THE INFLUENCE OF TEMPERATURE ON THE FRICTIONAL RESISTANCE EXPERIENCED BY PLANE SURFACES MOVING IN A FLUID.

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Karl E. Schoenherr.

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PREFACE.

The first part of this paper is a brief summary of our present knowledge of the general laws of frictional resistance. The second part contains a description and the results of systematic experiments carried out at the United States Experimental Model Basin, Washington, D. C. to determine the influence of the variation of temperature on the frictional resistance experienced by thin straight planes towed ~~by~~ through water.

PART 1.

A solid body moving at constant velocity in a viscous fluid experiences resistance. When the body is of good streamline form and is completely surrounded by the medium, as for example an airship, it is found that 80 to 90% of the total resistance is frictional resistance. When the body is floating on a free surface of the medium, as for example a steamship, the frictional resistance is found to be 50 to 85% of the total resistance, depending on the shape and speed of the body. This brief reference indicates sufficiently that exact knowledge of the laws and nature of frictional resistance is of great practical importance as well as of theoretical interest.

The theory of motion in a viscous fluid was founded by Navier in 1827 (Ref.1) and independently developed by Stokes in 1845 (Ref.2). The differential equations describing the motions in two dimensions, known as the Navier-Stokes equations, are as follows:

Eqqation 1:
$$\begin{cases} u \dfrac{\partial u}{\partial x} + v \dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial x} + \nu \nabla^2 u \\ u \dfrac{\partial v}{\partial x} + v \dfrac{\partial v}{\partial y} = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial y} + \nu \nabla^2 v \end{cases}$$

The interpretation of the symbols in the above equations and in all following equations has been placed for convenience in the appended table of symbols. It is seen that these equations differ from the well known Euler equations for non-viscous flow by the additional term $\nu \nabla^2$ on the right side of each equation. The presence of these terms renders the integration extremely difficult. Stokes, in 1851, found a solution after making simplifying assumptions and was able to calculate the theoretical resistance of a sphere moving at low velocity in a medium of large viscosity (Ref.3).

The next material contribution to the theory of viscous fluids was made by Osborne Reynolds in 1883 (Ref.4). Reynolds experimented with the flow of water in capillary tubes. By the ingenious method of making the flow visible.

through the introduction of thin threads of colored liquid he discovered the fact that there are two distinct types of stable flow. One type, "laminar flow", stable at low velocities, the other type, "turbulent flow", stable at high velocities. "Laminar flow", as the name indicates, consists of a smooth gliding action between contiguous layers, giving rise to shearing forces only. "Turbulent flow" results when between contiguous layers of the fluid small clumps of particles are set in rotation forming tiny eddies, which in turn produce other eddies, the process continuing in this way, until the whole mass of fluid is in a state of confused, irregular motion.

Reynolds also discovered one of the important laws of similitude for the motion in a viscous fluid, which law is now generally known by his name. (Dr. Zahm called the author's attention to the fact that Reynolds' law had been deduced theoretically by Helmholtz as early as 1852 (Ref.18)). This law states, that if two systems moving in viscous media under the exclusion of free surfaces are to be dynamically similar, the pure number $R = \frac{v l}{\nu}$ must be the same in the two systems, where: v is any particular velocity, l a characteristic length and ν is the coefficient of kinematic viscosity of the medium. An expressive interpretation of "Reynolds' number " R was recently given by v.Karman, as the product of the ratios, velocity of translation to the velocity of the molecules, and the linear extension of the body to the mean molecular path (Ref.5). A proof of Reynolds' law can be found in many publications. A rigorous proof was given by Weber (Ref.6)

A further important contribution to the theory of viscous fluids was made by Prandtl in 1904 with his theory of the "boundary layer" (Ref.7). Prandtl considered the motion of a viscous medium around a completely submerged body. In order to simpify the Navier-Stokes equations and render them integrable he made the following assumptions: First, the internal friction between the particles of the fluid is small, but finite; second, the fluid adheres to the body; third, the pressure gradient normal to the boundary layer is zero. With these assumptions the general equations were reduced to the following single equation:

Equation 2: $$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

Two very important conclusions can be drawn from Prandtl's theory, viz: All frictional action takes place inside of a very narrow layer close to the surface of the body, and, outside of this "boundary layer" the laws of potential motion hold. In effect, these conclusions constitute a connecting link between classical hydrodynamics and actual phenomena.

While Prandtl's theory has thus closed a gap which for many years has kept practical engineers and theoreticians apart, it has not by any means solved all the difficulties involved in viscous flow. The solution of Prandtl's equation still presents great mathematical difficulties. But even more restricting is the fact that it applies to laminar flow only, which type of flow is of much less practical importance than turbulent flow.

A number of interesting problems were solved by means of Prandtl's theory. For example, the problem of the laminar frictional resistance of the thin straight plate moving with uniform velocity parallel to its own plane was completely solved by Blasius (Ref.8) who found the following theoretical formula:

Equation 3: $$F = 1.327\, A\, \frac{\rho}{2}\, v^2 \left(\frac{v l}{\nu}\right)^{-\frac{1}{2}}$$

in which the symbols have the meaning given in the table of symbols. This theoretical formula has been found to agree very well with experiments (Ref. 9).

Turning our attention to the turbulent frictional resistance of the thin straight plate moving with uniform velocity parallel to its own plane, we find very little theory but numerous empirical or semi-empirical equations. Only the most important formulae will be discussed.

Theory allows us to make the following deductions: When discussing turbulent flow we have seen, that under the action of the viscosity forces particles of contiguous layers are accelerated. This acceleration produces inertia forces according to Newton's general law of motion. When stationary conditions have been reached, the viscosity forces must be in equilibrium with the mass forces; hence, it is permissible to write the law of frictional resistance in the following general form:

Equation 4: $$F = \kappa \rho A v^2$$

From dimensional reasoning we may now conclude that the coefficient "k" may be a function of any number of variables, provided, that this combination forms a non-dimensional number. We have seen in discussing Reynolds' law of similitude that the ratio $R = \frac{v l}{\nu}$ fulfills this condition. Hence, we may write:

Equation 4a: $$\kappa = \psi\left(\frac{v l}{\nu}\right)$$

Introducing this expression in equation 4, we obtain:

Equation 5: $$F = \rho A v^2 \psi\left(\frac{v l}{\nu}\right)$$

This equation was published by Lord Rayleigh in 1892 and is sometimes referred to as "Rayleigh's Law"(Ref.15). We perceive that equation 3 is of the same general type as equation 5, but while in equation 3, which applies to laminar flow, the function $\psi\left(\frac{vl}{\nu}\right)$ was determined theoretically, we have at present no corresponding theory of turbulent flow that permits a similar evaluation of equation 5. Recourse must be had to experiment. The usual experimental procedure is to tow thin boards in a model basin or suspend them in a wind tunnel and to measure their resistance.

A different, very instructive, method was worked out by v.Karman and Prandtl which permits the determination of the unknown function $\psi\left(\frac{vl}{\nu}\right)$ from the measured velocity distribution close to the surface of the body (Ref.9, 10, 11). This method was based on Prandtl's theory of the boundary layer. It is assumed that the distribution of the velocity inside of the boundary layer undergoes a complete change, with the change of the character of the flow from laminar to turbulent, until a new distribution is reached at which equilibrium of forces is reestablished. Prandtl's theory furnished a general expression for the equilibrium of forces; experiments on the friction in pipes made by Blasius (Ref.12) furnished an expression for turbulent velocity distribution. Combining these two expressions by straightforward mathemathical methods, v.Karman and Prandtl, independently of each other, were led to the following semi-empirical formula:

Equation 6: $$F = .072 \, \rho/2 \, A \, v^2 \left(\frac{vl}{\nu}\right)^{-1/5}$$

Within the range of Reynolds' numbers $R = 5 \times 10^{5}$ to $R = 2 \times 10^{6}$ this equation agrees fairly well with experiments made by Wieselsberger in air (Ref.13).

