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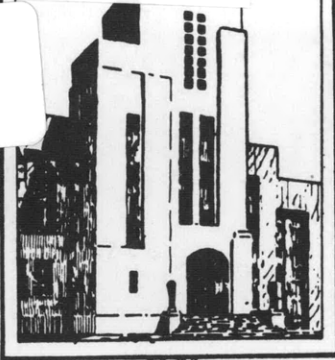
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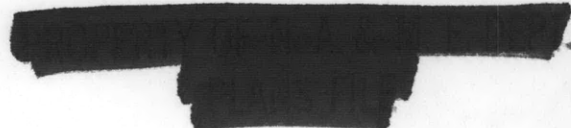


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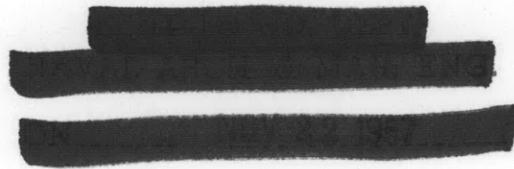
**CALCULATION OF TRANSIENT EXCITATION OF SHIP HULLS
BY FINITE DIFFERENCE METHODS**

AERODYNAMICS



by

Harry Polachek



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Calculation of Transient Excitation of Ship Hulls
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Abstract

A system of finite difference equations based on the non-uniform beam theory is developed for use in the calculation of the response of a ship hull to transient forces. The conditions for stability of these equations (and hence the conditions for validity of the numerical results) are derived. The feasibility of the method is tested by the solution of a vibration problem for a specific hull, the details of which are discussed in the present report and by Mr. R. T. McGoldrick ¹⁾ in TMB Report 1119, "Calculation of the Response of a Ship Hull to a Transient Load by a Digital Process". The use of this method lends itself to the solution of a wide class of problems related to the structural design of vessel hulls or other structures subject to transient forces. The solution has been programmed and carried out on the Bureau of Ships UNIVAC System, Applied Mathematics Laboratory, David Taylor Model Basin.

Introduction

The mathematical equations governing the solution of engineering problems in many broad areas of application are systems of partial differential equations. Supersonic and subsonic aerodynamics, nuclear reactor design theory, heat flow, propagation of electromagnetic and acoustic waves used in the design of radar and sonar equipment are but a few illustrative fields of engineering which fall in this category.

It is not generally realized that the solution of systems of differential equations by classical methods in terms of known mathematical functions or otherwise is extremely difficult and can only be carried out in the case of a few isolated and very simple problems. In general, it has not been possible in the past to obtain practical numerical results based on the fundamental systems of differential equations governing the behavior of material or devices under design study by engineers and scientists, although the basic equations have in most cases been derived many years ago. For this reason the design engineer must resort to many simplifying assumptions and approximations, which are not always valid, in order to find solutions based on theoretical considerations, and he must rely heavily on experimental data.

One of the basic functions of the Applied Mathematics Laboratory is to develop methods which will permit the direct numerical solution of systems of differential equations, particularly those types which govern

technical problems encountered in naval engineering and architecture. Success in this field will be the key to the solution of many outstanding naval engineering problems.

The rapid development of high-speed automatic digital calculators has given the mathematician a powerful tool which enables him to attack the direct solution of complex systems of partial differential equations with renewed vigor, and with greatly improved prospects for success. The most promising possibility for making progress in this effort is the use of finite difference methods. This is basically a very simple and direct approach to the solution of this problem, and for this reason has great merits. One additional advantage inherent in the method of finite differences is its broad generality.

The method of finite differences consists simply in replacing the derivatives of various orders in the differential system of equations by equivalent ratios of finite increments. For instance, the first derivative $\frac{dy}{dx}$ may be replaced by $\frac{\Delta y}{\Delta x}$, where Δy , Δx , are small but finite increments. This substitution results in a system of so called difference equations, which is algebraic in form, and hence easier to solve. It represents the original differential system of equations approximately. Although the finite difference equation system is only an approximate representation of the differential equations, its accuracy may under proper conditions which may be determined mathematically, be progressively improved by using increasingly smaller increments. In this manner, any desired accuracy may be achieved.

The mathematician has two major tasks in successfully applying finite difference methods to the solution of systems of partial differential equations, one theoretical and one practical. First, in order for the technique to be valid, he must find the conditions under which a decrease in the size of the increments will necessarily produce an improvement in the accuracy of the solution, so that any desired accuracy may be obtained. Unfortunately, this does not always occur. As a matter of fact, in many instances, the use of the method of finite differences produces entirely erroneous and irrelevant results. Secondly, since the sizes of the increments must in general be chosen small in order to obtain reasonably accurate results, the amount of computation required is usually extremely large, and in many cases far beyond the capacity of human endeavor. For this reason, the practical use of finite difference methods is in many areas only feasible by the use of a very rapid automatic high-speed calculator.

In this paper the method of finite differences is applied to the calculation of the motion of a ship hull which is subjected to a transient force. For the purpose of this study the ship hull is subdivided into twenty sections or increments and is assumed to behave like a free non-uniform beam. It is also assumed that a force which varies in time is applied to one or more of these sections. The final results obtained are a set of printed tables which give the locations, the deflections, the angular and translational velocities and accelerations, and the forces acting on each of the sections at any instant of time following the initiation of the transient excitation.

Governing Equations

The equations governing the motion of a ship hull based on uniform and non-uniform beam theory as developed by Timoshenko ²⁾ and others are discussed in considerable detail by R. T. McGoldrick and V. L. Russo ³⁾ in reference 3. A finite difference method for obtaining numerical solutions to these equations is presented here. For this purpose, the equations describing the damped vertical (or torsion-free horizontal) excitation of a ship hull subjected to a transient force will be used. The results may be directly extended to more general types of motion. The system of partial differential equations describing this type of motion, as given in reference 3 is:

$$(1) \quad M \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} + \frac{\partial V}{\partial x} = P(x, t)$$

$$(2) \quad I_{Mz} \frac{\partial^2 r}{\partial t^2} + V - \frac{\partial M}{\partial x} = 0$$

$$(3) \quad M = (EI) \frac{\partial r}{\partial x}$$

$$(4) \quad V = (KAG)r - (KAG) \frac{\partial y}{\partial x}$$

where,

t = time

x = distance coordinate along the longitudinal axis of a vessel

y = displacement normal to the longitudinal axis

γ = rotation of transverse section about an axis normal to the (x y) plane (z-axis)

M = bending moment

V = net shear force in y direction

\mathcal{M} = apparent mass (per unit length)

c = damping factor (per unit length)

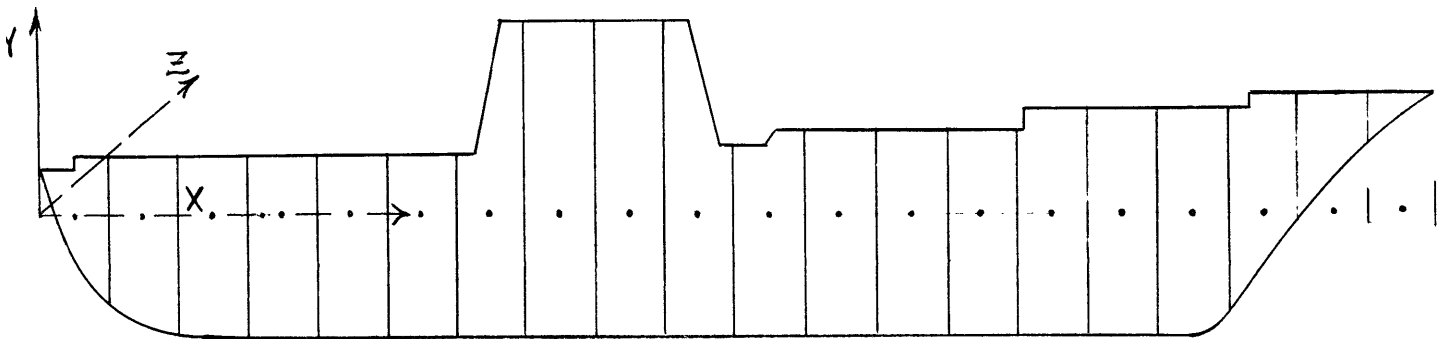
P = force (per unit length) acting upon ship hull

