain to reduce hull strength, a net gain might be achieved by virtue of the superior strength and other material properties inherent, at present, in thinner plating material. This approach could prove to be the only feasible method of hull construction using such higher strength materials as HY-steel, aluminum, titanium, and fiber-reinforced plastics. One other incidental advantage of this type of construction is that these hulls may under some conditions have superior acoustical properties due to frictional damping between the shell layers.

To use the multilayer concept, an exploratory investigation was instituted into the structural response of these shells, since little work had been done on this subject that is applicable to pressure hull design. Four rolled and welded steel cylindrical models were tested to collapse under external hydrostatic pressure. The results of these tests and several theoretical analyses are presented in this report. A preliminary report of the test results of the first three models has been published.¹

DESIGN AND DESCRIPTION OF MODELS

Four ring-stiffened cylindrical steel models designated ML-1, ML-2, ML-3, and ML-4 were tested. Models ML-1, ML-2, and ML-3 were nominally identical except that the shells consisted of one, two, and four layers, respectively. Dimensions and design details of the three models are shown in Figure 1. The weight, diameter, and overall length of Model ML-4 were the same; however, the overall shell thickness was reduced, the shell consisted of three layers, and the size and the number of frames were increased. This model is shown in Figure 2. The material used in all the models was HY-100 steel, with a nominal yield strength of 100,000 psi. Dimensions, actual material properties, and calculated parameters are summarized in Table 1.

The thickness-to-radius ratios were relatively high for these models, placing their dimensions in a possible range for an oceanographic research vehicle. It was expected that with the relatively high thickness-to-radius ratio, an efficient structure could be designed using a small ratio of frame area to shell area. Thus, a large proportion of the strength of the model in any mode of failure would be contributed by the shell, and it was expected that the effect of laminating the shell in Models ML-2 and ML-3 would be clearly defined. Model ML-4 represented an attempted optimum multilayered shell design, assuming the limitation that

¹References are listed on page 26.

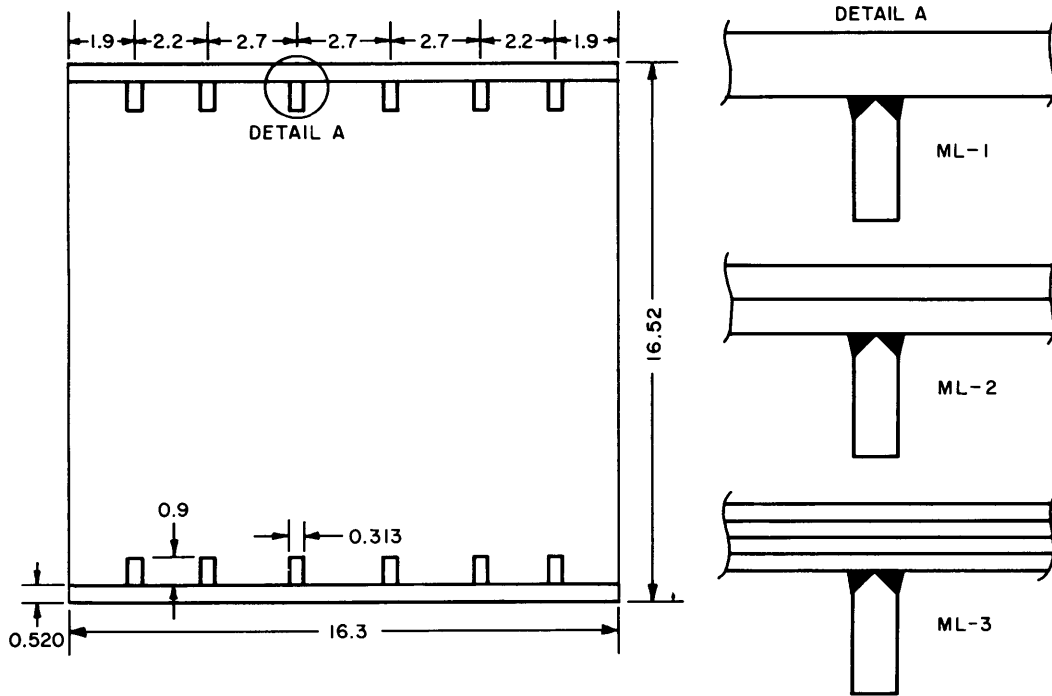


Figure 1 - Dimensions and Design Details of Models ML-1, ML-2, and ML-3

Note: All dimensions are in inches.

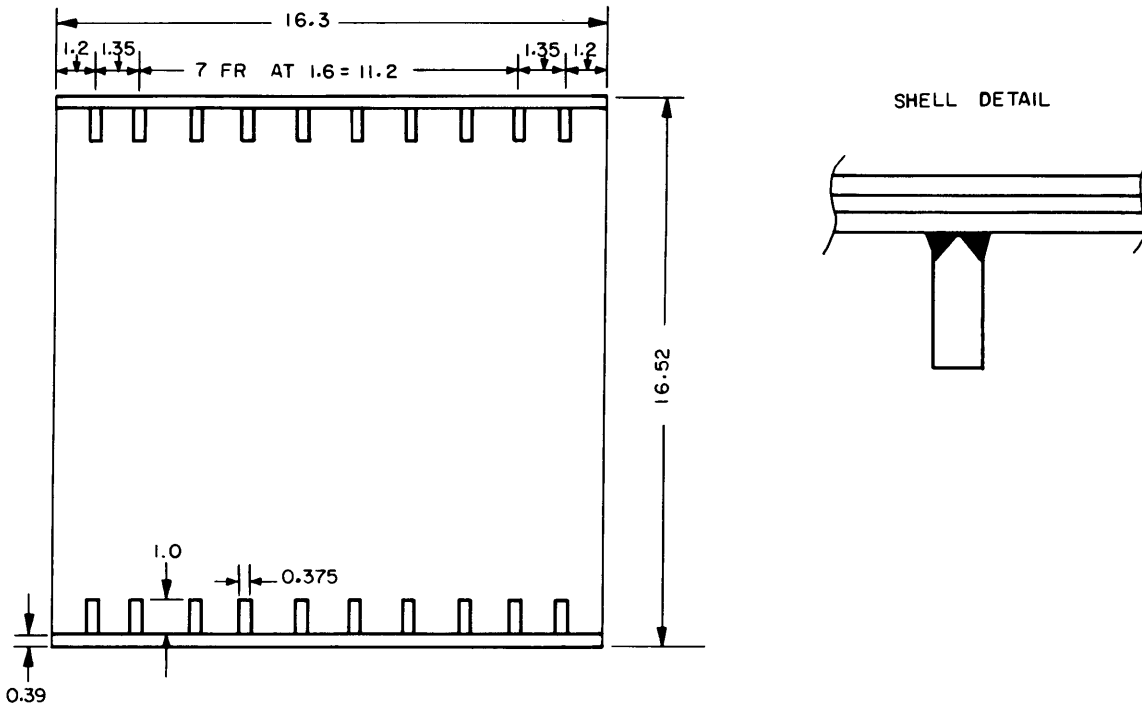


Figure 2 - Dimensions and Design Details of Model ML-4

Note: All dimensions are in inches.

TABLE 1
Physical and Material Properties of Models

	ML-1	ML-2	ML-3	ML-4
Number of layers	1	2	4	3
Total thickness, nominal inches	0.52	0.52	0.52	0.39
Thickness of individual layers, nominal inches	0.52	0.26	0.13	0.13
Actual total thickness, inches	0.521	0.514	0.520	0.396
Overall length of model, inches	16.3	16.3	16.3	16.3
Mean diameter of model, inches	16.0	16.0	16.0	16.13
Frame Spacing (typical bay) inches	2.7	2.7	2.7	1.6
A_f/A_s , typical bay	0.20	0.20	0.20	0.60
Yield strength of shell material, psi	99,200	88,600	110,000	107,700
Yield strength of frame material, psi	110,000	110,000	110,000	103,200
Weighted average yield strength, shell and frame material, psi	101,000	92,170	110,000	105,900
Weight/weight of displaced water	1.12	1.12	1.12	1.11

the thickness of the individual shell layers be not greater than that of the individual layers of Model ML-3. The spacing of the frames near the ends in all the models was varied somewhat to avoid an "end bay" failure.