The direct experimental method for finding an expression for turbulent frictional resistance was used by many investigators. Only the most important ones are mentioned. Froude (Ref.14) in 1871 towed through water a series of thin boards, up to 50 feet in length, at various speeds and expressed his results by the formula:

Equation 7: $$F = k A v^{m}$$

where m was found to vary somewhat but approached the constant value 1.83 for lengths approaching 50 feet. "k" was a coefficient varying with the length and the nature of the surface. Zahm (Ref.15) in 1902-03 tested thin plates in a wind tunnel and deduced the following expression:

Equation 8a: $$F = .00000778\ l^{.93}\ b\ v^{1.86}$$

It was shown by Lord Rayleigh that with slight modification Zahm's equation could be thrown into the following form:

Equation 8b: $$F = k A \rho v^{1.85} \left(\frac{l}{\nu}\right)^{-.15}$$

which equation is in harmony with the principle of dimensional homogeneity. Gebers (Ref.16), in 1919, repeated Froude's experiments with greatly improved apparatus and fitted the following equation to his results:

Equation 9: $$F = .0206\ \rho/2\ A\ v^{2} \left(\frac{v l}{\nu}\right)^{-\frac{1}{8}}$$

Wieselsberger (Ref.13) in 1920 tested plates in air and found the following formula;

Equation 10: $$F = .0375\ \rho/2\ A\ v^2 \left(\frac{vl}{\nu}\right)^{-.15}$$

Which has the form derived by Rayleigh from Zahm's experiments (Ref.15). Kempf (Ref.17) towed pipes up to 60 meters in length and later measured the frictional drag experienced by a piece of plating fitted neatly into an opening cut into the skin of a ship and based on these two kinds of tests proposed the following formula:

Equation 11: $$F = \rho/2\ A\ v^2 \left[.00111 + .072 \left(\frac{vl}{\nu}\right)^{-\frac{1}{5}}\right]$$

In all these investigations the temperature influence had been either neglected or, when appreciated, the temperature variation had been too small for any precise measurements of the resulting difference in resistance.

PART 2.

The influence of varistion of temperature of the medium on the measured resistance of models had been noted quite early in experimental model basin practice. In order to eliminate this influence most institutions had determined correction factors based on repeated tests with the same model. However, most of these data have remained unpublished and no uniform practice has been followed. After the laws of fluid friction had become more thoroughly understood it was apparent that the determining factor in the temperature influence was the viscosity of the medium.

For the present purposes, viscosity may be defined as that property of a medium which gives rise to shearing forces. The dimensions are force per unit area multiplied by time. In practice it is found more convenient to use the ratio viscosity coefficient μ divided by the density ρ which ratio has received the name "coefficient of kinematic viscosity" and is denoted by ν. Its dimensions are that of an area divided by time. In Table 1 are given the values of ν and the specific gravity γ for water at various temperatures in the English engineering system of units. This table is based on data published in the Smithsonian Tables, 7th edition of 1923.

Equation 5, repeated here for convenience, $F = \rho A v^2 \times \psi\left(\frac{vl}{\nu}\right)$ contains ν involved in the unknown function $\psi\left(\frac{vl}{\nu}\right)$. Inasmuch as the three variables, the speed v, the length l, and the coefficient of kinematic viscosity ν are independent of each other, a variation of any particular one will allow the evaluation of $\psi\left(\frac{vl}{\nu}\right)$ by experiment. For the reason of experimental simplicity most investigators had varied the speed v and the length l at constant or nearly constant viscosity ν. In the previous discussion, we have seen that the function $\psi(R)$, determined in this way, differs considerably in the various formulae, hence, it has seemed worth while to supplement existing data by systematic experiments, varying the viscosity ν and the speed v at constant length. Viscosity can be varied in two ways: First, by varying the medium itself. Second, by varying the temperature. For the present tests, the latter alternative was chosen.

Suitable for this purpose, the United States Experimental Model Basin possesses a small towing tank 35 feet long by 4 feet wide by 22 inches deep. The photograph, Plate 1, shows the water surface and one side wall of the tank. Attention is called to the test body in the foreground of the photograph. Objects are towed by means of a falling weight, accelerating, until the resistance is equal to the towing weight and traveling thereafter at constant speed. All the essential details of the towing apparatus are shown on Plate 2. In the foreground of this Plate may be seen the driving wheel mounted on sensitive bearings. A similar wheel is inside of the box at the farther end of the tank. The gravitational pull on the falling weight exerts a torque on the axle of the driving wheel. This torque produces a towing force at the periphery of the wheel equal to one tenth the falling weight. An endless silk cord stretched tightly between the two wheels transmits the towing force to the body.

The sensitivity of the apparatus is better than .0001 lb. The apparatus, by means of which the speed of the body is measured, may be seen at the left of the photograph. It consists of a strip of paper driven by an electric motor at constant speed on which are recorded electrically the distance traveled by the body and simultaneously the swings of a second pendulum.

At the suggestion of Captain Eggert, a catamaran friction plane was chosen for the test. By this term is meant, two thin flat boards of equal dimensions rigidly connected to each other above the water line. This type of body has the advantage of being completely waterborne, possessing great lateral stability and a large area of wetted surface. The drawing, Plate 3, and the photograph, Plate 3a, show all the essential details of the body. In fixing the dimensions of the plane two things had to be considered, first, not to make the plane larger than could be accomodated in the basin, second, not to make it too small, so as to obtain forces of reasonable magnitude. The dimensions of the plane finally chosen, were: 36 inches for the length, 12 inches for the depth, and one half inch for the thickness at the water line, tapering to one eighth inch at the bottom edges. The two individual planes were spaced 18 inches apart. This left a space of 15 inches between each plane and the side wall of the basin.

The plane was made in the shops of the Model Basin of clear white pine, built up in layers, and carefully worked down on both sides to the specified dimensions. After having been carefully smoothed and sandpapered, it was covered with several coats of paint and two coats of clear varnish. The weight of the finished plane was 4.56 lbs. At that displacement it floated at about two thirds of the intended draft. Five pieces of lead ballast weighing together 2.10 lbs. brought the plane to the designed draft of 12 inches. The ballast was arranged in such a way that it could be shifted longitudinally. In this way the trimming couple of the towing force was counteracted and the plane was on even keel at all speeds.

The first set of observations was made in September, 1929, and the results given in Table 3. This test brought out several important points: First, it was found inadvisable to exceed speeds of 2.7 feet per second on account of appreciable wave making at that speed. Second, the lowest practical speed was found to be .30 feet per second. Below this speed the readings became erratic. Third, it was found that in spite of the rounded forward edges of the plane mixed flow, i.e. part laminar and part turbulent flow, prevailed to almost the highest practical speed. The last objection was overcome in all subsequent tests by the following means: The forward edges of the plane were covered with fresh varnish and then coated with fine sand over a length of 4 inches. After the varnish had dried "Fixatif" was sprayed over the sand. The roughness obtained in this way corresponded approximately to that of number two sandpaper. This method proved entirely successful in breaking up the laminar flow at very much lower Reynolds' numbers and rendering it turbulent for the whole length of the plane.

A series of observations at high temperature were taken next. Inasmuch as the water was heated by means of a steam pipe placed along the bottom of the tank, any desired temperature could be reached. Nevertheless, it was found that at temperatures over 100 degrees F., the radiation from the surface became very rapid, even at a room temperature at 97 degrees F. Furthermore at temperatures of 120 degrees the varnish was found to have softened. Consequently, the data of the 120 degree run were later rejected, while the data of the 110 degree and 100 degree runs were given less value in the interpretation of the data. The data at low temperatures were taken in January and February, 1930. The heat was shut off from the room in which the tank was located and the windows were kept open. By these means the temperature of the water was reduced to 48 degrees F. Later ice and clean snow were added to the tank water, thereby reducing the temperature to 43.5 degrees F., the lowest temperature at which a set of observations was taken.

The procedure in each test was the same. A fixed weight was placed on the scale pan of the towing dynamometer and when the plane was traveling at constant speed, after having covered about one half the length of the tank, speed measurements were taken. Two or three runs were made with each load and the recorded speeds were averaged. The average deviation from the mean speed was .0027 ft. per second. The results of the runs are given in the appended tables 2 to 11. In these tables are set down side by side: The temperature of the water which was checked at frequent intervals; the measured speed in feet per second; the gross resistance which is equal to one tenth of the weight on the scale pan; the tare resistance which was found to vary according to the equation: Tare .00258 + .00029v; the net resistance F; the "mean specific resistance" c_f ; and Reynolds' number $R = \frac{v l}{\nu}$. The mean specific resistance is defined by the equation:

Equation 12: $$c_f = \frac{F}{\frac{\rho}{2} A v^2}$$

in which the wetted surface A was taken as 12 sq. ft. at a temperature of 67 degrees F., at which temperature the plane had been trimmed as closely as possible to a draft of 12 inches. For the other temperatures the assumption was made that the wetted surface varied inversely as the two thirds power of the density. This is strictly correct only in the case of similar models, but the error committed in assuming it here can not be large.

The mean specific resistance c_f was first plotted on speed as the abscissa, which plot is shown in Plate 4a. It is seen that each constant temperature group of spots lies along a separate and distinct curve.

Next, the data were plotted as shown in Plate 4. It may be recalled that during the course of a run the load on the towing dynamometer was held constant. Hence, for each constant load a series of spots was obtained, showing directly the variation of speed with viscosity. It is seen that the spots can well be represented by straight lines on logarithmic paper.