$I_{\mathcal{M}z}$ = mass moment of inertia about z axis (per unit length) - rotary inertia

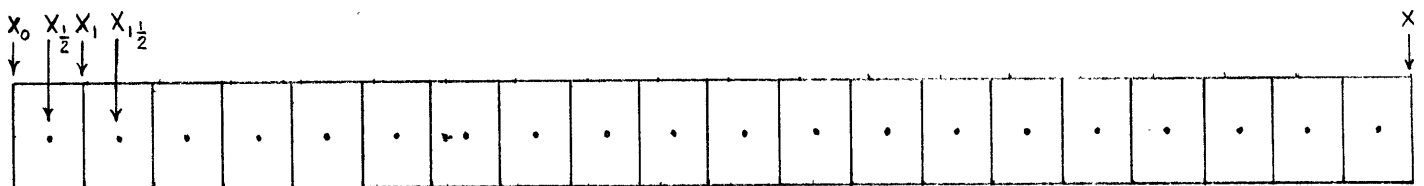
(EI) = bending rigidity factor

(KAG) = shearing rigidity factor.

Figure 1



1a - Ship hull



1b - Representation by non-uniform segmented beam

Finite Difference Representation

The set of equations (1) to (4) constitutes a system of partial differential equations which is assumed to govern the motion of the hull of a vessel, as simulated by a freely vibrating non-uniform beam. If the state of motion of the ship hull is known at any time t_0 , it is possible by obtaining the solution to these equations to determine its motion at any subsequent time. This system of equations may be represented approximately in finite difference form by replacing the partial derivatives by equivalent ratios of small finite increments. In making this substitution we will use the following notation:

t_0 ——— initial time

x_0 ——— initial position

Δt = increment in time

Δx = increment in length

$\Delta y, \Delta \gamma, \Delta M, \Delta V$ = corresponding increments in the dependent variables y, γ, M and V , respectively.

$$x_n = x_0 + n \Delta x$$

$$y_{n + \frac{1}{2}}^m = y \left[x_0 + \left(n + \frac{1}{2} \right) \Delta x, t_0 + m \Delta t \right] = \text{value of } y \text{ at the position}$$

$$x_0 + \left(n + \frac{1}{2} \right) \Delta x \text{ and at the time } t_0 + m \Delta t; \begin{cases} n = 0, 1, 2, \dots \\ m = 0, 1, 2, \dots \end{cases}$$

$$M_n^{m+1} = M \left[x_0 + n \Delta x, t_0 + (m+1) \Delta t \right] = \text{value of } M \text{ at the position}$$

$$x_0 + n \Delta x \text{ and at the time } t_0 + (m+1) \Delta t; \begin{cases} n = 0, 1, 2, \dots \\ m = 0, 1, 2, \dots \end{cases}$$

Similar notation will be used for other variables and factors in the equations.

For quantities which do not vary with time, the superscript will be omitted.

Figures 1a and 1b show the representation of a ship hull by means of a non-uniform beam and its division in twenty increments, as used in the initial solution. Using the above notation, the displacement of the hull at the position x_2 (the boundary point between the second and third increments) at the end of the m th time increment, for instance, will be represented by y_2^m ; its displacement at the center of the second increment will be represented by $y_{1\frac{1}{2}}^m$. The average mass per unit length at the center of the twentieth increment will similarly be represented by $M_{19.5}$.

To solve the above system of equations we propose to make the following specific finite difference substitutions in the system of equations (1) to (4):

$$(5) \quad \left(\frac{\partial^2 y}{\partial t^2} \right)_{n+\frac{1}{2}}^m = \frac{y_{n+\frac{1}{2}}^{m+1} - 2y_{n+\frac{1}{2}}^m + y_{n+\frac{1}{2}}^{m-1}}{(\Delta t)^2}$$

$$(6) \quad \left(\frac{\partial^2 \gamma}{\partial t^2} \right)_{n+\frac{1}{2}}^m = \frac{\gamma_{n+\frac{1}{2}}^{m+1} - 2\gamma_{n+\frac{1}{2}}^m + \gamma_{n+\frac{1}{2}}^{m-1}}{(\Delta t)^2}$$

$$(7) \quad \left(\frac{\partial y}{\partial t} \right)_{n+\frac{1}{2}}^m = \frac{y_{n+\frac{1}{2}}^m - y_{n+\frac{1}{2}}^{m-1}}{(\Delta t)}$$

$$(8) \left(\frac{\partial V}{\partial x} \right)_{n+\frac{1}{2}}^m = \frac{V_{n+1}^m - V_n^m}{\Delta x}$$

$$(9) \left(\frac{\partial M}{\partial x} \right)_{n+\frac{1}{2}}^m = \frac{M_{n+1}^m - M_n^m}{\Delta x}$$

$$(10) \left(\frac{\partial y}{\partial x} \right)_n^{m+1} = \frac{y_{n+\frac{1}{2}}^{m+1} - y_{n-\frac{1}{2}}^{m+1}}{\Delta x}$$

$$(11) \left(\frac{\partial \gamma}{\partial x} \right)_n^{m+1} = \frac{\gamma_{n+\frac{1}{2}}^{m+1} - \gamma_{n-\frac{1}{2}}^{m+1}}{\Delta x}$$

The resulting system of finite difference equations is given below:

$$(12) y_{n+\frac{1}{2}}^{m+1} = (2y_{n+\frac{1}{2}}^m - y_{n+\frac{1}{2}}^{m-1}) - \left(\frac{C_{n+\frac{1}{2}}}{M_{n+\frac{1}{2}}} \right) (y_{n+\frac{1}{2}}^m - y_{n+\frac{1}{2}}^{m-1}) \Delta t \\ - \frac{(V_{n+1}^m - V_n^m)}{M_{n+\frac{1}{2}}} \frac{(\Delta t)^2}{\Delta x} + P_{n+\frac{1}{2}}^m \frac{(\Delta t)^2}{M_{n+\frac{1}{2}}}$$

$$(13) \gamma_{n+\frac{1}{2}}^{m+1} = (2\gamma_{n+\frac{1}{2}}^m - \gamma_{n+\frac{1}{2}}^{m-1}) + \frac{M_{n+1}^m - M_n^m}{(I_{\mu z})_{n+\frac{1}{2}}} \frac{(\Delta t)^2}{\Delta x} - \frac{V_{n+\frac{1}{2}}^m (\Delta t)^2}{(I_{\mu z})_{n+\frac{1}{2}}}$$

$$(14) M_n^{m+1} = (EI)_n \frac{\gamma_{n+\frac{1}{2}}^{m+1} - \gamma_{n-\frac{1}{2}}^{m+1}}{\Delta x}$$

$$(15) V_n^{m+1} = (KAG)_n Y_n^{m+1} - (KAG)_n \frac{y_{n+\frac{1}{2}}^{m+1} - y_{n-\frac{1}{2}}^{m+1}}{\Delta x}$$

where

$$Y_n^m = \frac{1}{2}(Y_{n+\frac{1}{2}}^m + Y_{n-\frac{1}{2}}^m), \quad V_{n+\frac{1}{2}}^m = \frac{1}{2}(V_n^m + V_{n+1}^m) .$$

In order to define completely the motion of the ship hull (non-uniform beam) it is required that initial and boundary conditions be fixed. If we begin our computation when the vessel is at rest (just prior to subjecting it to any force) we have the condition at $t_0 = 0$, $y = \gamma = \frac{dy}{dt} = \frac{d\gamma}{dt} = V = M = 0$. At all time, t , we have the condition $M_0 = V_0 = M_{20} = V_{20} = 0$, on the basis of our assumption that the hull is vibrating freely.