All the models were of rolled and welded construction, having rectangular frames cut from plating material in three sections and welded together. Then, the inner shells were rolled to a cylindrical shape, the longitudinal seams were welded, and the frames were welded in place. For Models ML-2, ML-3, and ML-4 the outer layers were rolled, slipped over the inner layers, clamped in place, and welded along their longitudinal seams. These seams were staggered around the circumference, care being taken to avoid bonding the layers together along the longitudinal welds.

INSTRUMENTATION AND TEST PROCEDURE

Electrical resistance strain gages were installed on the models to study strain behavior and to facilitate interpretation of the mode of failure and the collapse pressure. Each model was tested in two runs, with water as the pressurizing medium. The first run was to 5000 psi and the second was to the collapse pressure. Strains were recorded during each run and the strain sensitivities, based on each second run, are given in Figure 3.

TEST RESULTS

Collapse pressures of all models are given in Table 2. All the models collapsed with the appearance of a single lobe running the entire length of the model. The models are shown after failure in Figure 4. Substantial yielding took place in all models before failure occurred. Pressures at which most strain versus pressure plots became markedly nonlinear are also given in Table 2.

The measured strain sensitivity distribution plots for the multi-layered models (ML-2, ML-3, and ML-4) appear to be rather erratic; nevertheless, certain significant features emerge quite consistently. First, the circumferential strains measured on the external shell surfaces were higher than those measured on the internal surfaces, except where the measurements were taken directly over a frame. Secondly, longitudinal strains, both on the external and internal shell surfaces, were higher