As the next step, the mean specific resistance c_f was plotted on Reynolds' number $R = \frac{v l}{\nu}$ as the abscissa, as is shown in Plate 5. It appears at once, that on this basis all the spots for the roughened plane define one single curve which is divided into three parts. From this fact we may draw the general conclusion, that: THE EFFECT ON THE "MEAN SPECIFIC FRICTIONAL RESISTANCE", $c_f = \frac{F}{\frac{\rho}{2} A v^2}$ CAUSED BY A VARIATION OF THE TEMPERATURE OF THE MEDIUM AND BY A VARIATION OF THE SPEED OF THE BODY, AT CONSTANT LENGTH OF THE BODY, IS CORRECTLY EXPRESSED BY THE EQUATION:

Equation 13: $$c_f = \frac{F}{\frac{\rho}{2} A v^2} = \psi\left(\frac{v l}{\nu}\right)$$

This is perhaps the only definite conclusion we are justified in drawing from the rather limited scope of the tests.

When, nevertheless, the data are evaluated further this is done with the thought in mind of finding a convenient means of comparing them with other test results, rather than of setting up a new formula for extrapolating beyond the range of the tests.

Plate 6 allows us to make several general deductions. It is seen, that at low Reynolds' numbers the spots lie along a straight line which has a negative slope of one half. Comparison with equation 3 suggests that the flow at these low speeds is purely laminar. It is specially interesting to note that in this region there is practically no distinction between the roughened and the smooth plane, which is what one might expect from the definition of laminar flow. Turbulence begins at $R = 1.5 \times 10^5$ for the roughened plane and apparently is established at $R = 3 \times 10^5$. For the smooth plane, turbulence begins at $R = 3 \times 10^5$ and appears to be barely established at about $R = 7.5 \times 10^5$. After turbulence has been fully established, the spots for the roughened plane lie again along a straight line. This means that the function $\psi\left(\frac{v l}{\nu}\right)$ is of the simple exponential type or nearly so. Before evaluating the function, however, the influence of the roughened forward edges will be considered more in detail.

Plate 6 shows, that at $R = 7 \times 10^5$ the c_f values of the roughened plane and those of the smooth plane differ approximately by .00085 . It seems fair to assume that this differnce is constant at all Reynolds' numbers within the turbulent range. This is equivalent to making the assumption that the "roughness resistance" varies as the square of the speed, or that the law of turbulent frictional resistance of the roughened plane can be expressed by the following equation:

Equation 14: $$F = \left[.00085 + c\left(\frac{vl}{\nu}\right)^m\right]\frac{\rho}{2} A v^2$$

With this assumption we obtain the mean specific frictional resistance for a hypothetical smooth plane by subtracting .00085 from each spot within the turbulent region. The results are shown in Plate 6. The spots lie now along a straight line which has the following equation:

Equation 15: $$C_f = .0242\left(\frac{vl}{\nu}\right)^{-.12}$$

Referring now again to Plate 4, we have seen that for constant resistance the speeds were an exponential function of the viscosity. Considering turbulent flow only, the average slope of the nine lines drawn is -.0654. Writing the equation for the frictional resistance in the simple exponential form:

Equation 16: $$F = \frac{\rho}{2} A v^2 k\left(\frac{vl}{\nu}\right)^m$$

and transforming, we get: $v = K\, \nu^{\frac{m}{2+m}}$

Hence, $-.0614 = \frac{m}{2+m}$ from which, $m = -.1227$.

This agrees very closely with the value found in equation 15.

In order to facilitate comparison, lines representing Gebers' and the v.Karman-Prandtl formulae were drawn in Plates 5 and 6. It is seen from Plate 6 that the line representing the v.Karman-Prandtl formula crosses the corrected experimental spots, but has a much steeper slope. On the other hand, the slope of Gebers' line is almost equal to that representing the experimental spots, but is consistently lower. Part of the difference is undoubtedly due to the fact that Gebers in the interpretation of his test results has purposely given preference to the lowest values measured and also has made an allowance for form resistance, as well as, for edge resistance, both of which effects have been neglected up to now in the evaluation of our data. In order to estimate the magnitude of these resistances, Dr. Gebers' published figures were taken (Ref.16) and applied to the present tests. The form resistance thus calculated amounted to about 4%, or more precisely the constant quantity .00019 which is to be subtracted from c_f. The edge resistance was estimated from Gebers' figures to be 2.2%. Making these allowances, the equation representing the test data finally became:

Equation 16: $$C_f = .0243\left(\frac{v l}{\nu}\right)^{-.125}$$

It may be observed that the exponent in the equation has exactly the same value as the exponent in Gebers' formula, but that the coefficient is 18% greater. It is also interesting to note, that in the laminar region the test results are 13% above the theoretical line.

In order to ascertain the cause of this discrepancy, the possibility of "wall effect" i.e. the disturbing influence of the basin walls was considered. Also, the possibility of the streamlines of one fin of the plane interfering with the streamlines of the other fin.

The possibility of a direct interference between the boundary layers of the two fins is remote, inasmuch as the maximum width of the boundary layer, estimated by the v.Karman-Prandtl formula, is of the order of .85 inch. The total width of the two boundary layers is hence of the order of 1.7 inches, which is less than one tenth the distance between the two fins.

The motion of any body through water, continually displacing fluid at the bow, necessarily causes a backward flow along the sides of the body. If the motion takes place in a restricted channel, instead of in a fluid of infinite extent, this back flow is naturally increased, causing an increase of the resistance of the body. In model basin practice it is ordinarily assumed that this increase in resistance is negligible when the cross sectional area of the basin is three hundred times the cross sectional area of the body. This, however, must not be regarded as a hard and fast rule, inasmuch as the wave making characteristics of the body and its speed are also important factors.

In our case, the cross sectional area of the basin is only 177 times the cross sectional area of the plane, but, as been pointed out before, the speed of the plane was not pushed beyond the point where wave making became appreciable and even that speed is only about one third the speed of the translational wave, which is calculated by the formula: $v = \sqrt{gH}$, where, g is the acceleration of gravity and H is the depth of the basin in feet. Hence, we need only consider the increase in frictional resistance caused by the back flow. This was done, and, without going into the details of the calculation, it was found that this increase did not exceed 1% when considering the cross section of the basin as a whole, and 1.5% when considering the section between the two fins of the plane and the section surrounding the plane separately.

From the above analysis we may conclude, that the experimental arrangement, which was largely dependent on the apparatus available, did not introduce material errors.

SUMMARY.

In this paper are described tests which were made at the United States Experimental Model Basin with a 3'x 1' catamaran friction plane, in order to determine the influence of temperature on the frictional resistance of plane surfaces, towed in water. Both, the speed of the plane and the temperature of the water were varied. The mean specific resistance was found to vary with the speed of the plane and the temperature of the water according to Rayleigh's law. A formula interpreting the test data was deduced which was compared with similar formulae by other experimenters. Finally, possible experimental errors were discussed and their magnitude estimated.

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TABLE OF SYMBOLS.

x, y, z : Coordinates in a trirectangular system. x-coordinate in the direction of the motion.

u, v, w : Components of velocity in the x, y, z directions.

p : Pressure.

∇^2 : Differential operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

v : Velocity in the direction of the motion, in ft. per sec.
V_1 : " " " " " " in knots.

l : Length of the body in feet.
b : Breadth " " " "

A : Area of the wetted surface of the body in feet^2.

F : Frictional resistance of the body in lbs.

ρ : Density of the medium.
γ : Specific gravity of the medium.
μ : Coefficient of viscosity of the medium.
ν : Coefficient of kinematic viscosity of the medium.

φ : Function of.

R : Reynolds number = $\frac{v\,l}{\nu}$

k, K : Constant coefficients.
m, n : Exponents.

TABLE 1.

Weight of a cubic foot of water at 4° C. equals 62.425 lbs.

Acceleration of gravity "g" equals 32.174 feet per sec.^{2}

Density in foot-pound-second units at 4° C. equals 1.94016

Density at other temperatures equals 1.94016 × specific gravity.

T in °C.	Sp. Gr.	Visc. x 10^5	T in °C.	Sp. Gr.	Visc. x 10^6
1	.99993	1.8638	29	.99598	.[illegible]406
2	.99997	1.8007	30	568	.86563
3	.99999	1.7429	31	537	.84786
4	1.0000	1.7429	32	506	.83070
5	.99999	1.6349	33	473	.81407
6	.99997	1.5854	34	440	.79791
7	993	1.5377	35	406	.78237
8	988	1.4921	36	371	.76747
9	981	1.4494	37	336	.75279
10	973	1.4080	38	300	.73864
11	963	1.3690	39	263	.72492
12	952	1.3314	40	225	.71166
13	940	1.2925	41	187	.69876
14	927	1.2613	42	147	.68626
15	913	1.2258	43	107	.67416
16	897	1.1972	44	066	.66248
17	880	1.1670	45	025	.65091
18	862	1.1382	46	.98982	.63977
19	843	1.1104	47	940	.62906
20	823	1.0837	48	896	.61856
21	802	1.0581	49	852	.60849
22	780	1.0324	50	807	.59852
23	757	1.0098	51	762	.58899
24	733	198669	52	715	.57956
25	708	.96481	53	669	.57045
26	682	.94346	54	621	.56167
27	656	.92298	55	573	.55298
28	627	.90326			

TABLE 2

Forward edge smooth.

