APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

PROJECT WHIRLWIND

SUMMARY REPORT NO. 30
SECOND QUARTER 1952

Submitted to the

OFFICE OF NAVAL RESEARCH
Under Contract N5ori60
Project NR 048-097

DIGITAL COMPUTER LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Massachusetts

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

TABLE OF CONTENTS

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are the development of an electronic digital computer of large capacity and very high speed, and its application to problems in mathematics, science, engineering, simulation, and control.

The Whirlwind Computers

The Whirlwind computer is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the electrostatic storage tube, which uses an electron beam for storing digits as positive or negative charges on a storage surface.

Whirlwind I (WWI) may be regarded as a prototype from which other computers will be evolved. It is being used both for a study of circuit techniques and for the study of digital computer applications and problems.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access electrostatic storage initially had a capacity of 4096 binary digits, sufficient for some actual problems and for preliminary investigations in most fields of interest. This capacity is being gradually increased toward the design figure of 32,768 digits. Present speed of the computer is 20,000 single-address operations per second, equivalent to about 6000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

Reports

Quarterly reports are issued to maintain a supply of up-to-date information of the status of the Project. Detailed information on technical aspects of the Whirlwind program may be found in the R-, E-, and M-series reports and memorandums that are issued to cover the work as it progresses. Of these, the R-series are the most formal, the M-series the least. A list of the publications issued during the period covered by this Summary, together with instructions for obtaining copies of them, appears in the Appendix.

1. QUARTERLY REVIEW (AND ABSTRACT)

Operation of the computer during this quarter was increased from 89 hours per week to 146 hours. Part of the additional time was assigned to another project, part to the Applications Group, who have been working on the solution of a number of engineering and scientific problems. Twenty-four of the most interesting problems are described in Section 4.1 of this report.

During the quarter the computer has yielded 80% useful time.

The reduction from the 85% figure of the previous quarter does not indicate an actual reduction in the reliability of the computer, as explained in Section 2.

A Raytheon magnetic-tape drive being used on an interim basis has proved highly reliable. Final circuits are being installed to replace the temporary units. A new control desk has facilitated operation of the computer.

Performance and reliability of the power supplies have been significantly improved.

Failures of vacuum tubes increased somewhat over the last quarter. Part of the additional failures can be attributed to the increased hours of operation of the computer. Analysis of tube life and research on tube reliability continue.

Work on storage tubes, magnetic-memory devices, transistors, and terminal equipment formerly treated in these reports has been taken over by another project under Contract AF(122)-458 and from now on will be covered in the reports of that project.

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

2. OPERATION OF WHIRLWIND I

The computer schedule has been revised during this quarter to increase the time of operation from 89 to 146 hours per week and the applications hours from 33 to 73 hours per week. The computer has shown a general reliability of 80% useful time during the period. The reduction from the 85% figure of the preceding quarter may be accounted for by three factors:

1. The higher use factor of the computer, and the increased operation by inexperienced personnel during the third shift and week-ends.
2. The keeping of a more careful and complete record of ES alarms, and the additional time spent in determining their causes.
3. The rash of failures of resistors described in Section 3.2.

2.1 TERMINAL EQUIPMENT

A Raytheon magnetic-tape drive using our control and amplifier circuits has been installed on an interim basis and is being used by the Applications Group. Final circuits are being used to replace the temporary units, and the final system will be installed by the end of the next quarter. Except for small troubles encountered during replacement of temporary units and modification of some of the associated circuits, the interim system has given excellent reliability.

A new control desk (Fig. 2-1) was designed and installed. Its purpose is to consolidate in a convenient arrangement the controls and terminal equipment required to operate the computer. The following units are available at the desk.