Figure 3 - Experimental Strain Sensitivities

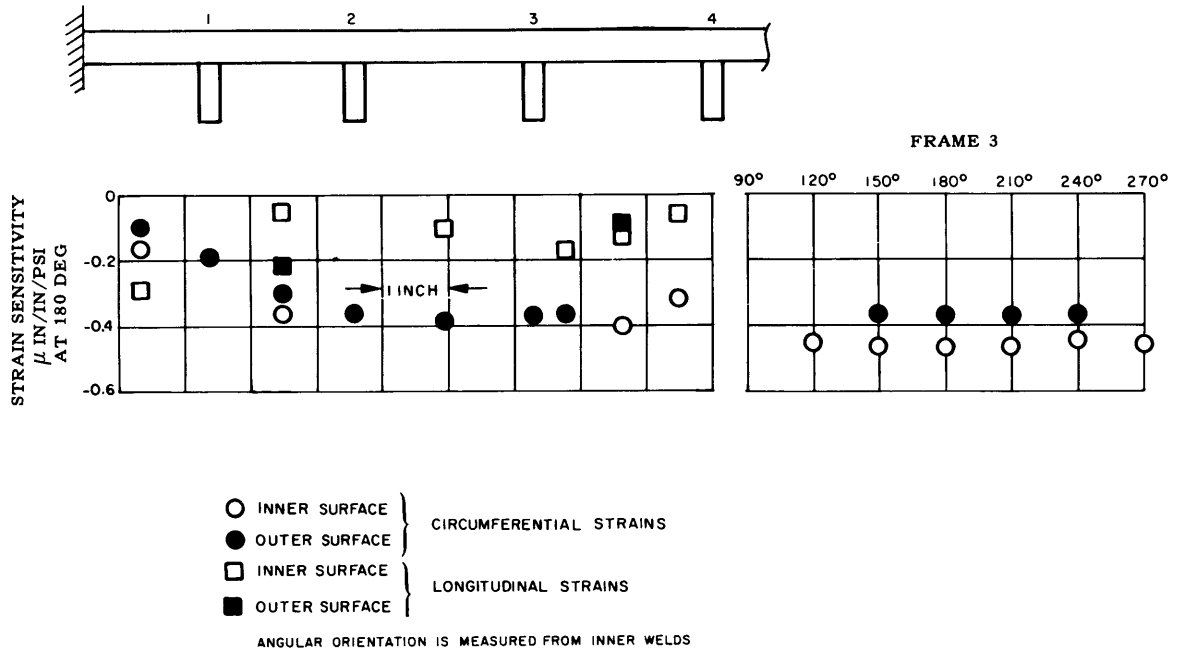


Figure 3a - Model ML-1

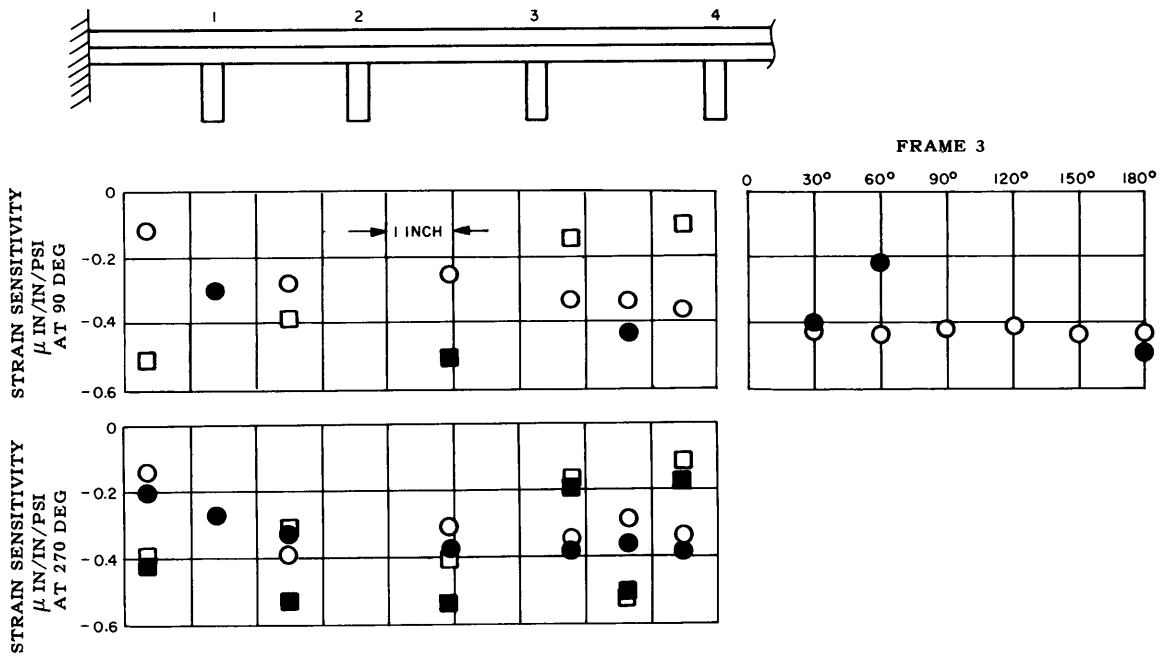


Figure 3b - Model ML-2

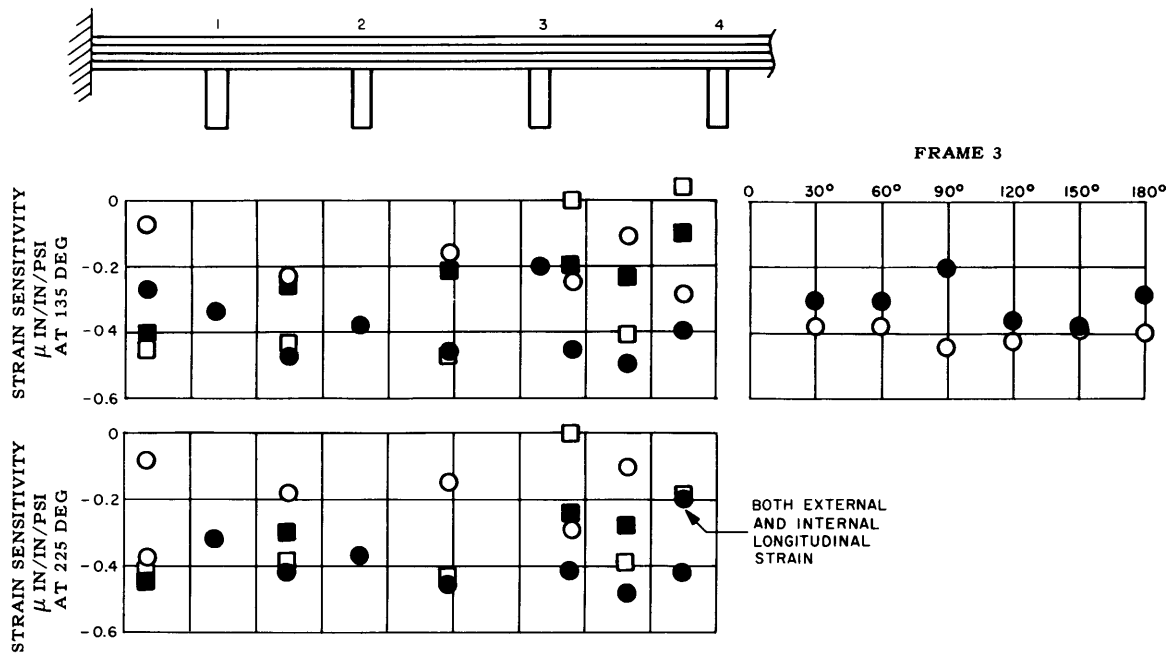


Figure 3c - Model ML-3

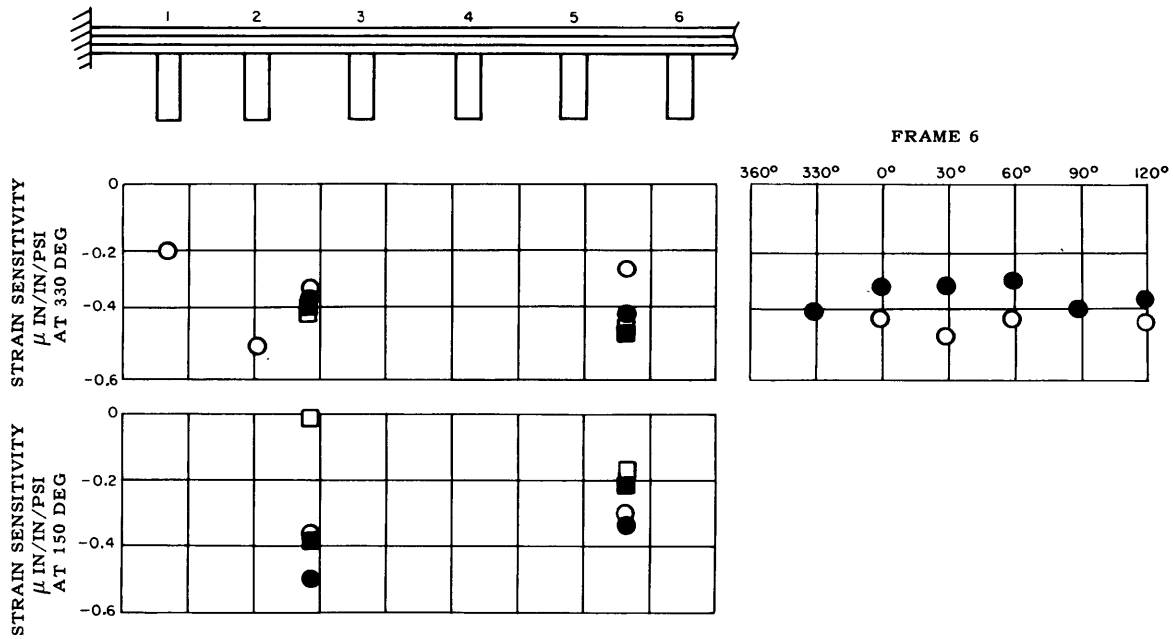


Figure 3d - Model ML-4

TABLE 2

Experimental and Theoretical Collapse Pressures of Models

	ML-1	ML-2	ML-3	ML-4
Number of layers	1	2	4	3
Experimental collapse pressure p_e , psi	8,560	7,440	7,460	7,300
Experimental collapse pressure corrected linearly for nominal thickness and yield strength,* psi	8,460	8,170	6,780	6,790
Theoretical elastic general instability collapse pressure p_g , psi	34,700(3)**	25,000(3)	18,700(3)	28,900(3)
Theoretical shell buckling pressure p_s , psi	203,000(5)	46,300(8)	14,500(10)	31,900(18)
p_g/p_e	4.1	3.4	2.5	4.0
βl , typical bay	1.50	2.12	3.00	1.55
Theoretical pressures required for initial yielding,† psi, based on:				
Stress at midbay on external surface, with Hencky-Von Mises criterion	8,710	7,670	9,120	8,460
Stress at midbay on external surface, with Hencky-Von Mises criterion	8,490	7,310	8,500	7,750
Circumferential stress at midbay on external surface	7,450	6,450	7,530	7,210
Circumferential stress†† adjacent to frames, on internal surface	7,380	6,450	7,870	7,060
Approximate experimental pressure at observed onset of yielding, psi	6,700	6,000	6,700	6,300
<p>* The weighted average yield strength of shell and frame material was used.</p> <p>** The numbers in parentheses indicate the number of lobes.</p> <p>† The yield strength of the shell material was used here.</p> <p>†† Theoretical circumferential stresses were higher than longitudinal stresses at all locations.</p>				

Figure 4 - Models After Collapse



Figure 4a - Model ML-1

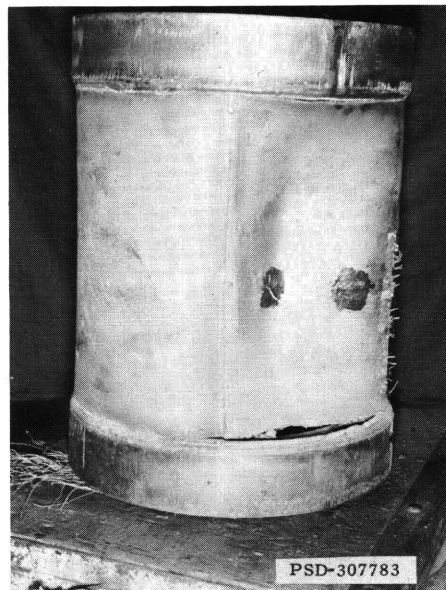


Figure 4b - Model ML-2

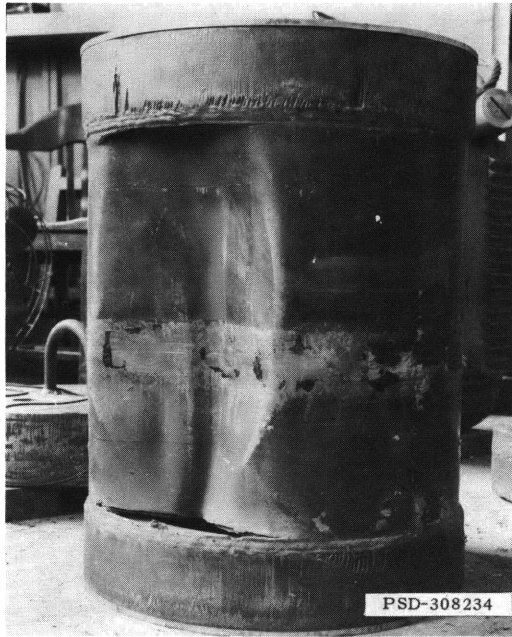


Figure 4c - Model ML-3



Figure 4d - Model ML-4

in compression midway between frames than at locations near the frames. (This was not shown for Model ML-4 because the distribution of strains between frames was not measured.) In addition, for Models ML-2 and ML-3, a distinct change of slope was discernible in most pressure-strain plots for inside longitudinal gages and for circumferential gages at approximately 2500 to 3000 psi. Typical pressure-strain plots for midbay locations of each model are given in Figure 5. The strain data indicate that there was probably some separation of the shell layers of Models ML-2 and ML-3 between the frames, principally because of distortion of the inner shells when the frames were welded in place. Some separation of the shell layers of Model ML-4 is also indicated by the difference in external and internal circumferential strain sensitivities, although it is not apparent in the behavior of the longitudinal strains. Close contact between the shell layers at the frames of the multilayered models is indicated by a comparison of external and internal circumferential strains. The internal surface strains are higher by approximately the amount that would be accounted for by the difference in radii of the inner frame and the outer shell surface.