Date: September 16, 1929.

Run No.	Temp. Deg. F. T	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. R	$\frac{R}{\frac{\rho}{2} A V^2}$	Reyn'ds No. R
1	74.2	.3976	.01	.00226	.00774	.004219	1.194×10^5
2	74.2	.6797	.02	.00227	.01773	.003307	2.041 "
3	74.2	.9061	.03	.00229	.02771	.002892	2.728 "
4	74.2	1.043	.04	.00231	.03769	.002980	3.132 "
5	74.2	1.132	.05	.00232	.04767	.003203	3.402 "
6	74.3	1.160	.06	.00233	.05767	.003690	3.487 "
7	74.3	1.290	.07	.00236	.06764	.003502	3.877 "
8	74.4	1.403	.09	.00239	.08761	.003834	4.214 "
9	74.4	1.557	.12	.00242	.11758	.004209	4.679 "
10	74.4	1.708	.15	.00245	.14755	.004358	5.133 "
11	74.2	1.954	.20	.0025	.1975	.004466	5.872 "
12	74.2	2.152	.25	.0025	.2474	.004601	6.466 "
13	74.2	2.333	.30	.0026	.2974	.004710	7.012 "
14	74.3	2.510	.35	.00265	.3474	.004749	7.542 "
15	74.3	2.684	.40	.0027	.3973	.004747	8.062 "
16	74.3	2.836	.45	.00272	.[illegible]775	.004780	8.523 "

TABLE 5

Forward edge roughened.

Date; September 28, 1929.

Run No.	Tem. Deg. F	Speed Ft. Sec. v	Gross Resist.	Tare	Net Resis. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reyn'ds No. R
1	110	2.971	.55	.00344	.54656	.004889	1.324×10^5
2	109.8	2.832	.50	.00340	.49660	.006340	12.622 "
3	109.5	2.682	.45	.00336	.44664	.005353	11.956 "
4	109.1	2.522	.40	.00331	.39669	.005580	11.24 "
5	109.0	2.362	.35	.00328	.34672	.005358	10.535 "
6	110.5	2.193	.30	.00322	.29678	.005324	9.282 "
7	110.0	1.945	.25	.00314	.24686	.005624	8.670 "
8	109.5	1.750	.20	.00309	.19691	.005552	7.803 "
9	110.1	1.514	.15	.00302	.14698	.005528	6.750 "
10	109.9	1.174	.10	.00292	.09708	.006140	5.233 "
11	109.4	.8061	.05	.00283	.04717	.006258	3.592 "
12	109.1	.7107	.04	.00279	.03721	.006560	3.155 "
13	109.0	.6097	.03	.00278	.02722	.006313	2.718 "
14	108.9	.4872	.02	.00272	.01728	.006277	2.172 "
15	108.8	.3304	.01	.00268	.00732	.006788	1.473 "

TABLE 4

Forward edge roughened.

Date: September 30, 1929.

Run No.	Temp. Deg.F.	Speed Ft.Sec. v	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reyn'ds Num. R
1	100	.3519	.010	.00268	.00732	.00510	1.4025×10^5
2	100	.4973	.020	.00272	.01728	.006020	1.984 "
3	99.9	.6058	.030	.00277	.02723	.006406	2.417 "
4	99.8	.7043	.040	.00279	.03721	.006463	2.911 "
5	100.4	.8273	.050	.00282	.04718	.005940	3.298 "
6	100.4	.9621	.070	.00286	.06714	.006260	3.837 "
7	100.2	1.148	.100	.00291	.09709	.006355	4.579 "
8	100	1.435	.150	.00300	.14700	.006153	5.72[illegible] "
9	100	1.682	.200	.00307	.19693	.006007	6.710 "
10	99.9	1.905	.250	.00313	.24687	.005860	7.600 "
11	100.9	2.1175	.300	.00320	.29680	.005713	8.442 "
12	100.8	2.2945	.350	.00325	.34675	.005675	9.159 "
13	100.1	2.4635	.400	.00329	.39671	.005680	9.800 "
14	100.4	2.613	.450	.00334	.44666	.005640	10.420 "
15	100.1	2.772	.500	.00338	.49662	.005272	11.060 "
16	100	2.938	.550	.00343	.54657	.005463	11.710 "

TABLE 4

Forward edge roughened.

Date: September 29, 1939.

Run No.	Temp. Deg. F	Speed Ft./Sec. V	Cross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A V^2}$	Reyn'ds Num. R
1	73	.3792	.010	.00269	.00731	.004361	1.422×10^5
2	"	.5403	.020	.00274	.01726	.004996	2.029 "
3	"	.6657	.030	.002784	.02724	.006814	2.364 "
4	"	.7281	.040	.00279	.03721	.006043	2.732 "
5	"	.8193	.050	.00282	.04718	.006052	3.073 "
6	"	.9895	.070	.00287	.06713	.005913	3.710 "
7	"	1.1915	.100	.00293	.09707	.005898	4.472 "
8	"	1.482	.150	.00302	.14698	.005762	5.562 "
9	"	1.7282	.200	.00308	.19692	.005686	6.482 "
10	"	1.952	.250	.00315	.24685	.005580	7.323 "
11	"	2.1485	.300	.00320	.29680	.005542	8.060 "
12	"	2.331	.350	.00326	.34674	.005498	8.748 "
13	"	2.501	.400	.00331	.39669	.005462	9.390 "
14	"	2.618	.450	.00335	.44665	.005448	9.[illegible] "
15	"	2.802	.500	.00340	.49660	.005448	10.513 "
16	"	2.936	.550	.00343	.54657	.005463	11.012 "

Table 6

Forward edge roughened.

Date: September 29, 1929.

Run No.	Temp. Deg. F	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A V^2}$	Reyn'ds Num. R
1	82.2	.3943	.010	.00269	.00731	.004262	1.2965×10^5
2	"	.5461	.020	.00274	.01726	.004985	1.806 "
3	"	.6428	.030	.00277	.02723	.005678	2.127 "
4	"	.7245	.040	.00280	.03720	.006100	2.379 "
5	"	.8133	.050	.00282	.04718	.006188	2.869 "
6	"	.9741	.700	.00286	.06714	.006098	3.219 "
7	"	1.182	.100	.00292	.09708	.005982	3.907 "
8	"	1.469	.150	.00301	.14699	.005860	4.857 "
9	"	1.716	.200	.00308	.19692	.005760	5.675 "
10	"	1.929	.250	.00314	.24686	.005712	6.377 "
11	"	2.131	.300	.00320	.29680	.005628	6.739 "
12	"	2.303	.350	.00325	.34675	.005522	7.620 "
13	"	2.453	.400	.00329	.39671	.005672	8.118 "
14	"	2.632	.450	.00334	.44666	.005548	8.703 "
15	"	2.785	.500	.00338	.49662	.005622	9.205 "
16	"	2.917	.550	.00343	.54657	.005522	9.644 "

TABLE 7

Forward edge roughened.

Date: September 21, 1929.

Run No.	Temp. Deg. F	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reynolds Number R
1	66.8	.3638	.010	.0024	.0076	.004943	.9758 $\times 10^5$
2	"	.5844	.020	"	.0176	.004433	1.569 "
3	"	.6815	.030	"	.0276	.005117	1.8265 "
4	"	.7629	.040	"	.0376	.005157	2.047 "
5	"	.8369	.050	"	.0476	.005048	2.242 "
6	"	.9797	.070	.0025	.0675	.006059	2.628 "
7	"	1.111	.090	"	.0875	.00610	2.980 "
8	"	1.2935	.120	"	.1175	.006040	3.468 "
9	"	1.457	.150	.0026	.1474	.005972	3.908 "
10	"	1.650	.190	"	.1874	.005970	4.427 "
11	"	1.871	.240	.0027	.2373	.005838	5.018 "
12	"	2.068	.290	"	.2873	.005788	5.545 "
13	"	2.2495	.340	"	.3373	.005732	6.036 "
14	"	2.4135	.390	.0028	.3872	.005718	6.478 "
15	"	2.611	.450	"	.4472	.005642	7.003 "
16	"	2.7545	.500	.0029	.4971	.005638	7.393 "
17	"	2.881	.550	"	.5471	.005640	7.[illegible]12 "

TABLE 8

Forward edge roughened.

Date: September 21,1929.