1. The most frequently used push-button and switch controls
2. Two output scope display units
3. A photoelectric paper-tape reader

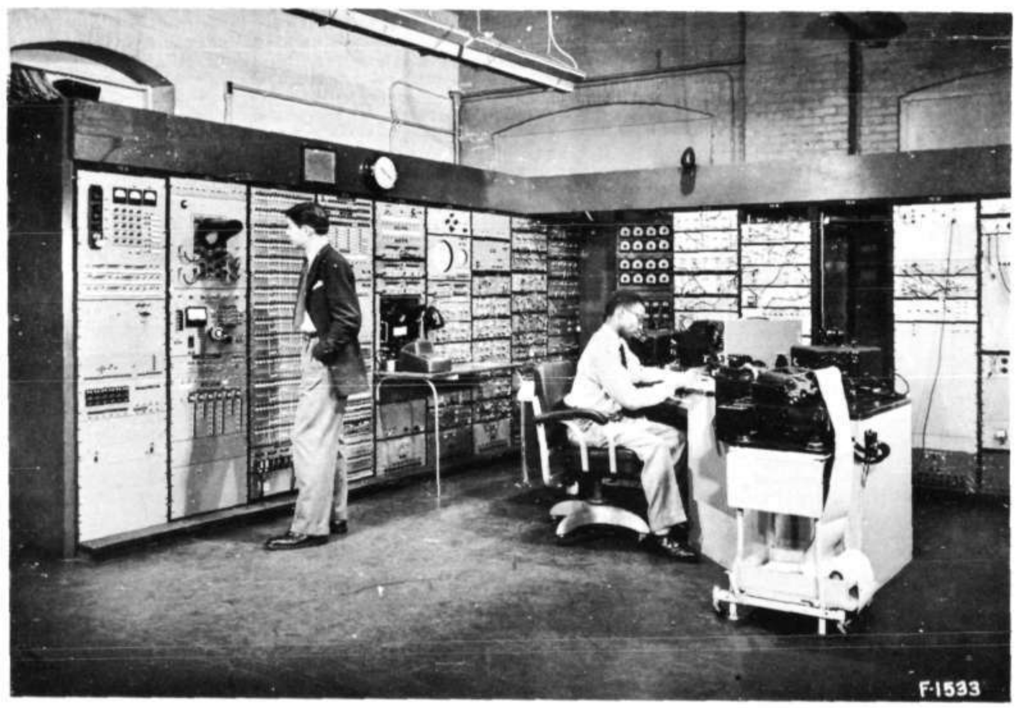

Fig. 2-1. New control desk.

4. An output printer
5. Mechanical paper-tape reader and punch
6. Telephone.

The desk is arranged for easy viewing of various indicator lights and for access to the more specialized controls of the operational control center.

2.2 POWER SUPPLIES

During the past few months considerable progress has been made in improving performance, increasing reliability, and facilitating servicing of the power supplies.

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

3. CIRCUITS AND COMPONENTS

3.1 VACUUM TUBES

3.11 Vacuum-Tube Life

During this past quarter the operating schedule of WWI was changed so that the total operation time was nearly 1700 hours instead of the previous 1000 hours per quarter. As a result, more tube failures should be expected during this period than previously. In many cases this increase has occurred. It is therefore better to analyze the data of Fig. 3-1 on the basis of the average failure rate rather than on simply the total number of failures.

The three most common tube types in WWI are the 7AD7, the 7AK7, and the 6SN7GT. Information on the average failure rates of these types was calculated as of August 1951 for service prior to that time. This data, as well as the failure rates reported in Summary Report 29, are summarized in the table below with current data.

Tube Type	Failure Rate, percent per 1000 hours		
	To August 1951	First Quarter 1952	Second Quarter 1952
7AD7	3.4	1.0	1.8
7AK7	0.4	0.06	0.3
6SN7GT	0.9	2.25	0.7

The variations in failure rate for the 7AK7 and 6SN7GT are probably influenced greatly by statistical fluctuations because of the relatively small number of failures. However, the changing emphasis of maintenance on different parts of the computer also may have had its effect. Marginal checking always is carried on, but troubles in one part of the computer cause closer looks at that part, with the result that weak tubes only remotely associated with any trouble are replaced. Variations from such a cause are difficult to evaluate quantitatively, and no attempt will be made to evaluate them here.

These variations in emphasis, as well as a change in the marginal-checking procedure for flip-flops, have no doubt caused the lower failure rate observed recently for 7AD7 tubes. It will be interesting to watch this failure rate over the coming months to see whether it climbs back to the old figure or remains at a considerably lower level.