INTERPRETATION OF RESULTS AND DISCUSSION

The appearance of the models after collapse and the fact that substantial yielding had occurred previously indicate that plastic-general instability was the mode of failure of all the models. A satisfactory method of computing theoretical plastic general instability collapse pressures for fabricated steel cylinders is not available at present. These collapse pressures are related to the collapse pressures in the elastic buckling and yielding modes, however, and calculated pressures for these modes of collapse are given in Table 2. The theoretical bases for these calculations are discussed in the Appendix. The calculated pressures must be considered merely as estimates, however, principally for comparative purposes, because the actual models did not completely satisfy the simplifying assumptions which were made to facilitate the analyses. In particular, it was assumed that the shell layers were in contact with each other over their entire mating surfaces. The test results indicate that for Models ML-2 and ML-3 there was some separation of the shell

Figure 5 - Typical Pressure-Strain Diagrams

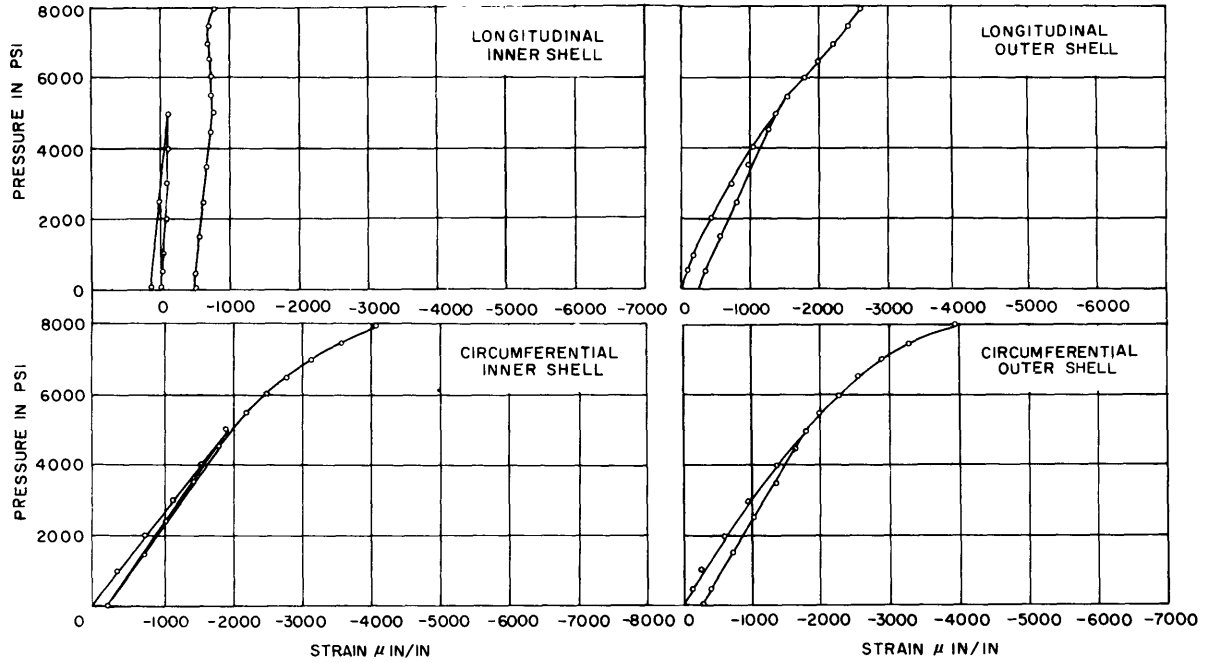


Figure 5a - Model ML-1 at Station 1 1/2 at 180 Degrees

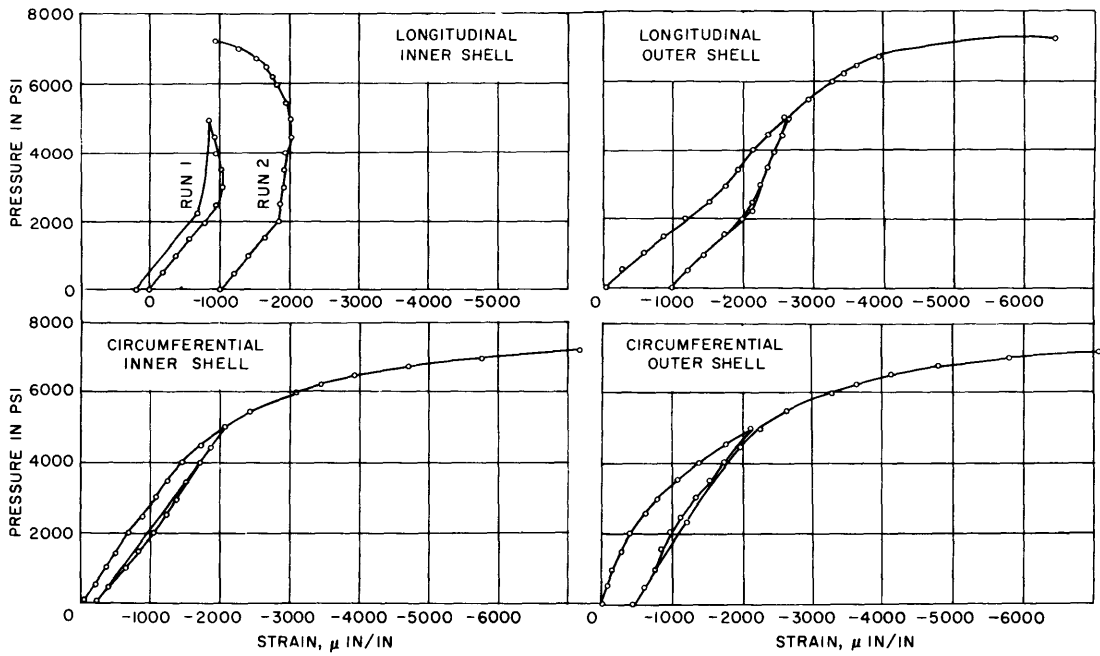


Figure 5b - Model ML-2 at Station 2 1/2 at 270 Degrees

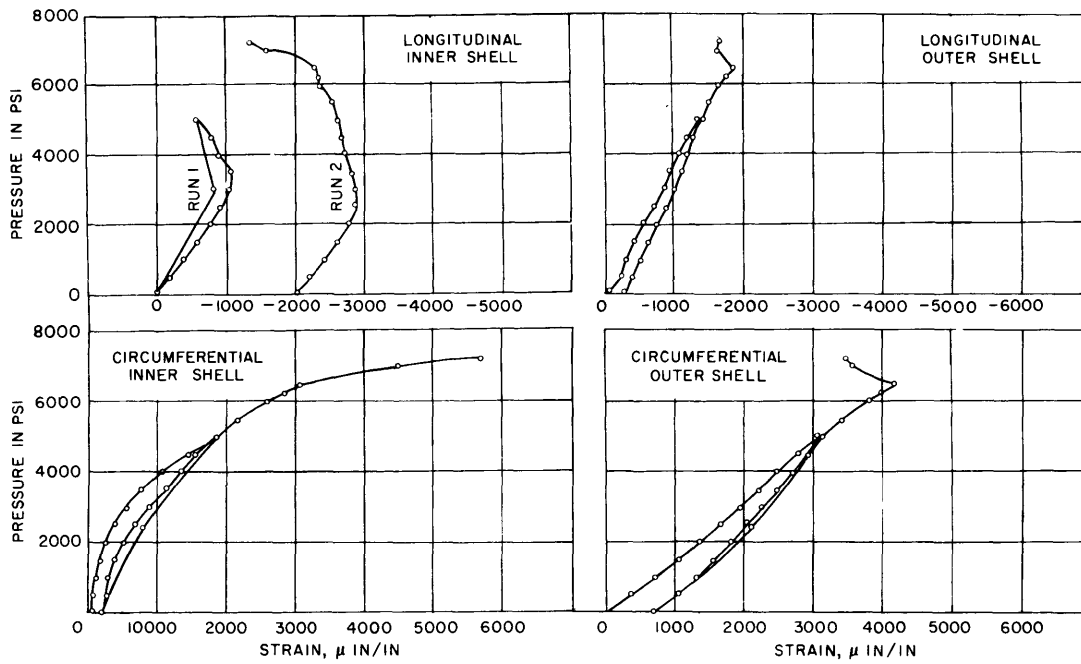


Figure 5c - Model ML-3 at Station 3 1/2 at 135 Degrees

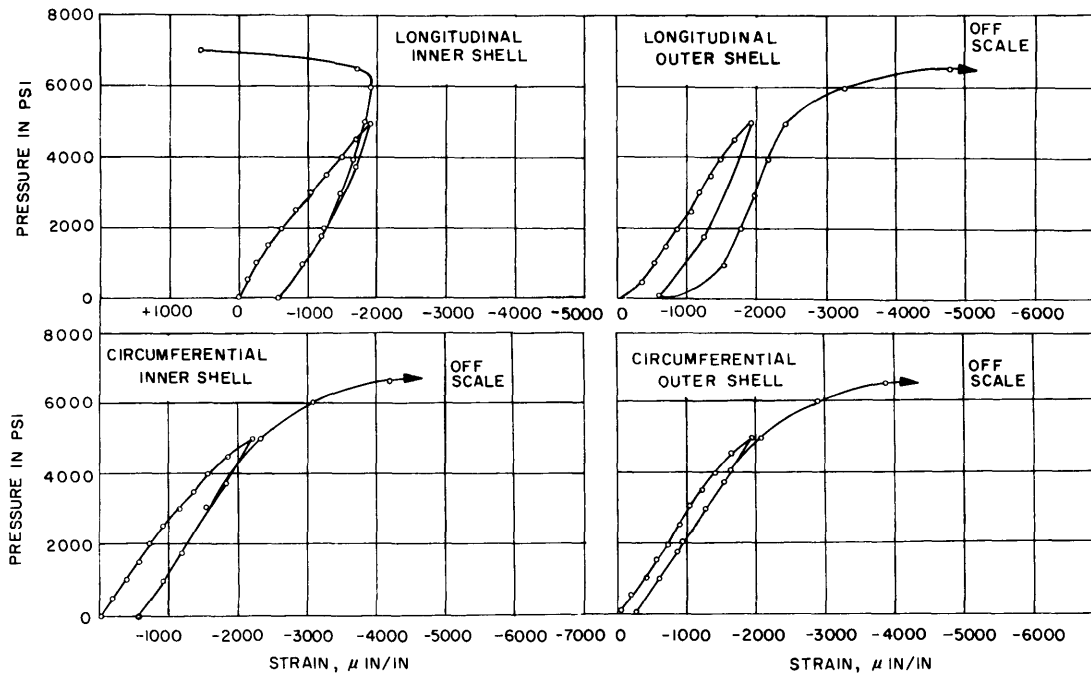


Figure 5d - Model ML-4 at Station 5 1/2 at 150 Degrees

layers until a pressure of about 2500 to 3000 psi was reached. There was also evidently some separation of the shell layers of Model ML-4. It was not apparent from the test results whether or not the layers were forced into complete contact during the test of this model. One other assumption which may not have been completely satisfied, particularly for the outer shell layers, was that the frames acted effectively as boundaries, in the longitudinal direction, of any lobes formed in the process of shell buckling. Probably the largest errors from these sources would appear in the calculated shell buckling and shell yielding pressures.