Run No.	Temp. Deg. F	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reynolds Number R
1	61.9	.5980	.020	.00265	.01735	.004174	1.511×10^5
2	"	.7861	.040	.00268	.03732	.005190	1.987 "
3	"	.9199	.060	.00271	.05729	.005822	2.324 "
4	"	1.047	.080	.00273	.07727	.006066	2.648 "
5	"	1.1745	.100	.00275	.09725	.006062	2.969 "
6	"	1.399	.140	.00279	.13721	.006028	3.534 "
7	"	1.5515	.180	.00282	.17718	.005975	3.922 "
8	"	1.779	.220	.00287	.21713	.005960	4.497 "
9	"	1.947	.260	.00289	.25711	.005837	4.925 "
10	"	2.099	.300	.00292	.29708	.005802	5.308 "
11	"	2.241	.340	.00295	.33705	.005768	5.663 "
12	"	2.369	.380	.00297	.37703	.005779	5.985 "
13	"	2.501	.420	.00299	.41701	.005735	6.321 "
14	"	2.626	.460	.00301	.45699	.005700	6.638 "
15	"	2.740	.500	.00303	.49697	.005693	6.924 "

TABLE 8

Forward edge roughened.

Date: January 7, 1930.

Run No.	Temp. Deg. F	Speed Ft. Sec. v	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reynolds Number R
1	56.4	.3311	.010	.00268	.00732	.005745	.7800 × 10^6
2	56.5	.5737	.020	.00275	.01725	.004510	1.351 "
3	56.7	.6574	.030	.00277	.02723	.005261	1.511 "
4	"	.7223	.040	.00279	.03721	.006130	1.702 "
5	56.6	.8852	.050	.00284	.04716	.005178	2.084 "
6	"	.8770	.050	.00283	.04717	.005276	2.067 "
7	56.5	1.002	.070	.00287	.06713	.005752	2.362 "
8	"	1.176	.100	.00292	.09709	.006040	2.771 "
9	"	1.437	.150	.00300	.14700	.006120	3.384 "
10	"	1.6665	.200	.00306	.19694	.006078	3.927 "
11	"	1.869	.250	.00312	.24688	.006078	4.403 "

TABLE 10

Forward edge roughened.

Date: January 22, 1930.

Run No.	Temp. Deg. F	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A v^2}$	Reynolds Number R
1	42.1	.3187	.010	.00267	.00733	.006208	$.6594 \times 10^5$
2	"	.5685	.020	.00275	.01725	.004361	1.176 "
3	"	.7425	.030	.00280	.02720	.004246	1.536 "
4	"	.83282	.040	.00282	.03717	.004560	1.732 "
5	"	.9104	.050	.00284	.04715	.004892	1.884 "
6	"	1.0305	.070	.00288	.06712	.005438	2.132 "
7	"	1.189	.100	.00292	.09707	.005908	2.461 "
8	"	1.2875	.120	.00295	.11705	.006152	2.665 "
9	"	1.4395	.150	.00300	.14700	.006100	2.980 "
10	"	1.5428	.170	.00303	.16697	.006031	3.193 "
11	"	1.6755	.200	.00307	.19693	.006032	3.468 "
12	"	1.877	.250	.00312	.24688	.006025	3.883 "
13	"	2.070	.300	.00318	.29682	.005968	4.282 "
14	"	2.2435	.350	.00323	.34677	.005868	4.653 "
15	"	2.402	.400	.00328	.39672	.005898	4.971 "
16	"	2.5545	.450	.00332	.44668	.005885	5.285 "
17	"	2.702	.500	.00336	.49664	.005845	5.590 "

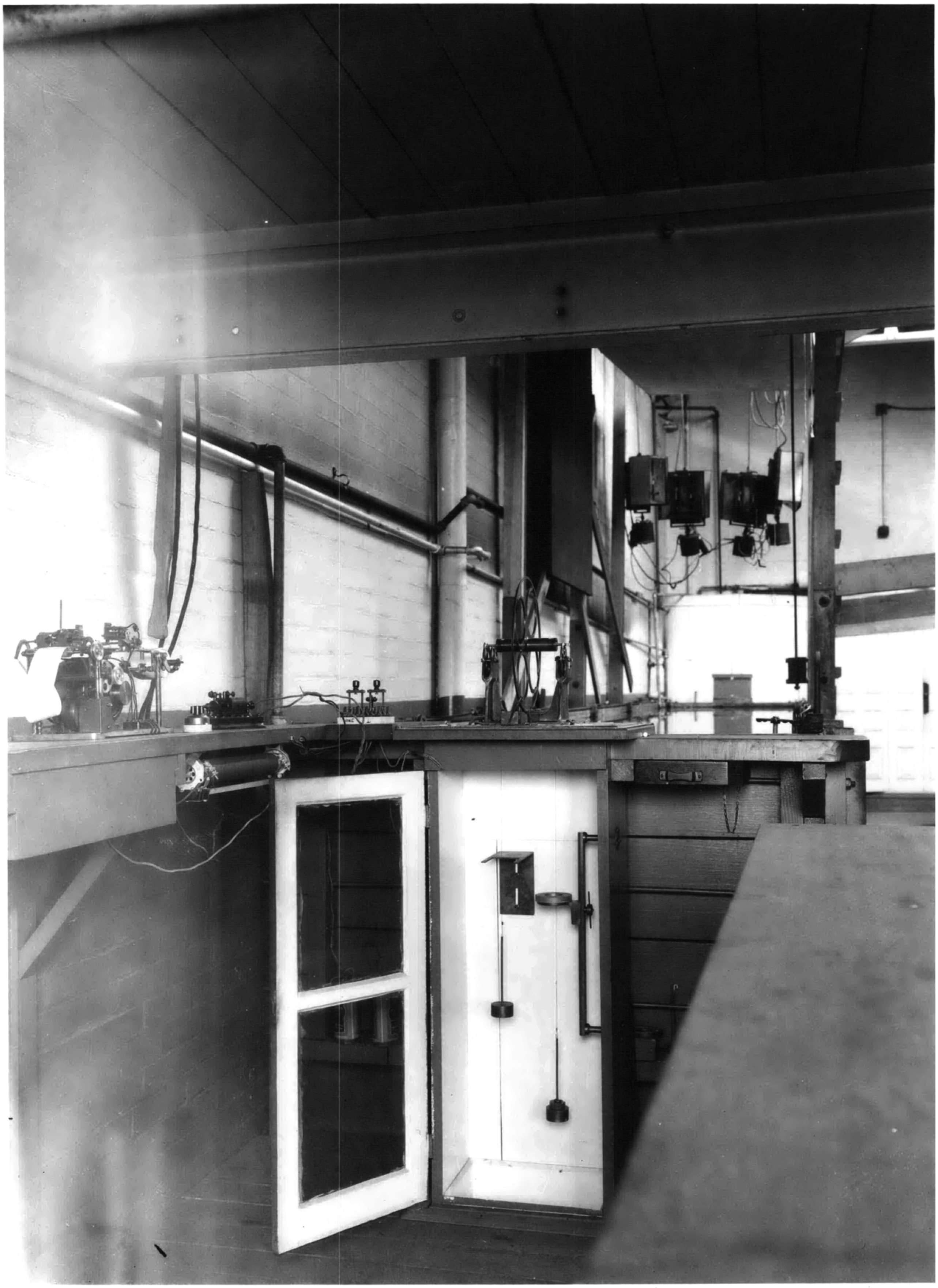
TABLE 11.

Forward edge roughened.

Date: February 1, 1930.

Run No.	Temp. Deg. F	Speed Ft. Sec. V	Gross Resist.	Tare	Net Resist. F	$\frac{F}{\frac{\rho}{2} A V^2}$	Reynolds Number $\frac{VL}{\nu}$
1	43.5	.3687	.010	.00269	.00731	.004627	$.7285 \times 10^6$
2	"	.5752	.020	.00275	.01725	.004483	1.101 "
3	"	.6863	.030	.00278	.02722	.004972	1.374 "
4	"	.8637	.040	.00284	.03716	.004091	1.590 "
5	"	.9462	.050	.00285	.04716	.004832	1.811 "
6	"	1.070	.070	.00289	.06711	.005043	2.049 "
7	"	1.217	.100	.00293	.09707	.005700	2.330 "
8	"	1.306	.120	.00296	.11704	.005897	2.500 "
9	"	1.446	.150	.00300	.14700	.006042	2.769 "
10	"	1530	.180	.00304	.17696	.006017	3.043 "
11	"	1.677	.200	.00307	.19693	.006025	3.211 "
12	"	1.877	.250	.00312	.24688	.006022	3.590 "
13	"	2.066	.300	.00318	.29682	.005965	3.950 "
14	"	2.236	.350	.00323	.34677	.005980	4.277 "
15	"	2.399	.400	.00328	.39672	.005920	4.590 "
16	"	2.551	.450	.00332	.44667	.005900	4.882 "
17	"	2.703	.500	.00336	.49664	.005842	5.175 "