The above system of equations (equations (12) to (15)) taken together with the supplementary initial and boundary conditions may be summarized in the following form used in programming the solution on a high-speed digital calculator (UNIVAC System):

$$(12') y_{n+\frac{1}{2}}^{m+1} = (1 + K_1)(y_{n+\frac{1}{2}}^m - y_{n+\frac{1}{2}}^{m-1}) + y_{n+\frac{1}{2}}^m + K_2(V_{n+1}^m - V_n^m) + K_3$$

$$(13') \gamma_{n+\frac{1}{2}}^{m+1} = (\gamma_{n+\frac{1}{2}}^m - \gamma_{n+\frac{1}{2}}^{m-1}) + \gamma_{n+\frac{1}{2}}^m + K_5(M_{n+1}^m - M_n^m) + K_6(V_{n+1}^m + V_n^m)$$

$$(14') M_n^{m+1} = K_4(\gamma_{n+\frac{1}{2}}^{m+1} - \gamma_{n-\frac{1}{2}}^{m+1})$$

$$(15') V_n^{m+1} = K_7(\gamma_{n+\frac{1}{2}}^{m+1} + \gamma_{n-\frac{1}{2}}^{m+1}) + K_8(y_{n+\frac{1}{2}}^{m+1} - y_{n-\frac{1}{2}}^{m+1})$$

where K_1, K_2, \dots, K_8 are multiplying factors which are, with the exception of K_3 , functions only of the characteristics of the vessel hull and the selected time and space intervals, and which may be precomputed prior to the main calculation. K_3 may also be precomputed in the case of the application of a constant force; in the case the acting force is a function of time a new set of values K_3 must be computed for each time interval. Specifically,

$$(16) \quad \begin{cases} K_1 = -\frac{C_{n+\frac{1}{2}}}{M_{n+\frac{1}{2}}} \Delta t, & K_2 = -\frac{1}{M_{n+\frac{1}{2}}} \frac{(\Delta t)^2}{\Delta x}, & K_3 = \frac{P_{n+\frac{1}{2}}^m}{M_{n+\frac{1}{2}}} (\Delta t)^2, \\ K_4 = \frac{(EI)_n}{\Delta x}, & K_5 = \frac{1}{(IMZ)_{n+\frac{1}{2}}} \frac{(\Delta t)^2}{\Delta x}, & K_6 = -\frac{(\Delta t)^2}{2(IMZ)_{n+\frac{1}{2}}}, \\ K_7 = \frac{(KAG)_n}{2}, & K_8 = \frac{(KAG)}{\Delta x} \end{cases}$$

The initial and boundary conditions are given by the relations,

$$(17) \quad \begin{cases} y_{n+\frac{1}{2}}^0 = y_{n+\frac{1}{2}}^{-1} = \gamma_{n+\frac{1}{2}}^0 = \gamma_{n+\frac{1}{2}}^{-1} = V_n^0 = M_n^0 = 0, \\ M_0^m = M_{20}^m = V_0^m = V_{20}^m = 0. \end{cases}$$

From the above it is also possible to derive the following useful relations

for the end-point values of y , :

$$(18) \quad \gamma_0^m = \gamma_{\frac{1}{2}}^m, \quad \gamma_{20}^m = \gamma_{19.5}^m, \quad y_0^m = y_{\frac{1}{2}}^m - \frac{\Delta x}{2} \gamma_{\frac{1}{2}}^m, \quad y_{20}^m = y_{19.5}^m + \frac{\Delta x}{2} \gamma_{19.5}^m.$$

Numerical Stability

In order to carry out successfully the solution of a system of partial differential equations such as equations (1) to (4) by finite difference methods, the finite difference equivalent system (12) to (15) must be stable in the sense discussed in References 5, 6 and 7. We will now derive the conditions under which this system of equations will satisfy these stability requirements. The conditions of stability are satisfied if the amplitude of a small disturbance, introduced at any time, t , does not increase with successive time steps. This condition may be stated as follows: If $\delta F(x, t)$ and $\delta F(x, t + \Delta t)$ are values of a variation (or perturbation) of any of the dependent variables y, γ, M and V in the system, then it is said to be stable provided

$|\delta F(x, t + \Delta t) / \delta F(x, t)| \leq 1$. To determine the conditions for stability we will introduce perturbations $\delta y, \delta \gamma, \delta M$ and δV in the independent variables y, γ, M and V , respectively. Substituting in equations (12) to (15) we obtain the variational equation system:

$$(19) \quad M_{n+\frac{1}{2}} (\delta y_{n+\frac{1}{2}}^{m+1} - 2\delta y_{n+\frac{1}{2}}^m + \delta y_{n+\frac{1}{2}}^{m-1}) + C_{n+\frac{1}{2}} (\delta y_{n+\frac{1}{2}}^m - \delta y_{n+\frac{1}{2}}^{m-1}) \Delta t + \frac{(\Delta t)^2}{\Delta x} (\delta V_{n+1}^m - \delta V_n^m) = 0$$

$$(20) \quad \frac{(I_{pz})_{n+\frac{1}{2}}}{(\Delta t)^2} (\delta \gamma_{n+\frac{1}{2}}^{m+1} - 2\delta \gamma_{n+\frac{1}{2}}^m + \delta \gamma_{n+\frac{1}{2}}^{m-1}) - \frac{\delta M_{n+1}^m - \delta M_n^m}{\Delta x} + \delta V_{n+\frac{1}{2}}^m = 0$$

$$(21) \quad \frac{(EI)_n}{\Delta x} (\delta \gamma_{n+\frac{1}{2}}^{m+1} - \delta \gamma_{n-\frac{1}{2}}^{m+1}) - \delta M_n^{m+1} = 0$$

$$(22) \quad \frac{\delta y_{n+\frac{1}{2}}^{m+1} - \delta y_{n-\frac{1}{2}}^{m+1}}{\Delta x} - \delta \gamma_n^{m+1} + \frac{\delta V_n^{m+1}}{(KAG)_n} = 0.$$

We will assume in this analysis that within a small region in the (x, t) plane the coefficients $(\mu, I_{rz}, \text{etc.})$ of the variational functions can be treated as constants. A solution of the system of equations (19) to

(22) can then be obtained in the form

$$(23) \begin{cases} \delta y_n^m = a e^{i\beta n + \alpha m \Delta t} \\ \delta r_n^m = b e^{i\beta n + \alpha m \Delta t} \\ \delta M_n^m = c e^{i\beta n + \alpha m \Delta t} \\ \delta V_n^m = d e^{i\beta n + \alpha m \Delta t} \end{cases}$$

where a, b, c, d are real constants and α complex. Substituting (23) in (19) to (22) we obtain a system of linear homogeneous equations for the quantities a, b, c and d which has a non-trivial solution provided the determinant D of the coefficients is identically zero, where

$$(24) D = \begin{vmatrix} \mu(\lambda - 2 + \lambda^{-1}) + c\Delta t(1 - \lambda^{-1}) & 0 & 0 & \frac{\Delta t^2}{\Delta x}(e^{\frac{i\beta}{2}} - e^{-\frac{i\beta}{2}}) \\ 0 & \frac{I_{rz}}{(\Delta t)^2}(\lambda - 2 + \lambda^{-1}) & -\frac{(e^{\frac{i\beta}{2}} - e^{-\frac{i\beta}{2}})}{\Delta x} & 1 \\ 0 & \frac{EI}{\Delta x}(e^{\frac{i\beta}{2}} - e^{-\frac{i\beta}{2}}) & -1 & 0 \\ \frac{(e^{\frac{i\beta}{2}} - e^{-\frac{i\beta}{2}})}{\Delta x} & -1 & 0 & \frac{1}{(KAG)} \end{vmatrix}$$

and where $\lambda = e^{\alpha \Delta t}$.