Stress calculations were carried out for all the models, and the methods used are also described in the Appendix. Calculated stresses at the critical locations were used for computing the various theoretical yielding pressures given in Table 2. Longitudinal stress and circumferential strain sensitivity distributions are also given in Figure 6. These calculated stresses and strains provide a useful comparison of the theoretical elastic behavior of the models. The experimental results did not provide an evaluation of these calculated values, largely because of the aforementioned imperfections. Also, there was too little bending between the frames of all the models, because of the relatively small size and close spacing of the frames, to evaluate the bending analysis.

It is also shown in Table 2 that yielding was observed in all models at pressures below those at which it should theoretically begin, even at the most highly stressed points. For Model ML-1, this may be attributed to residual stresses due to rolling and welding and possibly to out-of-roundness. For the other models, correspondingly high stresses probably existed for somewhat different reasons. Residual stresses in these models due to rolling would be expected to be lower because of the thinner plating and, of course, stresses caused by the circumferential frame welds would not exist in the outer layers. However, the outer layers underwent higher-than-theoretical initial circumferential strains until they were in complete contact with the inner layers. Thus, the total circumferential strains (and consequently stresses) in the outer layers were always greater than the theoretical values. In all cases frame yielding would be expected to follow closely behind shell yielding because of the relative lightness of the frames. It has been shown for

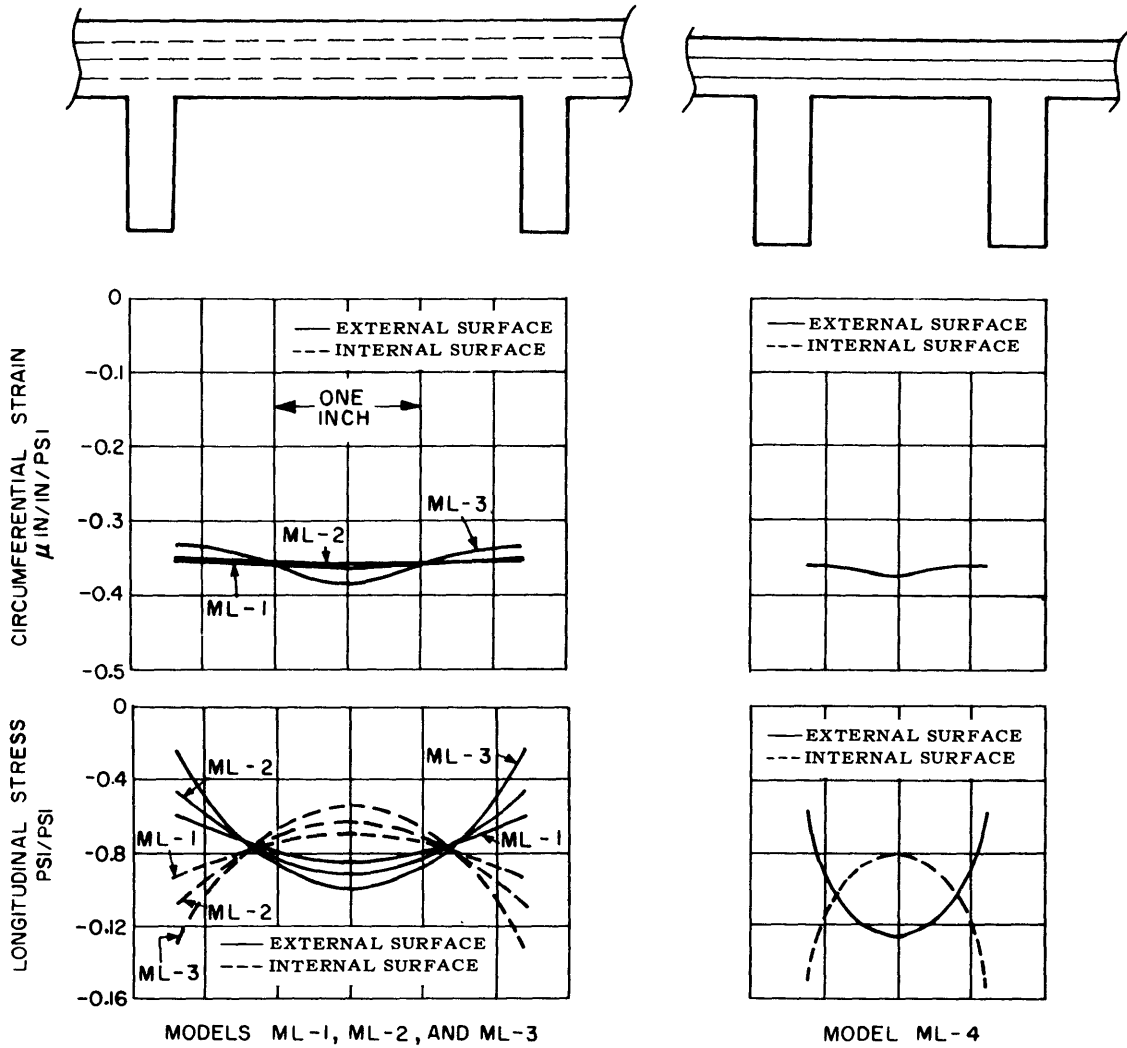


Figure 6 - Theoretical Stress and Strain Sensitivity Distribution for Typical Bays