Bt



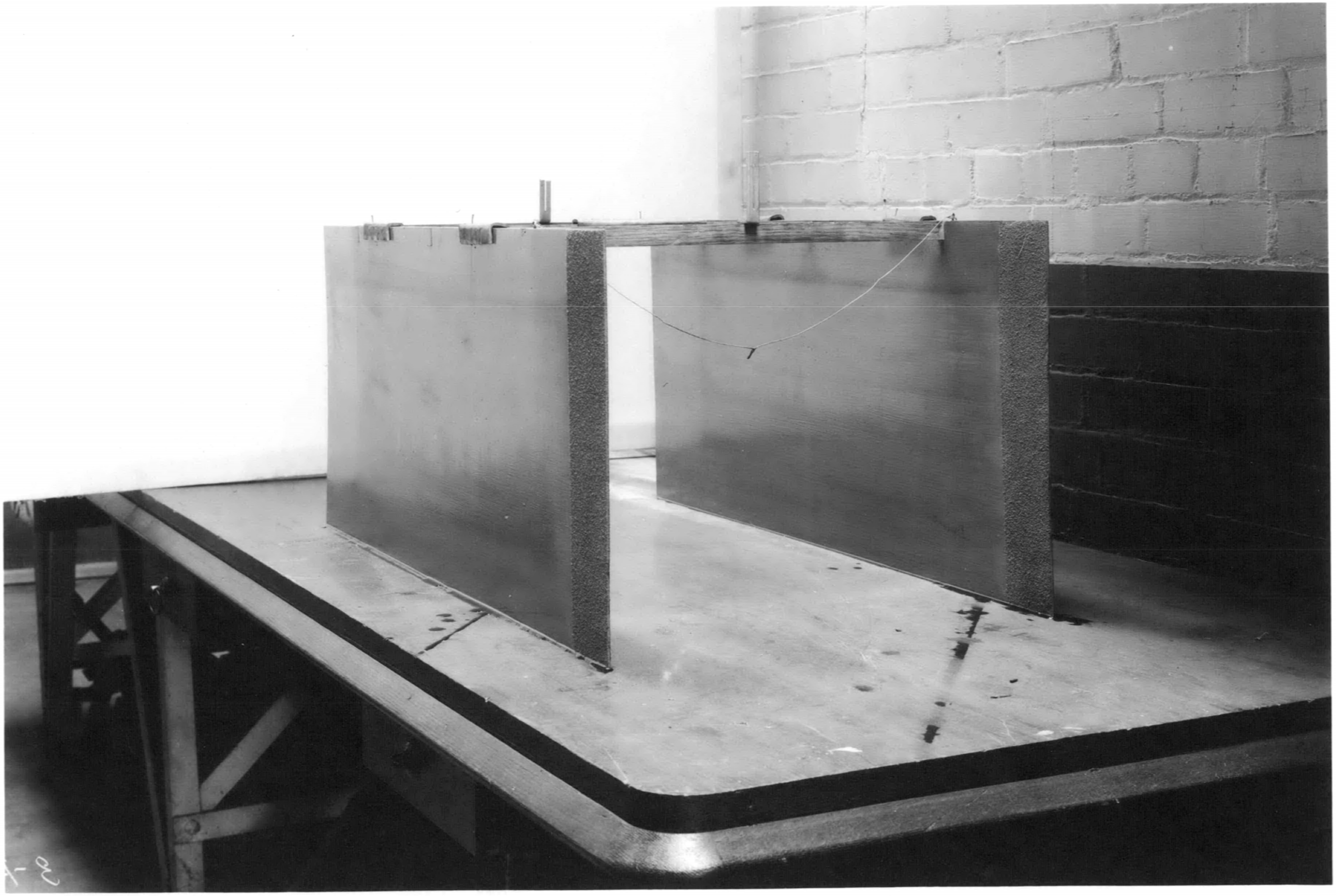
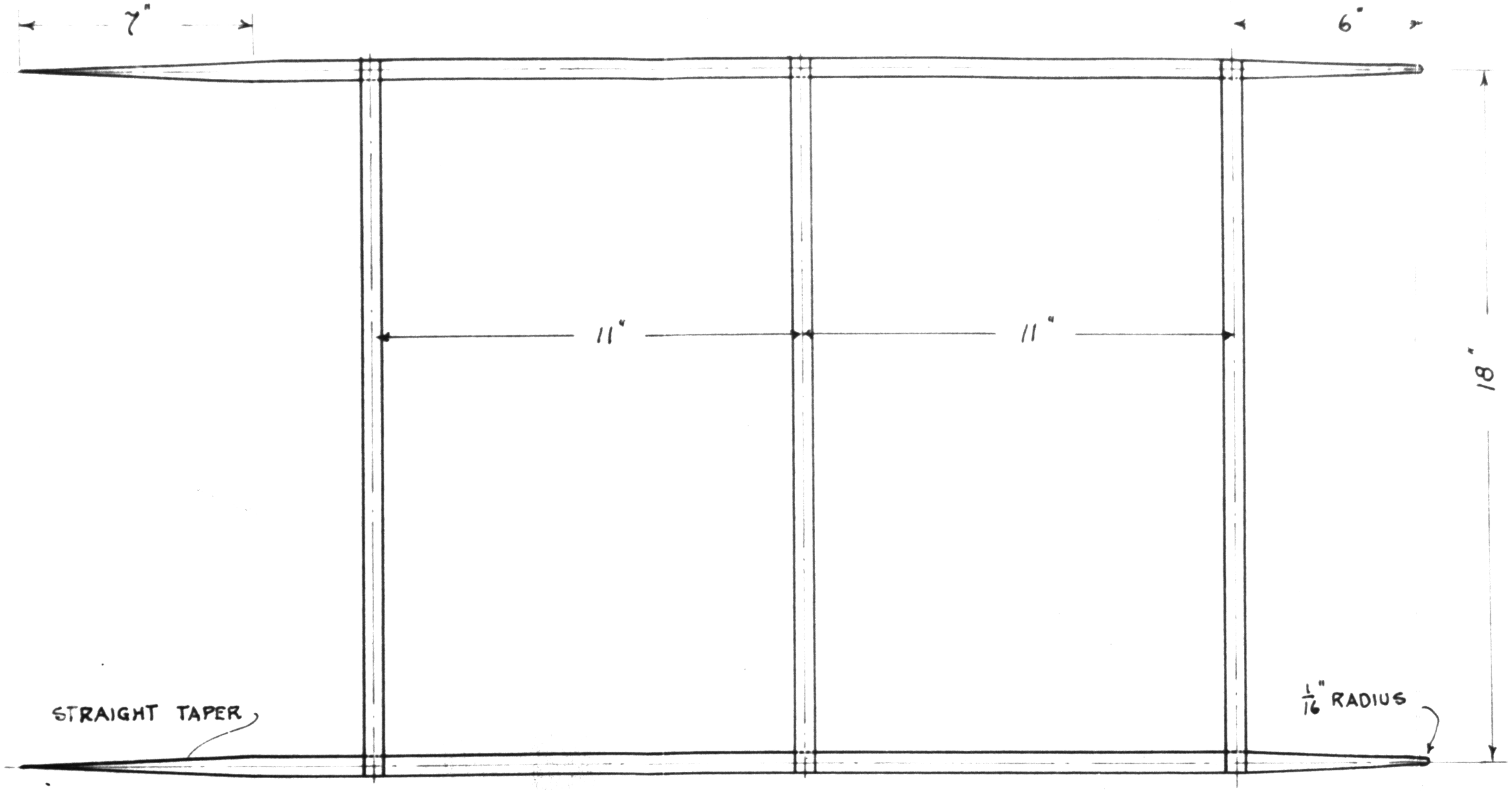


PLATE No 3

CATAMARAN FRICTION PLANE

SCALE: 1" = 4"