From the above we obtain

$$(25) \quad [M(\lambda - 2 + \lambda^{-1}) + c \Delta t (1 - \lambda^{-1})] \left[-1 + \frac{EI}{(KAG)(\Delta x)^2} (2i \sin \frac{\beta}{2})^2 - \frac{I_{Mz}(\lambda - 2 + \lambda^{-1})}{(KAG)(\Delta t)^2} \right] \\ - \frac{(\Delta t)^2}{(\Delta x)^2} (2i \sin \frac{\beta}{2})^2 \left[\frac{EI}{(\Delta x)^2} (2i \sin \frac{\beta}{2})^2 - \frac{I_{Mz}(\lambda - 2 + \lambda^{-1})}{(\Delta t)^2} \right] = 0.$$

If we further assume $\Delta t \ll \frac{M}{c}$
we obtain,

$$(25') \quad \left(\frac{M\xi}{KAG} + 4r \sin^2 \frac{\beta}{2} \right) (4r(EI) \sin^2 \frac{\beta}{2} + I_{Mz} \xi) + M\xi(\Delta t)^2 = 0$$

or

$$(26) \quad \frac{M I_{Mz}}{KAG} \xi^2 + \left[4r \sin^2 \frac{\beta}{2} (I_{Mz} + \frac{M(EI)}{KAG}) + M(\Delta t)^2 \right] \xi + 16r^2(EI) \sin^4 \frac{\beta}{2} = 0$$

where $\xi = \lambda - 2 + \lambda^{-1}$, $r = \frac{(\Delta t)^2}{(\Delta x)^2}$.

Equation (26) above is a fourth degree polynomial equation in λ .

If we designate one of its roots λ_1 we draw the conclusion from the relation $\xi = \lambda - 2 + \lambda^{-1}$ that $\lambda_2 = \frac{1}{\lambda_1}$ will be another root.

For stability both $|\lambda_1| \leq 1$ and $|\lambda_2| = \left| \frac{1}{\lambda_1} \right| \leq 1$. It follows that $|\lambda_1| = |\lambda_2| = 1$. Likewise, $|\lambda_3| = |\lambda_4| = 1$.

Let $\lambda_1 = \cos \gamma + i \sin \gamma$; $\lambda_2 = \cos \gamma - i \sin \gamma = \frac{1}{\lambda_1}$, then,

$$(27) \quad \xi = 2(\cos \gamma - 1),$$

$$\text{or} \quad -4 \leq \xi \leq 0.$$

On the other hand, from equation (26) we obtain

$$(28) \quad \xi = -\frac{KAG}{2MI_{Mz}} \left[(US + M(\Delta t)^2) + (U^2T^2 + 2M(\Delta t)^2US + M^2(\Delta t)^4)^{\frac{1}{2}} \right]$$

and

$$(29) \quad \xi = -\frac{KAG}{2MI_{Mz}} \left[(US + M(\Delta t)^2) - (U^2T^2 + 2M(\Delta t)^2US + M^2(\Delta t)^4)^{\frac{1}{2}} \right]$$

where,

$$S = \left(I_{Mz} + \frac{M(EI)}{KAG} \right), \quad T = \left(I_{Mz} - \frac{M(EI)}{KAG} \right), \quad U = 4r \sin^2 \frac{\beta}{2}.$$

It follows that, for stability, the inequalities

$$(30) \quad -4 \leq -\frac{KAG}{2MI_{Mz}} \left[(US + M(\Delta t)^2) + (U^2T^2 + 2M(\Delta t)^2US + M^2(\Delta t)^4)^{\frac{1}{2}} \right] \leq 0$$

and

$$(31) \quad -4 \leq -\frac{KAG}{2MI_{Mz}} \left[(US + M(\Delta t)^2) - (U^2T^2 + 2M(\Delta t)^2US + M^2(\Delta t)^4)^{\frac{1}{2}} \right] \leq 0$$

must hold. It may be seen by examining the expression for ξ that its value is always less than or at most equal to zero and that the left hand inequalities (30) and (31) are satisfied if

$$(32) \quad (U_S + M(\Delta t)^2) \frac{KAG}{2M I_{Mz}} \leq 2$$

In addition, from the previous discussion it follows that Δt must be chosen so that

$$(33) \quad \Delta t \ll \frac{M}{C}$$

The above result may be stated as follows:

Theorem:

The finite difference system, consisting of equations (12) to (15), governing the transient motion of a ship hull, is numerically stable provided the time increment Δt is chosen sufficiently small so that $\Delta t \ll \frac{M}{C}$ and

$$(32') \quad (\Delta t)^2 \leq \frac{M I_{Mz} (\Delta x)^2}{I_{Mz} (KAG) + M(EI) + .25 M (\Delta x)^2 (KAG)}$$

throughout the range of solution.