various types of cylindrical shell structures^{2,3} that the extent of yielding a structure may endure before buckling in an inelastic mode is influenced by the ratio of theoretical elastic to inelastic buckling pressures in that mode. These ratios may be estimated for this series of models by taking the actual collapse pressures p_e as the inelastic general instability pressures and by comparing them with the calculated elastic general instability buckling pressures p_g . These p_g/p_e ratios, which vary from 2.5 to 4.1, are given in Table 2. Test results² indicate that the strength of fabricated (rolled and welded) cylindrical shell structures designed to fail in a buckling mode and having p_g/p_e ratios less than about 6 is likely to be reduced by residual stresses, out-of-roundness, and other imperfections. It appears likely, therefore, that the reasons for the rather low actual collapse pressures, compared with the various predicted yielding pressures (which are indicative of the lower limits of inelastic buckling pressures), are premature yielding and the lack of ability to sustain much yielding because of the moderately low p_g/p_e ratios.

The estimated shell buckling pressure for Model ML-3 is relatively quite low and leads to the speculation that the failure of this model could have been initiated by shell buckling. Then, because of the small size of the frames and a moderately large amount of energy in the test tank, the frames could have buckled and masked the original shell buckling failure. There is nothing in the strain data or in the appearance of the model after failure to verify this, however, and as stated previously, the reliability of the modified shell buckling formula used for this estimate must be regarded as questionable.

Model ML-4 was designed after tests of the other models were completed. It represented an attempted optimum design of a model having the same weight as the others and with the arbitrary limitation that the thickness of the individual shell layers be no greater than that of Model ML-3. The depth and closeness of spacing of the frames was limited to some extent by practical fabrication considerations. Since a need for more frame strength was apparent, it was decided to use only three layers for the shell and to replace the material with larger and more numerous frames. The shorter frame spacing would offset the tendency of the thinner

shell to fail, and also it was expected to minimize the shell separation problem. One other consideration was that the frames not be too large since, to some extent, the same material procurement and fabrication problems would apply to them as to the shell in a prototype vehicle.

The collapse strength of Model ML-4 was slightly less than that of Model ML-3. When a linear correction is made for the actual yield strengths of the materials and shell thicknesses of the models, the collapse pressures of the two models are practically identical. The ratios of elastic buckling pressure in both the general instability and the shell buckling modes to the actual collapse pressures are higher for Model ML-4 than for Model ML-3. The relatively low collapse pressure for Model ML-4 can probably be attributed partly to the fact that shell stresses, both predicted and observed, were higher than those in Model ML-3. The p_g/p_e ratio for Model ML-2 was somewhat lower than for Model ML-4, however, and the observed initial yielding pressure was lower (because of the lower shell material yield strength); nevertheless, its collapse strength was slightly higher.

A meaningful comparison of collapse pressures of models which fail in a plastic mode is often obtained by applying a linear correction to all the actual collapse pressures for a single common yield strength, and for actual shell thicknesses. This comparison is shown in Table 2, wherein all the experimental collapse pressures are corrected for a yield strength of 100,000 psi, but is probably relevant here only for Models ML-1 and ML-2. The collapse pressure of Model ML-2 would almost certainly have been higher if its yield strength had been increased to 100,000 psi, but it is doubtful that it would reach the 8170 psi given in Table 2 because the p_g/p_e ratio would have decreased somewhat. As discussed previously, the value of this ratio is significant if it is less than about six. Since one of the primary objectives of the multilayer concept is to permit the use of higher strength material, a comparison of the actual collapse pressures of Models ML-3 and ML-4 with those of Models ML-1 and ML-2 is probably most appropriate. Considering Models ML-2, ML-3, and ML-4, which failed at essentially the same pressures, it can be concluded that the strength of a two-layered shell of nearly optimum design, and having a yield strength of 88,600 psi could be matched by the thinner-layered models

by a sufficient increase of their yield strengths. The similarity of the collapse pressures of Models ML-3 and ML-4 indicates their designs are both probably nearly optimum, with the scantlings being not too critical within that range of collapse pressure and material yield strength. Obviously their yield strengths must have exceeded that of Model ML-1 by a much greater margin if they were to achieve the same collapse strength, and it has not been proven that this is possible. For a higher yield strength, a design similar to that of Model ML-4, which allows higher shell stresses, but has a greater margin of stability, would probably be superior to that of Model ML-3. It must be remembered, however, that the material procurement and fabrication of the larger frames in a prototype vehicle could be a problem. The deeper frames also may reduce the usable space inside the vehicle.

In further tests of multilayered cylinders, fabrication of all models and proposed prototypes should incorporate a solution of the problem of initial separation of the shell. Probably the most simple solution would be to inject a plastic filler material between the layers after the cylinder is fabricated. This material need not necessarily provide any significant bonding but would merely be required to transmit a share of the pressure load from the outer to the inner shell layers.

The feasibility and effects of various degrees of bonding, both continuous and intermittent, should also be investigated. For instance, if the shell layers could be welded or keyed together at each frame location, the bending moment of inertia of the shell-frame combination in the circumferential direction, and hence, the elastic general instability collapse strength, would be increased. Also, positive boundaries would be provided in the longitudinal direction for the outer shell layers against shell buckling.

CONCLUSIONS

1. The strength of the single-layer cylinder having a yield strength of 99,200 psi would be reduced by somewhat more than 3.4 percent by replacing the shell with one of two equal layers having the same yield strength.

2. The strength of the two-layered cylinder with a shell yield strength of 88,600 psi was practically achieved in the four- and three-layered cylinders having individual layers of one-half the thickness of that of the individual layers of the two-layered cylinder. This was done by increasing the yield strength of the shell material of the four- and three-layered cylinders by 12.4 and 12.1 percent, respectively.

3. The strengths of the multilayered cylinders were significantly reduced by the lack of perfect contact of the shell layers along their mating surfaces.

4. A systematic series of models should be tested to determine the increase in yield strength required to duplicate the strength of single-layered cylinders with multilayered cylinders, in various yield-strength ranges. Various weight-to-displacement ratios should be investigated and the number of layers used should be held to the maximum estimated to be required in the prototype.

5. More adequate theories should be developed for predicting the structural behavior of multilayered cylinders. In particular, reliable methods of calculating buckling pressures are needed.

APPENDIX
THEORETICAL CALCULATIONS*

AXISYMMETRIC STRESS ANALYSIS

It will be assumed that the extensional rigidity of a shell is unaffected by lamination of the shell. Except for special conditions of loading and restraint, however, the bending rigidity is altered. It will be seen by inspection that if the layers are free to slide over each other, the flexural rigidity is proportional to the sum of the moments of inertia of the individual layers. For a monolithic shell:

$$D = \frac{E h^3}{12(1-\nu^2)} \quad [1]$$

where D is the flexural rigidity,
E is Young's modulus,
h is total thickness, and
 ν is Poisson's ratio

For a laminated shell:

$$D_e = \frac{D}{j^2} \quad [2]$$

where D_e is effective flexural rigidity of a laminated shell and j is the number of layers.