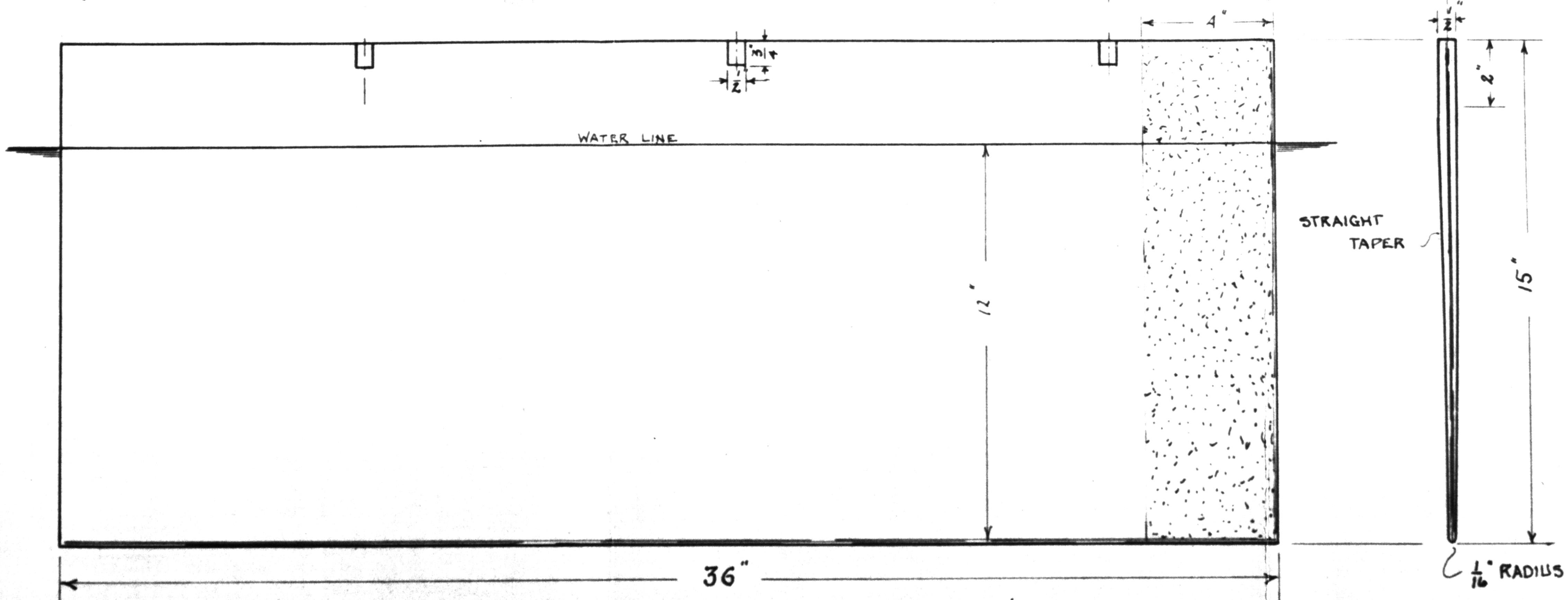
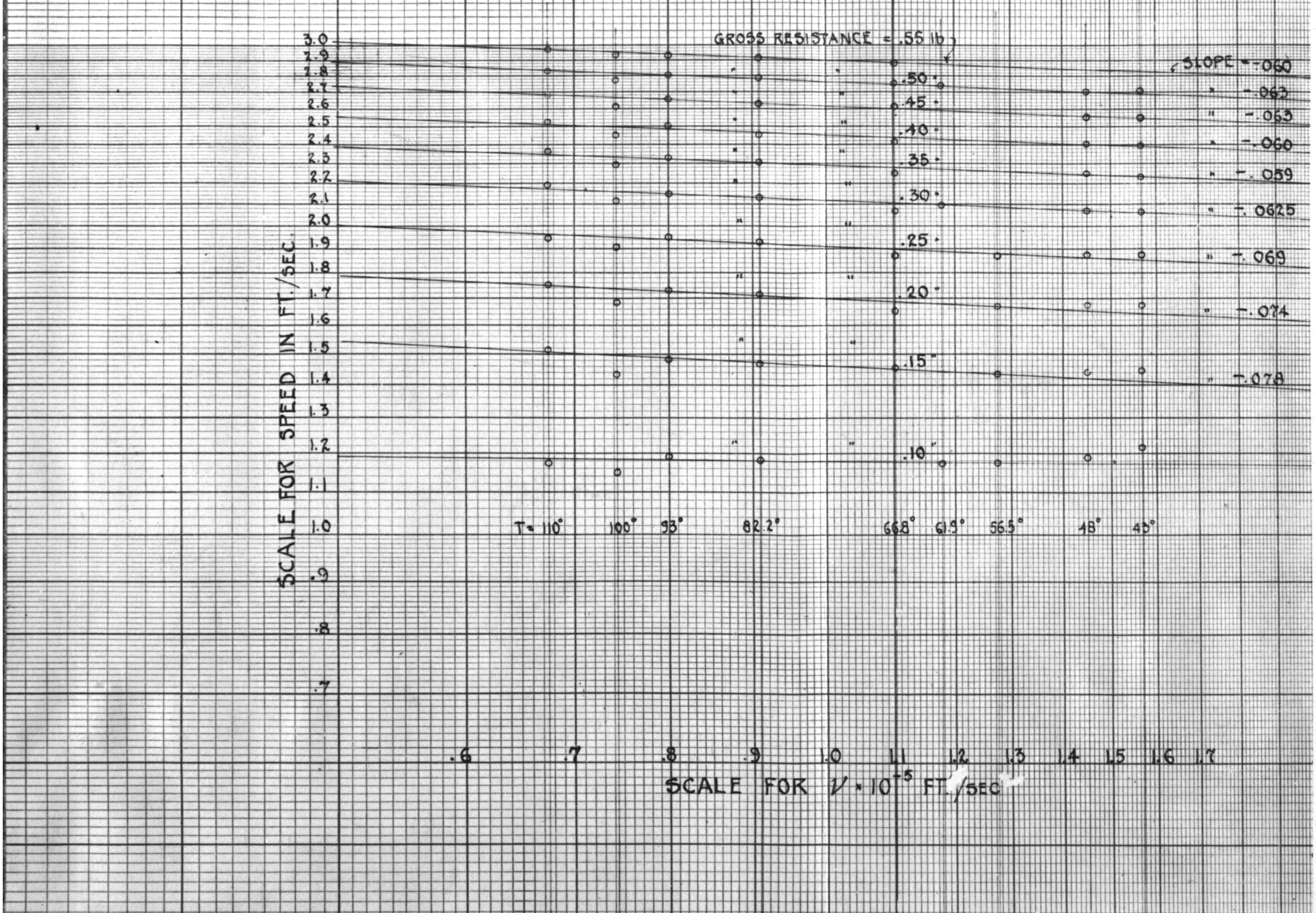


PLATE Nº 4



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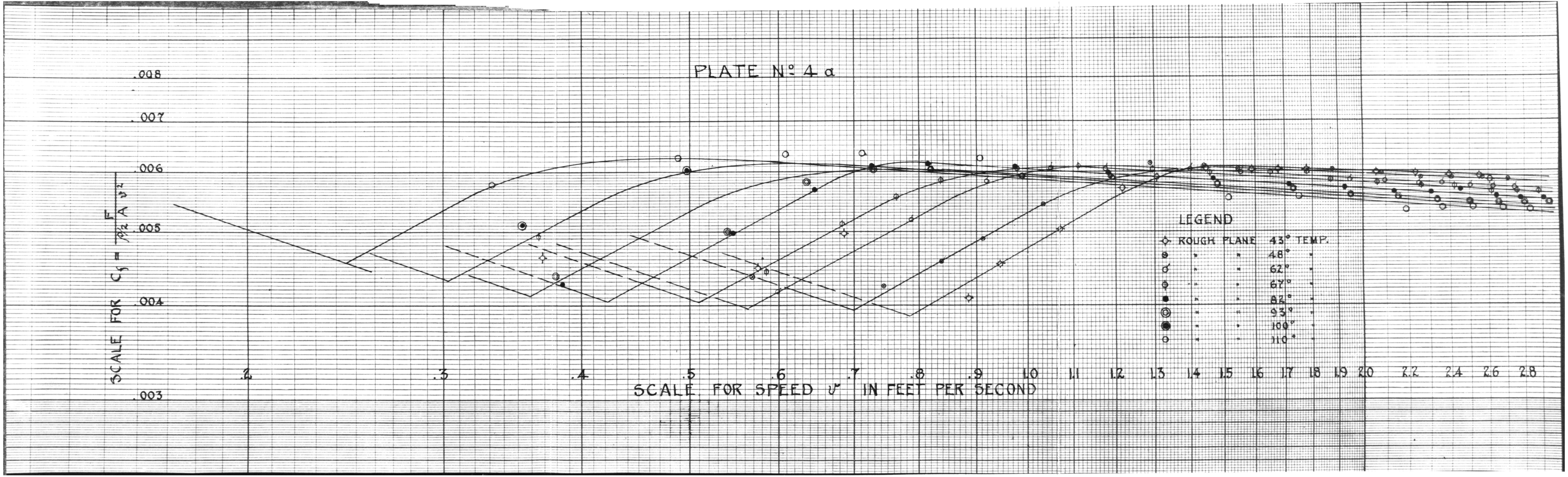
.40
.35
.30
.25
.20
.15
.10
SLOPE = -.060
-.063
-.063
-.060
-.059
-.0625
-.069
-.074
-.078
T = 110°
100°
93°
82.2°
66.8°
61.9°
56.5°
48°
43°
SCALE FOR SPEED IN FT./SEC.
SCALE FOR ν × 10⁻⁵ FT./SEC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.004
.003
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.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.2
2.4
2.6
2.8
SCALE FOR SPEED v IN FEET PER SECOND
LEGEND
ROUGH PLANE 43° TEMP.
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" " 82° "
" " 93° "
" " 100° "
" " 110° "