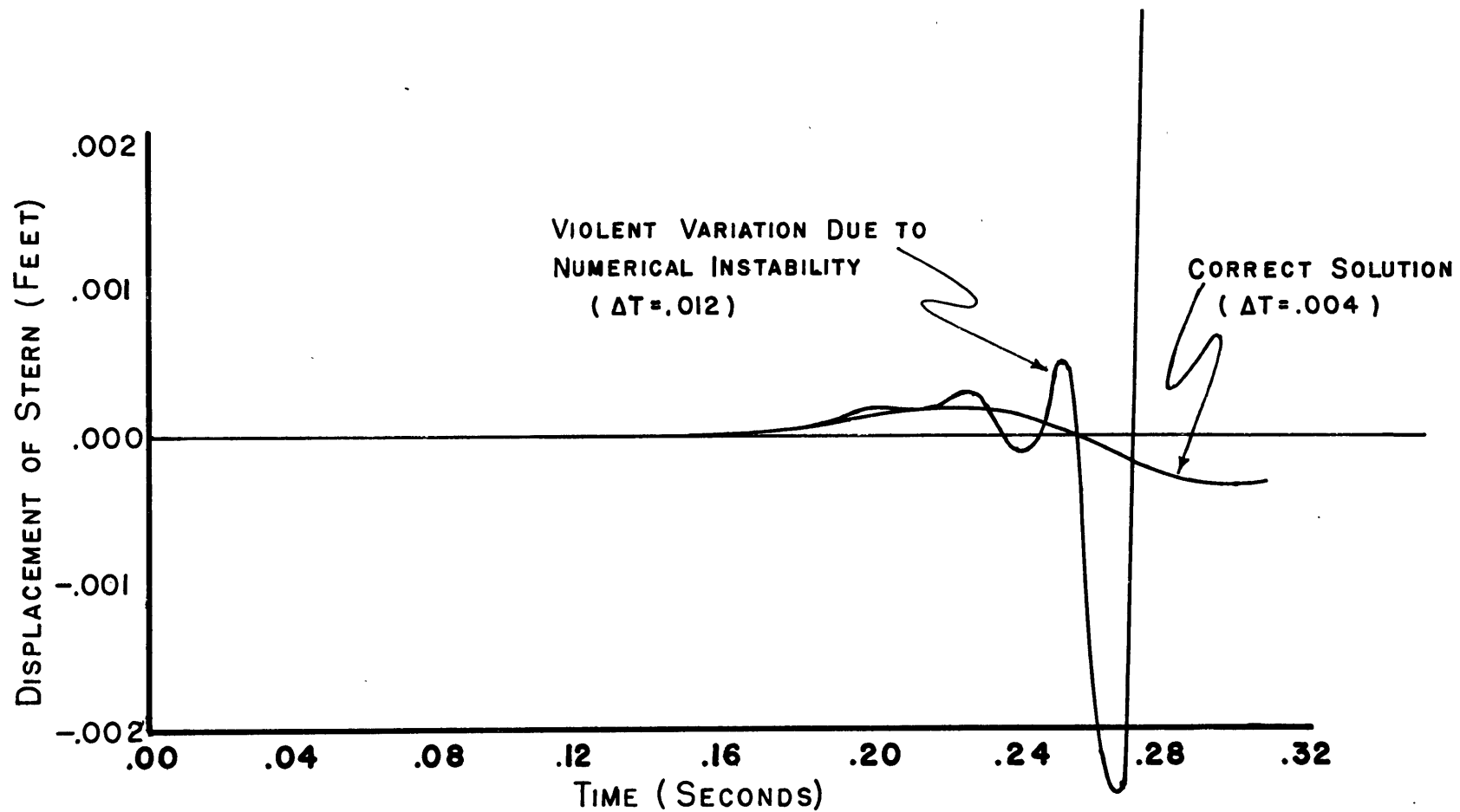
Numerical instability is usually accompanied by violent variations in the computed functions which invalidate the solution. This effect is shown in Figure 2. Whereas a time interval satisfying inequalities (32) and (33) guarantees stability, it should be pointed out that the interval need not necessarily be as small as indicated by these inequalities.

Computational Procedure

We are now prepared to use the system of finite difference equations ((12) to (15) and (16) to (18)) to calculate the motion of a ship hull in response to a force acting upon it. The physical characteristics of the ship hull and the magnitude of the forces acting upon it (i. e. μ , C, P, $I_{\mu z}$, (EI) and (KAG)) are calculated on the basis of the theory of elasticity from experimentally determined physical quantities. (For instance, see Table 7, Ref. 4 for values of these parameters in the case of the SS Gopher Mariner.) From these the maximum value that can be assigned to Δt , the time increment, in order to insure numerical stability may be calculated on the basis of the inequalities (32) and (33). Then the coefficients K_1, K_2, \dots, K_8 are calculated by use of equations (16). These should be listed at the full-interval positions or at the half-interval positions (See Fig. 1 b) as follows:

K_1, K_2, K_3, K_5, K_6 ——— listed at half-interval positions
 K_4, K_7, K_8 ——— listed at full (integral) interval positions.

From the known conditions ($y, \gamma, M,$ and V) at time $t = 0$ (equation 17) we proceed to calculate the values of y and γ at $t = \Delta t$,



**FIGURE 2- NUMERICAL INSTABILITY RESULTING FROM
INCORRECT CHOICE OF INTEGRATION INTERVAL**

by use of equations (12) and (13). We then obtain the values of M and V at $t = \Delta t$ from equations (14') and (15'), also using the boundary relations $M_0 = M_{20} = V_0 = V_{20} = 0$. We may now repeat this cycle any number of times, obtaining the values of the variables y , γ , M and V at $t = 2 \Delta t, 3 \Delta t, \dots$, etc. — until we reach any desired value of time, t .

The above computation procedure was programmed for solution on the UNIVAC system, and a trial solution carried out. The results appear successful in every respect. Calculations at varying time intervals demonstrate the feasibility of producing accurate solutions much beyond engineering requirements.

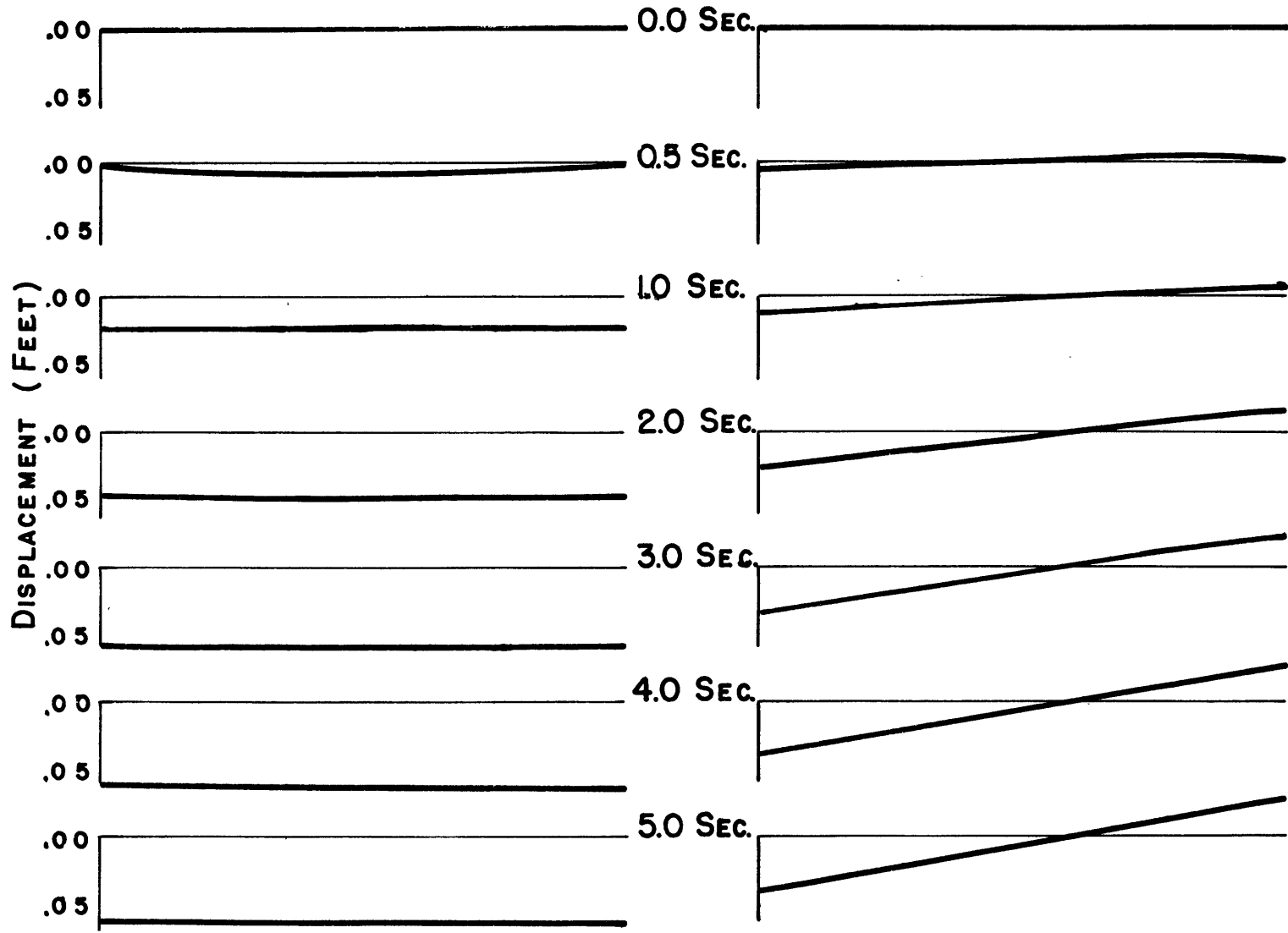
Figure 3 shows the results of two sample calculations for the transient excitation of a ship hull due to a large force applied for the duration of a second a) amidship and b) at the bow. The vertical displacements of a cross section of the hull are plotted in each case as a function of time. Appendix A contains detailed procedure for use by a mathematician or programmer in order to carry out additional solutions on the UNIVAC system in the future. It also contains a) a memory allocation chart, b) the contents of the instruction tape and c) sample input data.