If the beam-on-an-elastic-foundation approach is followed to determine the axisymmetric behavior of the cylindrical shells, we note that the foundation modulus, k, being an extensional term, is unaffected by lamination of the shell.

$$k = \frac{Eh}{R^2} \quad [3]$$

where R is the shell radius.

* Figure 7 illustrates the nomenclature used here.

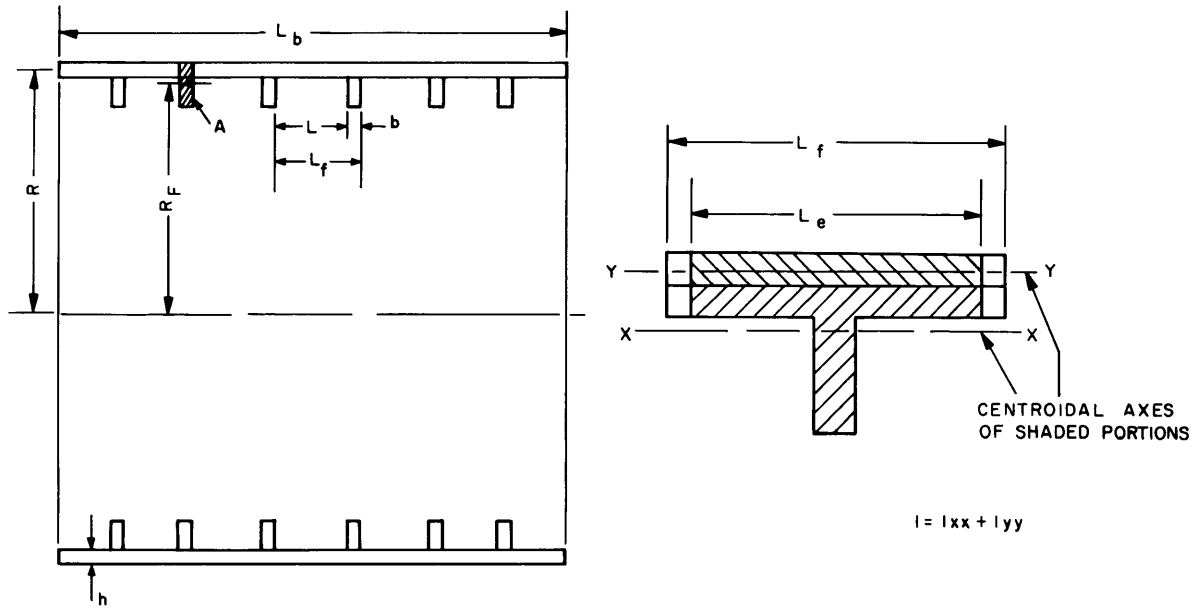


Figure 7 - Nomenclature for Theoretical Calculations

Thus the parameter

$$\beta = \sqrt[4]{k/4D} \quad [4]$$

for a monolithic shell is modified by the factor \sqrt{j} for a laminated shell.

$$\beta_e = \sqrt[4]{\frac{3j^2(1-\nu^2)}{R^2 h^2}} \quad [5]$$

where β_e is the effective β for a laminated shell.

The differential equation of equilibrium for a laminated cylindrical shell is the same as that for a monolithic shell, except that β is replaced with β_e . Thus, the homogeneous form of the equation is

$$\frac{d^4 w}{dx^4} + 4 \beta_e^4 w = 0 \quad [6]$$

where w is radial deflection and x is distance along the shell. The stress analysis of the monolithic cylinder may therefore be used by replacing D and β with D_e and β_e .

Formulas for the relevant stresses and strains thus derived may be expressed in the following form to facilitate the calculations for cylinders with close frame spacing ($\beta_e \ell \leq 3$).

At the edge of the shell, adjacent to the frame:

$$\varepsilon_\theta = -\frac{pR}{Eh} (1 - \nu/2) + \frac{2 HR}{E h \ell} \frac{S_2}{S_1} \quad [7]$$

$$\sigma_x = -\frac{pR}{2h} \pm \frac{\ell H j}{h^2} \frac{S_3}{S_1}$$

and at midbay:

$$\varepsilon_\theta = -\frac{pR}{Eh} (1 - \nu/2) + \frac{2 RH}{Eh \ell} \frac{S_4}{S_1}$$

$$\sigma_x = - \frac{pR}{2h} \mp \frac{\ell H_j}{2h^2} \cdot \frac{S_5}{S_1}$$

where

$$H = \frac{p\ell}{2} \frac{(1 - \nu/2) (1 - bh/A \cdot R_F/R)}{h\ell/A \cdot R_F/R + S_2/S_1},$$

ε_θ is the circumferential strain, tension positive,

σ_x is the longitudinal stress, tension positive,

p is the external pressure,

ℓ is the unsupported length of the shell,

A is the cross-sectional area of the frame and faying width of the shell combination,

b is the faying width of the shell, and

R_F is the radius of the centroid of the frame and faying width of the shell combination.

$$S_1 = 1 + 2 \frac{(\beta_e \ell)^4}{6!} + 2 \frac{(\beta_e \ell)^8}{10!} + \dots$$

$$S_2 = 1 + \frac{(\beta_e \ell)^4}{5!} + \frac{(\beta_e \ell)^8}{9!} + \dots$$

$$S_3 = 1 + \frac{6(\beta_e \ell)^4}{7!} + \frac{6(\beta_e \ell)^8}{11!} + \dots$$

$$S_4 = 1 - \frac{(\beta_e \ell)^4}{4 \cdot 5!} + \frac{(\beta_e \ell)^8}{4^2 \cdot 9!} - \dots$$

$$S_5 = 1 - \frac{3}{2} \frac{(\beta_e \ell)^4}{7!} + \frac{3}{2^3} \frac{(\beta_e \ell)^8}{11!} - \dots$$

and the upper signs in the expression σ_x apply to the external surface, and the lower signs to the internal surface. The other stresses and strains may be determined from Hooke's law.

The results of this method as applied to a typical bay (where the frames do not rotate) are identical to those obtained by considering a shell composed of only one of the layers, reinforced by frames of $1/j$ times the cross-sectional area of the actual frame, and subjected to $1/j$ times the actual pressure applied.

BUCKLING ANALYSIS

An equation developed by Bryant⁴ may be modified to give an estimated general instability collapse pressure of a multilayered cylinder. This equation consists of two terms: one extensional term, unaffected by lamination of the shell, and one bending term. This equation is

$$p_{cr} = \frac{Eh}{R} \left[\frac{\alpha^4}{(n^2 + \frac{1}{2}\alpha^2 - 1)(n^2 + \alpha^2)^2} \right] + \frac{I_e E}{L_f R^3} (n^2 - 1) \quad [8]$$

where n is the mode of collapse ($n = 2, 3, 4$, etc.),

$$\alpha = \frac{\pi R}{L_b},$$

L_b is the bulkhead spacing in inches,

L_f is the frame spacing in inches,

I_e is the moment of inertia of the combined section of one frame plus an effective length of shell L_e , and

$$L_e = \frac{S_1}{S_2} L + b.$$

To adapt formula [8] to laminated shells, I_e may be taken as the sum of the moments of inertia of the frame-inner shell layer combination, and of each of the remaining layers. An effective length, L_e , of shell is taken in all cases and the moment of inertia of each component is taken about its own centroid.

The procedure for estimating the elastic shell buckling pressures was somewhat simpler. The Von Mises⁵ formula may be adapted to multilayered shells in a way similar to the method used for the general instability pressure. This formula, in a somewhat simplified version, also may be

considered to consist of an extensional and a bending term. The bending term contains an h^3 in its numerator, which represents the moment of inertia of a unit length of shell about its centroid. This term may be modified to represent the sum of the moments of inertia of the individual layers by dividing it by j^2 . Thus we have

$$p = \left\{ \frac{1}{3} \left[n^2 + \left(\frac{\pi R}{L} \right)^2 \right]^2 \frac{2E}{(1 - \nu^2)j^2} \left(\frac{h}{2R} \right)^3 + \frac{E h/R}{\left[n^2 \left(\frac{L}{\pi R} \right)^2 + 1 \right]^2} \right\} \frac{1}{n^2 + \frac{1}{2} \left(\frac{\pi R}{L} \right)^2} \quad [9]$$

For the multilayered models, it may be somewhat more realistic to use L_f in place of L , since the outer shell layers are not attached to the frames.