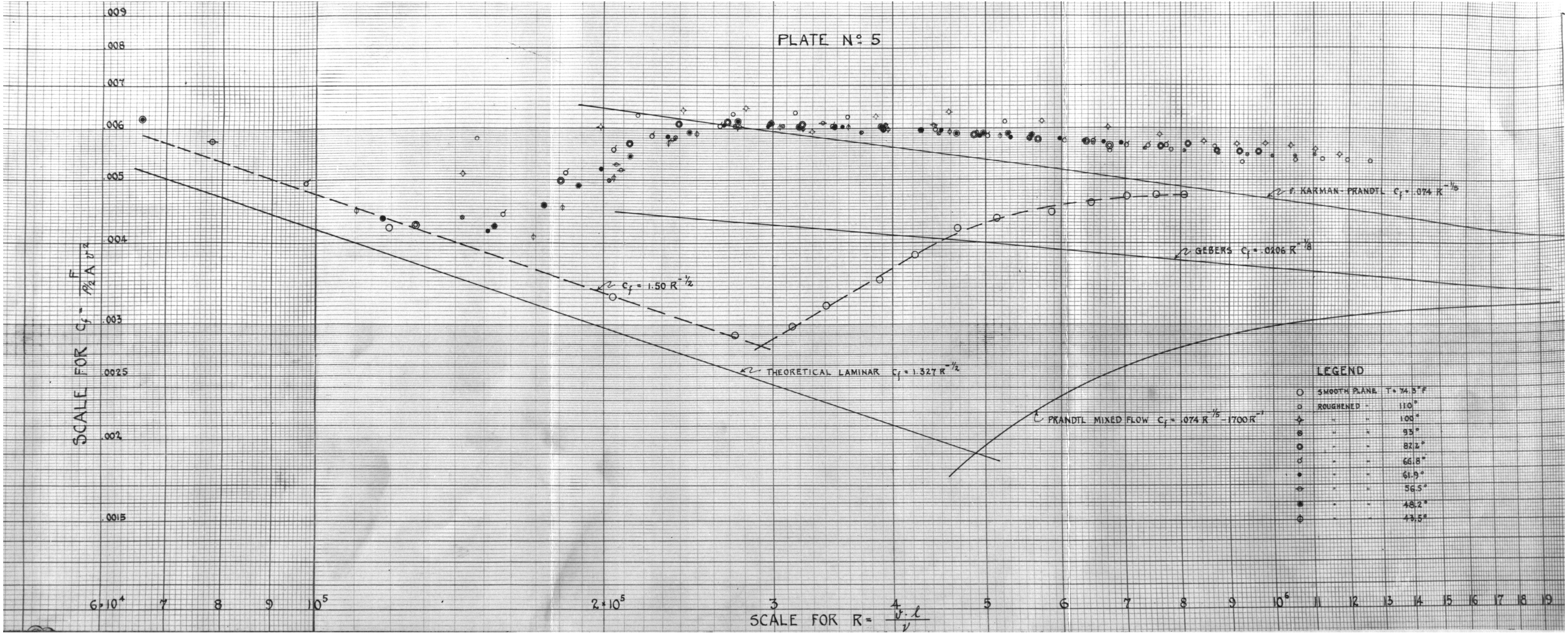

PLATE No 5
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.007
.006
.005
.004
.003
.0025
.002
.0015
SCALE FOR $R = \frac{v \cdot l}{\nu}$
$6 \cdot 10^4$ 7 8 9 10^5 $2 \cdot 10^5$ 3 4 5 6 7 8 9 10^6 11 12 13 14 15 16 17 18 19
v. KARMAN - PRANDTL $C_f = .074\, R^{-1/5}$
GEBERS $C_f = .0206\, R^{-1/8}$
$C_f = 1.50\, R^{-1/2}$
THEORETICAL LAMINAR $C_f = 1.327\, R^{-1/2}$
PRANDTL MIXED FLOW $C_f = .074\, R^{-1/5} - 1700\, R^{-1}$
LEGEND
SMOOTH PLANE T = 74.3°F
ROUGHENED " 110°
" " 100°
" " 93°
" " 82.2°
" " 66.8°
" " 61.9°
" " 56.5°
" " 48.2°
" " 43.5°

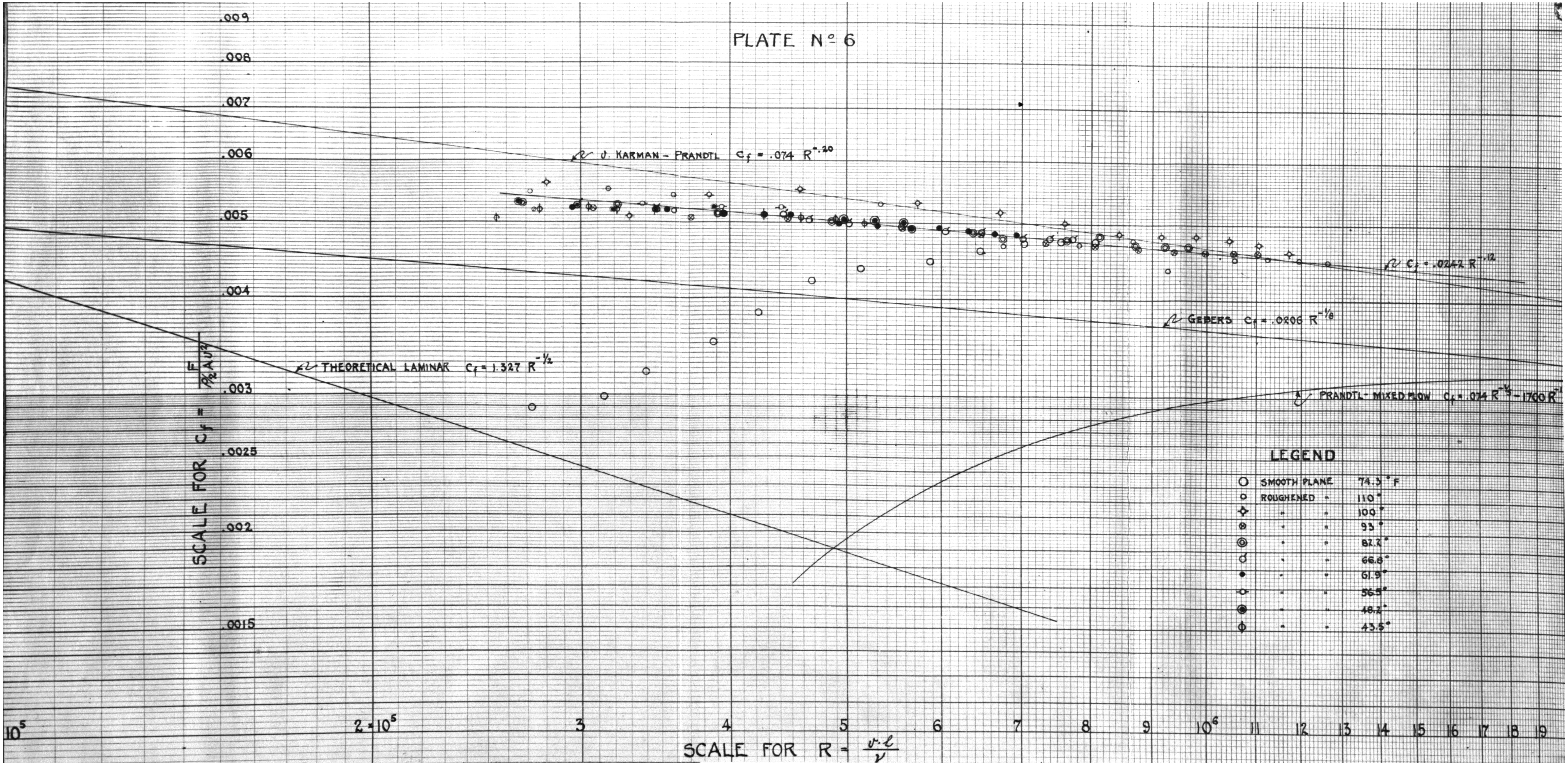

PLATE N° 6
.009
.008
.007
.006
.005
.004
.003
.0025
.002
.0015
V. KARMAN - PRANDTL $C_f = .074\ R^{-.20}$
$C_f = .0242\ R^{-.12}$
GEBERS $C_f = .0206\ R^{-1/8}$
THEORETICAL LAMINAR $C_f = 1.327\ R^{-1/2}$
PRANDTL - MIXED FLOW $C_f = .074\ R^{-1/5} - 1700\ R^{-1}$
SCALE FOR $C_f = \frac{F}{\rho/2\ A U^2}$
LEGEND
SMOOTH PLANE 74.3° F
ROUGHENED " 110°
" " 100°
" " 93°
" " 82.2°
" " 66.8°
" " 61.9°
" " 56.5°
" " 48.2°
" " 43.5°
10^5
2×10^5
3
4
5
6
7
8
9
10^6
11
12
13
14
15
16
17
18
19
SCALE FOR $R = \frac{v \cdot \ell}{\nu}$