Conclusion and Acknowledgement

The successful calculation of the transient motion of a ship hull by finite differences (on the basis of the fundamental system of differential equations) and the use of a high-speed calculator system constitutes a

FIGURE 3- CALCULATED DISPLACEMENT OF HULL
FORCE OF ONE SECOND DURATION
APPLIED AMIDSHIP

FORCE OF ONE SECOND DURATION
APPLIED AT BOW



significant forward step in the continual search by the applied mathematician to develop practical numerical techniques for the solution of engineering problems governed by complex systems of differential equations. It establishes the feasibility of utilizing this technique for the analysis of structural vibrations of systems subjected to external forces on the basis of theoretical considerations. This method has many advantages over others:

a) In many areas of application, it is the only practical method available to obtain numerical solutions of engineering problems governed by systems of differential equations.

b) It produces a detailed description of the motion of the structure under investigation and the forces acting upon it, as a function of time.

c) It is extremely simple in concept and relies entirely on the fundamental differential equations which govern the motion — and not on an approximate version thereof.

d) It can readily be generalized and extended in many directions. For instance, various forms of the basic equations may be solved and compared with experimental results to determine the most adequate theory; or, a solution may be carried out in two or even three dimensions without encountering any substantial additional basic difficulties in the application of the method.

The solution of structural vibration problems by the method outlined here was made possible only by the development of the mathematical theory for the numerical stability of finite difference methods and by the advent of high-speed electronic digital calculators.

The author is indebted to Mr. R. T. McGoldrick for proposing the problem, for suggesting the use of finite differences as well as for carefully reviewing the manuscript, and to Dr. E. H. Kennard for many helpful discussions. He would also like to thank Miss Jacqueline Kitchens for valuable assistance in carrying out the computations and in preparing the report.

Appendix A

Procedure for Solution of Transient Ship Hull Excitation

Problem on UNIVAC System

The present UNIVAC program for the solution of the transient ship hull excitation problem was designed primarily to test the feasibility of using finite difference methods for this purpose. Consequently it should be considered an interim program. However, it may be used effectively to solve problems in this field, pending the development of a more general program. Because of its limited scope it has the advantage of being simple to use and efficient. The machine procedure consists of only one run which carries out the computation, produces the "edited" output and contains an automatic rerun or restart feature.

The program is based entirely on equations (12) to (15) and (16), (17) in the main text. It assumes twenty intervals for representation of the vessel hull and a constant (square shaped) force which acts on the hull from time $t = 0$ for some specified duration $t = t_1$. The force may act upon any number of the sections of the hull, located anywhere along its length. This problem may readily be reprogrammed to incorporate a larger number of intervals, if desired, and a force function which is an arbitrary function of location and time. A number of additional extensions and improvements should be made in reprogramming this problem for production purposes.

The procedure for the solution of a problem on the UNIVAC system consists of three phases, a) preliminary computation, b) preparation of

input data and c) machine operation. Detailed instructions for each of these phases will be given in the order listed.

A. Preliminary Computation

1. Form of originating data

The proposer will provide the following information, which he will obtain experimentally, or estimate:

μ , C, P and $I_{\mu z}$ — listed at half-interval positions
(EI) and (KAG) — listed at full (integral) interval positions.

In addition he will give the duration, t_1 , of the force, P, acting on the vessel, and will indicate the approximate time interval, δt , at which he desires tabular listings, and the total time, t_2 , during which the motion is to be calculated. If the data is not centered as required above, proper centering can be obtained simply by averaging.

2. Determination of the time interval Δt

The first computational task is to determine the maximum value Δt for which the stability criteria will be satisfied. This value is computed on the basis of the inequalities (32) and (33), which must be satisfied at each value of x . Once the maximum allowable Δt is determined a convenient value Δt is chosen which is smaller than this maximum and which is an integral divisor of 0.1. If it is chosen too small, however, it will unnecessarily increase the amount of computation.

3. Computation of Coefficients $K_1 \dots K_8$ and $k_1 \dots k_8$

The coefficients $K_1 \dots K_8$ are then computed on the basis of

equations (16) in the main text, and centered as indicated in the section on computational procedure (main text). Specifically, K_1, K_2, K_3, K_5, K_6 are computed for values of $n = 0.5, 1.5, \dots, 19.5$; and K_4, K_7, K_8 are computed for $n = 1.0, 2.0, \dots, 19.0$. (See Fig. 1b). Then compute k_1, k_2, \dots, k_8 as follows: $k_1 = K_1, k_2 = 10^5 K_2, k_3 = K_3, k_4 = 10^{-9} K_4, k_5 = 10^9 K_5, k_6 = 10^8 K_6, k_7 = 10^{-8} K_7, k_8 = 10^{-5} K_8$. The program assumes that $k_1, k_2, \dots, k_6, \frac{k_7}{10}, k_7, \frac{k_8}{10}$ are less than unity.

4. Determination of pseudo-time factor f and computation of time values

In order to facilitate tabular listings at desired time intervals, it was convenient to use pseudo-time values, T , in place of actual time values in the control phases of the data input. Pseudo-time, T , is equal to a factor f times real time, t , i. e. $T = ft$. The program was so arranged so as to obtain a listing every interval $\delta T = 0.1$ in pseudo-time, i. e. for $T = 0.1, 0.2, 0.3 \dots$ etc. The factor f is thus used as a means for obtaining listings at any desired real time interval δt .

The interval δt , at which listings occur should be taken so that $\delta t = n \Delta t$, where n is an integer. The factor f is then computed $f = \frac{0.1}{\delta t}$. Then compute $\Delta T = f \Delta t, T_1 = ft_1$ and $T_2 = ft_2$. The pseudo-times $\Delta T, T_1$ and T_2 are entered in preparing the tabulating control input data, rather than the real time. These should be rounded off to the nearest integral multiple of ΔT .

B. Preparation of Input Data

The input data consists of ten blocks. These are prepared as

described in Table 1, which is a reproduction of a sample set of input data together with notations which indicate the locations of all quantities used in its formation.

C. Machine Operating Instructions

1. Machine Procedure

Place instruction tape on Uniservo 1, data tape on Uniservo 2, blank tapes on Uniservos 3 and 8. Uniservo 3 should be set to write in blockette form. This Uniservo will contain the results. Initial read Uniservo 1. To facilitate following the progress of the computation the supervisory control typewriter will print pseudo-time at each interval printed in the final result. Upon completion the machine will stop.

To print results on High Speed Printer use double space control tape with first skip to line 7.

2. Rerun Procedure

If the machine malfunctions, or if a problem is not completed during any one period at the computer, remove rerun tape from Uniservo 8, place on Uniservo 2, place blank tape on servo 8, rewind Uniservo 1 and initial read Uniservo 1.