REFERENCES

1. Raetz, R. V., "Multilayered Shells for Pressure Hulls," Proceedings of the David Taylor Model Basin Workshop on Deep-Submergence Hulls (U) 3, 4, 5 Dec 1962, David Taylor Model Basin Report C-1327, Vol. I (Mar 1963) CONFIDENTIAL.
2. Krenzke, M. A., et al., "Potential Hull Structures for Rescue and Search Vehicles of the Deep Submergence Systems Project," David Taylor Model Basin Report 1985 (Mar 1965).
3. Hom, K. and Blumenberg, W. F., "Hydrostatic Tests of Structural Models for Preliminary Design of a Web-Stiffened Sandwich Pressure Hull," David Taylor Model Basin Report 1763 (Sep 1963).
4. Ball, W. E., Jr., "Formulas and Curves for Determining the Elastic General-Instability Pressures of Ring-Stiffened Cylinders," David Taylor Model Basin Report 1570 (Jan 1962).
5. Windenburg, D. F. and Trilling, C., "Collapse by Instability of Thin Cylindrical Shells Under External Pressure," Experimental Model Basin Report 385 (Jul 1934).

INITIAL DISTRIBUTION

Copies		Copies	
17	CHBUSHIPS	1	SUPSHIP, Groton
	2 Sci & Res (Code 442)	1	EB Div, Gen Dyn Corp
	1 Chief Scientist for R&D (Code 305)	1	SUPSHIP, Newport News
	1 Lab Mgt (Code 320)	1	NNSB & DD Co
	3 Tech Lib (Code 210L)	1	SUPSHIP, Pascagoula
	1 Struc Mech, Hull Mat & Fab (Code 341A)	1	Ingalls Shipbldg Corp
	1 Struc & Ship Protection Sec (Code 342C)	1	SUPSHIP, Camden
	1 Prelim Des Br (Code 420)	1	New York Shipbldg
	1 Prelim Des Sec (Code 421)	1	DIR DEF, R&E, Attn: Tech Lib
	1 Ship Protec (Code 423)	1	CO, USNROTC & NAVADMINU, MIT
	1 Hull Des Br (Code 440)	1	O in C, PGSCOL, Webb
	1 Struc Sec (Code 443)	1	CO, PGSCOL, Monterey
	2 Sub Br (Code 525)	1	DIR, APL, Univ of Washington, Seattle
	1 Pres Ves Sec (Code 651F)	1	NAS, Attn: Comm on Undersea Warfare
3	CHONR	1	Prof. J. Kempner, Polytechnic Inst of Brooklyn
	1 Res Coordinator (Code 104)	1	Dr. R. DeHart, SWRI
	1 Struc Mech Br (Code 439)	1	Mr. Leonard P. Zick, Chicago Bridge & Iron Co.
	1 Undersea Programs (Code 466)	1	V. L. Salerno, Applied Technology Assoc.
1	CHBUWEPS, Res Div (Code RRRE)	1	Prof. E. O. Waters, Yale Univ
4	CNO	2	Mr. C. F. Larson, Sec, Welding Res Council
	1 Plans, Programs & Req Br (Op 311)	1	Prof. Bernard Budiansky, Harvard Univ
	1 Tech Anal & Adv Gr (Op 07T)	1	Mr. J. Mavor, WHOI
	1 Sub Program Br (Op 713)		
	1 Tech Support Br (Op 725)		
20	CDR, DDC		
1	CO & DIR, USNMEL		
1	CDR, USNOL		
1	DIR, USNRL (Code 2027)		
1	CO & DIR, USNUSL		
1	CO & DIR, USNEL		
1	CDR, USNOTS, China Lake		
1	CDR, USNOTS, Pasadena		
1	CO, USNUOS		
2	NAVSHIPYD PTSMH		
2	NAVSHIPYD MARE		
1	NAVSHIPYD CHASN		

DOCUMENT CONTROL DATA - R&D		
<i>(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)</i>		
1 ORIGINATING ACTIVITY <i>(Corporate author)</i> Department of the Navy David Taylor Model Basin Washington, D.C. 20007		2a REPORT SECURITY CLASSIFICATION Unclassified
		2b GROUP
3 REPORT TITLE TESTS OF FABRICATED MULTILAYERED RING-STIFFENED CYLINDRICAL MODELS UNDER EXTERNAL HYDROSTATIC PRESSURE		
4 DESCRIPTIVE NOTES <i>(Type of report and inclusive dates)</i> Final		
5 AUTHOR(S) <i>(Last name, first name, initial)</i> Raetz, Richard V.		
6 REPORT DATE April 1966	7a TOTAL NO. OF PAGES 29	7b NO. OF REFS 5
8a. CONTRACT OR GRANT NO.	9a ORIGINATOR'S REPORT NUMBER(S) 2173	
b. PROJECT NO. S-R011 01 01, Task 0401		
c.	9b. OTHER REPORT NO(S) <i>(Any other numbers that may be assigned this report)</i>	
d.		
10 AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited.		
11 SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY David Taylor Model Basin Director's Research	
13 ABSTRACT <p>An exploratory investigation was carried out to study the strength of ring-stiffened multilayered cylindrical shells under external hydrostatic pressure. Four HY-100 steel models, having relatively large shell thickness-to-diameter ratios, were tested to collapse. The weight-to-displacement ratios for all models were approximately equal. The tests indicated that for the weight-to-displacement ratio studied, the strength of a single-layered ring-stiffened cylindrical model could be attained by a model with a shell consisting of two layers, each layer one-half the single-layered shell thickness, with a moderate increase in shell yield strength. It was also indicated that the strength of the single-layered model could not be attained in a model with a shell consisting of layers of one-fourth the single-layered shell thickness without a very large increase in the yield strength of both shell and frame material.</p>		

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Multilayered Ring-Stiffened Cylindrical Shells						
Inelastic Buckling of Multilayered Ring-Stiffened Cylindrical Shells						
Collapse of Multilayered Ring- Stiffened Cylindrical Shells						
Stress Analysis of Multilayered Ring-Stiffened Cylindrical Shells						

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.
- 2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.
- 7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.
- 8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.
- 9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).
10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.
12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.
13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.
It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).
There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.
14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

David Taylor Model Basin. Report 2173.
TESTS OF FABRICATED MULTILAYERED RING-STIFFENED
CYLINDRICAL MODELS UNDER EXTERNAL HYDROSTATIC
PRESSURE, by Richard V. Raetz. Apr 1966. ii, 27p.
illus., graphs, refs. UNCLASSIFIED

An exploratory investigation was carried out to study the strength of ring-stiffened multilayered cylindrical shells under external hydrostatic pressure. Four HY-100 steel models, having relatively large shell thickness-to-diameter ratios, were tested to collapse. The weight-to-displacement ratios for all models were approximately equal. The tests indicated that for the weight-to-displacement ratio studied, the strength of a single-layered ring-stiffened cylindrical model could be attained by a model with a shell consisting of two layers, each

1. Cylindrical shells (Stiffened)--Hydrostatic pressure--Model tests
2. Cylindrical shells (Stiffened)--Collapse--Model tests
3. Cylindrical shells (Stiffened)--Structural analysis--Model tests
4. HY-100 steel--Applications
I. Raetz, Richard V.
II. S-R011 01 01;
Task 0401

layer one-half the single-layered shell thickness, with a moderate increase in shell yield strength. It was also indicated that the strength of the single-layered model could not be attained in a model with a shell consisting of layers of one-fourth the single-layered shell thickness without a very large increase in the yield strength of both shell and frame material.

III. T: Multilayered
ring-stiffened
cylindrical shells

MIT LIBRARIES DUPL

3 9080 02753 0648

Date Due		
JAN 25 2006		
		Lib-26-67