3. Interpretation of Printed Results

Each page of printed output will contain the results for one time interval. The pseudo-time (not real time) will be given in the upper right hand corner. The decimal point will be located between the sixth and seventh digits. The problem designation and run number will be printed in the upper left hand corner.

The columns headed by Y, G will list y and $10^3 \gamma$, respectively. The decimal point is located between the first and second digits. The values correspond to the half-interval positions, but are aligned to print at the subsequent integral value of N. For instance y_1 is listed on the line $N = 1$. It will be noted that the column heading for segment numbers uses capital N in place of n.

The columns headed by M and V will list $10^{-6} M$ and $10^{-5} V$, respectively. The decimal point is located between the first and second digits. The values correspond to the whole interval positions, and are aligned correctly.

If values for y_0, y_{20} or γ_0, γ_{20} are desired, these may be obtained easily from equations (18) in the main text. If intermediate whole interval values of y or γ are desired, such as γ_{12} , these may be obtained by averaging, i. e. $\gamma_{12} = \frac{\gamma_{11.5} + \gamma_{12.5}}{2}$.

4. Memory Allocation Chart and UNIVAC Instructions

As an aid to the mathematician programmer, in the event modification or extension of the program is desired, a memory allocation chart and the contents of the instruction tape is reproduced here.

An auxiliary program is available to change the time interval Δt , if desired.

MEMORY ALLOCATION CHART

000	010	020	030	040	050	060	070	080	090
PROGRAM									
100	110	120	130	140	150	160	170	180	190
200	210	220	230	240	250	260	270	280	290
EDITED					OUTPUT				
300	310	320	330	340	350	360	370	380	390
400	410	420	430	440	450	460	470	480	490
					450 = $\textcircled{4}$	n			
500	510	520	530	540	550	560	570	580	590
Δ				k_1			568 = Δ_1 569 = i_2	k_2	
600	610	620	630	640	650	660	670	680	690
k_3		629 = i_1		k_4		k_5			
700	710	720	730	740	750	760	770	780	790
k_6		k_7		k_8 / IO		y			
800	810	820	830	840	850	860	870	880	890
$y\Delta t$			g			\dot{g}			
900	910	920	930	940	950	960	970	980	990
M			V		930 = t_0	WORKING STORAGE	COPY: IC	39	

28

UNIVAC Instructions
(four blocks)

CODING SHEET
PRNC-TMB-487

PROBLEM 840-7-490
DATE 8-18-56

PAGE 1

00	11	000			
			31	060	Read instruction tape
01	30	120			
			Y	010	
02	Z	970			Copy (10-39) (970-999)
			Y	020	
03	Z	980			
			Y	030	
04	Z	990			
			12	000	
05	32	420			
			42	150	
06	32	210			
			U	090	
07	32	480			
			32	540	Read data tape
08	32	600			
			32	660	
09	32	720			
			32	780	
10	32	840			
			32	900	
11	B	930			Initial time to 099
			C	146	
12	L	<u>816</u>			
			M	<u>546</u>	
13	C	960			
			B	<u>936</u>	
14	S	<u>935</u>			
			K	000	
15	M	<u>576</u>			Compute y^{m+1} $\frac{m+1}{m+2}$
			A	960	
16	A	<u>606</u>			
			A	<u>816</u>	
17	H	<u>816</u>			
			A	<u>786</u>	
18	C	<u>786</u>			
			00	000	
19	00	000			
			00	000	

20	B	<u>906</u>			
			S	905	
21	K	000			
			M	<u>666</u>	
22	C	960			
			B	<u>936</u>	
23	A	<u>935</u>			
			K	000	Compute g_{n+1}^{m+1}
24	M	<u>696</u>			
			A	960	
25	A	<u>876</u>			
			H	<u>876</u>	
26	A	<u>846</u>			
			C	<u>846</u>	
27	00	000			
			00	000	
28	B	<u>846</u>			
			S	<u>845</u>	
29	K	000			
			M	<u>635</u>	Compute M_n^{m+1}
30	C	<u>905</u>			
			00	000	
31	B	<u>786</u>			
			S	<u>785</u>	
32	01	000			
			K	000	
33	M	<u>755</u>			
			C	960	
34	B	<u>846</u>			
			A	<u>845</u>	Compute v_n^{m+1}
35	K	000			
			M	<u>725</u>	
36	A	960			
			C	<u>935</u>	
37	00	000			
			00	000	
38	00	000			
			00	000	
39	00	000			
			00	000	

40	B	012			
			A	077	
41	C	012			
			B	013	
42	A	078			
			C	013	
43	B	014			
			A	079	
44	C	014			
			B	015	
45	A	079			
			C	015	
46	B	016			
			A	077	
47	C	016			
			B	017	Modify address locations
48	A	077			
			C	017	
49	B	018			
			A	079	
50	C	018			
			B	020	
51	A	077			
			C	020	
52	B	021			
			A	078	
53	C	021			
			B	022	
54	A	078			
			C	022	
55	B	023			
			A	079	
56	C	023			
			B	024	
57	A	079			
			C	024	
58	B	025			
			A	077	
59	C	025			
			B	026	

60	A	077			
			C	026	
61	B	028			
			A	077	
62	C	028			
			B	029	
63	A	078			
			C	029	
64	B	030			
			A	079	
65	C	030			
			B	031	
66	A	077			
			C	031	
67	B	033			
			A	079	Modify address locations
68	C	033			
			B	034	
69	A	077			
			C	034	
70	B	035			
			A	078	
71	C	035			
			B	036	
72	A	078			
			H	036	Check for completion of all space increments
73	L	076			
			Q1	080	
74	00	000			
			U	012	
75	00	000			
			00	000	
76	A	960			
			C	956	
77	000	001			
			000	001	
78	000	000			
			000	001	
79	000	001			
			000	000	

80	Y	970			
			Z	010	
81	Y	980			
			Z	020	Reset (10-39)
82	Y	990			
			Z	030	
83	B	146			Reset time from T^m to T^{m+1}
			H	960	
84	A	568			
			H	146	
85	L	629			
			Q	097	Test for end of force duration (T_i)
86	.5	000			
			K	000	
87	B	960			
			.5	000	
88	000	000			
			Q2	012	Test for print cycle
89	000	000			
			U	110	
90	42	270			
			32	330	
91	42	390			
			32	480	
92	00	000			
			U	007	
93	V	118			
			W	960	
94	V	120			
			W	962	
95	V	122			
			W	964	
96	00	000			
			U	118	
97	Y	100			
			Z	600	Insert zeros
98	Z	610			
			Z	620	
99	00	000			
			U	086	

	100	000	000			
		↑	↑			
				000	000	
	1			↑	↑	
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	4					
	5					
	6					
	7					
	8					
		↓	↓			
	10 ⁹	000	000	↓	↓	
				000	000	
	11 ⁰	B	146			
				C	930	
	11 ¹	58	420			
				58	480	
	11 ²	58	540			
				58	600	Rerun
	11 ³	58	660			
				58	720	
	11 ⁴	58	780			
				58	840	
	11 ⁵	58	900			
				58	480	
	11 ⁶	68	000			
				61	000	
	11 ⁷	000	000			
				U	093	
	11 ⁸	B	450			
				C	150	
	11 ⁹	B	780			
				C	152	

12 ⁰	B	840			
			C	154	
12 ¹	B	900			
			C	156	
12 ²	B	930			
			C	158	Edit
12 ³	000	000			
			00	000	
12 ⁴	B	118			
			A	149	
12 ⁵	C	118			
			B	119	
12 ⁶	A	149			
			C	119	
12 ⁷	B	120			
			A	149	
12 ⁸	C	120			
			B	121	
12 ⁹	A	149			
			C	121	
13 ⁰	B	122			
			A	149	
13 ¹	H	122			
			L	148	
13 ²	000	000			
			Q3	134	
13 ³	000	000			
			U	118	
13 ⁴	V	960			
			W	118	
13 ⁵	V	962			
			W	120	Edit
13 ⁶	V	964			
			W	122	
13 ⁷	53	150			
			53	210	
13 ⁸	53	270			
			53	330	
13 ⁹	53	390			
			50	158	

14 ⁰	B	146			
			L	569	
14 ¹	000	000			
			Q4	143	Test for end of problem
14 ²	000	000			
			U	012	
14 ³	B	147			
			C	480	
14 ⁴	53	480			
			62	000	Rewind Tapes
14 ⁵	63	000			
			90	000	
14 ⁶	00	000			
			00	000	T ⁿ⁺¹
14 ⁷	$\cancel{P} \Delta \Delta \Delta \Delta \Delta$				
			$\Delta \Delta \Delta \Delta \Delta \Delta$		
14 ⁸	B	960			
			c	458	
14 ⁹	000	001			
			000	010	
15 ⁰					
			Zeros to end of block		
15 ¹					
15 ²					
			Type block of Zs		
15 ³					
15 ⁴					
15 ⁵					
15 ⁶					
15 ⁷					
15 ⁸					
15 ⁹					

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1	↑				↑	↑				↑		
2												
3	↓				↓	↓				↓		
4												
5	0	0	0	0	0	0	0	0	0	0		
6	-	0	0	4	0	0	0	0	0	0	0	$k_1 = K_1$ for $n = 0.5$
7	↑				↑	↑				↑	1.5	
8											2.5	
9	↓				↓	↓				↓	3.5	
0											4.5	
1	↑				↑	↑				↑	5.5	
2											6.5	
3	↓				↓	↓				↓	7.5	
4											8.5	
5	↑				↑	↑				↑	9.5	
6											10.5	
7	↓				↓	↓				↓	11.5	
8											12.5	
9	↑				↑	↑				↑	13.5	
0											14.5	
1	↓				↓	↓				↓	15.5	
2											16.5	
3	↑				↑	↑				↑	17.5	
4											18.5	
5	-	0	0	4	0	0	0	0	0	0	0	$k_1 = K_1$ for $n = 19.5$
6	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	Decimal point
8	0	0	0	0	0	0	0	2	0	0	0	ΔT
9	0	0	0	0	2	5	0	0	0	0	0	T2

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	↑								↑	↑					↑	
2	↓								↓	↓					↓	
3	↑								↑	↑					↑	
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	-	1	4	9	1	8				0	0	0	0	0	0	$k_2 = 10^5 K_2$ for $n = 0.5$
7	↑	0	8	8	2	0			↑	↑					↑	1.5
8	↓	0	5	2	4	0			↓	↓					↓	2.5
9	↑	0	3	3	5	9			↑	↑					↑	3.5
0	↓	0	2	4	7	0			↓	↓					↓	4.5
1	↑	0	2	2	5	1			↑	↑					↑	5.5
2	↓	0	2	2	0	6			↓	↓					↓	6.5
3	↑	0	1	9	8	3			↑	↑					↑	7.5
4	↓	0	1	8	1	5			↓	↓					↓	8.5
5	↑	0	1	7	2	1			↑	↑					↑	9.5
6	↓	0	1	8	2	4			↓	↓					↓	10.5
7	↑	0	2	0	5	0			↑	↑					↑	11.5
8	↓	0	2	2	4	1			↓	↓					↓	12.5
9	↑	0	2	7	2	0			↑	↑					↑	13.5
0	↓	0	3	4	8	8			↓	↓					↓	14.5
1	↑	0	4	7	8	3			↑	↑					↑	15.5
2	↓	0	6	1	0	9			↓	↓					↓	16.5
3	↑	0	6	3	4	8			↑	↑					↑	17.5
4	↓	0	8	2	6	0			↓	↓					↓	18.5
5	↑	-	1	5	6	3	3		↑	0	0	0	0	0	0	$k_2 = 10^5 K_2$ for $n = 19.5$
6	↓	0	0	0	0	0	0		↓	0	0	0	0	0	0	
7	↑	0	0	0	0	0	0		↑	0	0	0	0	0	0	
8	↓	0	0	0	0	0	0		↓	0	0	0	0	0	0	
9	↑	0	0	0	0	0	0		↑	6	0	9	5	0	0	$(\Delta t)^2 / \Delta x$

0	0	0	0	0	0	0	0	0	0	
1	↑								↑	
2										
3										
4										
5										
6										$k_3 = K$ for $n = 0.5$
7										1.5
8										2.5
9										3.5
0										4.5
1										5.5
2										6.5
3	↓								↓	7.5
4	0	0	0	0	0	0	0	0	0	8.5
5	0	0	0	0	0	1	1	5	3	9.5
6	0	0	0	0	0	0	0	0	0	10.5
7	↑								↑	11.5
8										12.5
9										13.5
0										14.5
1										15.5
2										16.5
3										17.5
4	↓								↓	18.5
5										$k_3 = K$ for $n = 19.5$
6	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	Decimal point
9	0	0	0	0	0	5	0	0	0	T ₁

0	0	0	0	0	0	0	0	0	0	
1	↑								↑	
2										
3										
4	↓								↓	
5	0	0	0	0	0	0	0	0	0	
6	-	2	7	9	1	8	0	0	0	$k_G = 10^8 K_G$ for $n = 0.5$
7	-	1	4	8	3	5				1.5
8	-	1	0	4	9	2				2.5
9	-	0	6	9	3	0				3.5
0	-	0	4	6	5	3				4.5
1	-	0	4	0	1	7				5.5
2	-	0	3	9	0	9				6.5
3	-	0	3	4	4	3				7.5
4	-	0	3	0	5	3				8.5
5	-	0	2	7	6	9				9.5
6	-	0	2	9	7	5				10.5
7	-	0	3	5	2	3				11.5
8	-	0	3	9	3	8				12.5
9	-	0	4	9	4	9				13.5
0	-	0	6	7	7	3				14.5
1	-	1	0	1	2	1				15.5
2	-	1	4	0	2	9				16.5
3	-	1	7	4	9	4				17.5
4	-	2	8	7	4	6				18.5
5	-	6	7	4	2	5	0	0	0	$k_G = 10^8 K_G$ for $n = 19.5$
6	0	0	0	0	0	0	0	0	0	
7	↑								↑	
8	↓								↓	
9	0	0	0	0	0	0	0	0	0	

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3	△△△△△△	M △△△△△	
4	△△△△△△	△△△△△△	
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6	↑	↑	
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4	↓	↓	
5	0 0 0 0 0 0	0 0 0 0 0 0	
6	△△△△△△	△△△△△△	
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9	△△△△△△	△△△△△△	

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2	△	△	△	△	△	△	△	△	△	△	△	△	
3	△	△	△	△	△	△	v	△	△	△	△	△	
4	△	△	△	△	△	△	△	△	△	△	△	△	
5	0	0	0	0	0	0	0	0	0	0	0	0	
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4	↓					↓	↓					↓	
5	0	0	0	0	0	0	0	0	0	0	0	0	
6	△	△	△	△	△	△	△	△	△	△	△	△	
7	↑					↑	↑					↑	
8	↓					↓	↓					↓	
9	△	△	△	△	△	△	△	△	△	△	△	△	

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