Various factors have prevented the completion of the punched-card records in time for extensive analysis at this time. A very considerable amount of progress has been made, but certain details, including the checking of total operating time and the determination of operating functions for all tubes, have taken more time than expected. Listing and punching of some 5000 cards on 7AD7 tubes should be complete during the next quarter. Listing of another 5000 cards covering other tube types is complete except for minor details, but punching of this latter group is just getting started.

Examination of Fig. 3-1 will show that more tube types are listed than usual; many of these types are used in only small quantities in WWI. The longer operating time has contributed to the number of failures in all types. A few types, notably the 5U4G, have shown a rather large number of failures consistently over a long period. However, complete analysis of the behavior of these miscellaneous types must await more data than is easily available now.

3.12 Vacuum-Tube Research

Two life-test studies have been made during this period: a study of one lot of 6AS7G tubes for cathode flaking, and a study of plate-current changes in SR1407 tubes with age.

The 6AS7G tubes, of a lot made late in 1951, have not shown the cathode-peeling trouble associated with some of the earlier lots of these tubes. For the time being at least, these tubes can be used with somewhat more confidence than existed when serious trouble was found in the storage section of WWI. (See Section 3.12 of Summary Report 29 for details.)

In SR1407 tubes used in WWI, it frequently has been found that the level of plate current increased slightly after a thousand hours or so of operation. (The SR1407 is an

Type	Total in Service	Hours at Failure	Reason for Failure; number failed: Change in Characteristics	Mechanical	Burn-out	Gassy
7AD7	2190	1000-2000	1	1		
		2000-3000	5	3		
		3000-4000	7	4		
		5000-6000	1	1		
		6000-7000	3	1		
		7000-8000	3	2		
		8000-9000	2			
		9000-10000	5	6		
		10000-11000	9	8		
		11000-12000		2		
		12000-13000	2			
7AK7	1700	6000-7000	1			
		9000-10000	1			
		10000-11000		5		
		11000-12000		2		
6SN7GT	500	3605	1			
		8044	1			
		10000-11000		1	1	
		11000-12000	1		1	
2D21	40	10292	1			
2C51	20	0		5		
		10000-11000	2			
3E29	240	0	1			
		2000-3000	2			
		3000-4000	7			
		10852	1			
		11019				1
5651	34	0	1			
6AS7	165	356		1		
		1000-2000	1	3		
		3000-4000	1	3		
6AK5	88	2303	1	2		2
6AU6	8	4387	1			
6AL5	186	2967	1	1		
6AC7	10	6064				1
		10769			1	
		11298			1	
6AG7	120	4964	1			
		9515	1			
		12140	1			
6AS6	3	1081		1		
6AH6	15	0	1			
6Y6G	300	10000-11000		1		
		11000-12000	2	1	1	
6X5GT/G	28	11298	1			
6L6GA, G	111	607			1	
		3068	2			
6X4	17	0	1			
6J5	15	6550	1			
		11298	1			
715B	28	0	1			
C16J	12	1313	1			
		7000-8000	2			
3C	2	4335	1			
0D3	47	0	1			
5U4G	40	0	1			
		0	1			
		5037	2			
		7000-8000			2	

Fig. 3-1. Tube failures in WWI

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

improved version of the 7AD7.) This change had not been observed in previous life tests run primarily to study interface impedance, so it was decided to run another life test to check the trend of plate current on life. The data has not yet been checked completely, but a small decrease of plate current apparently occurred. It appears that a study of the differences between computer and life-test operation will be necessary, with perhaps a repeat of the life test under conditions more closely simulating computer operation.

The short-time instability previously mentioned (Summary Report 29) in conjunction with 715B troubles has been investigated more completely. Instabilities of the same kind have been found to exist in aged tubes of many different types. The most serious trouble is found when an interface impedance is associated with the cathode; however, the instability is not necessarily associated with interface impedance. This change in space-charge-limited current with time-constants in the one-second region can cause serious trouble in many different kinds of circuits including, for example, d-c amplifiers, precision gate generators, and digital-to-analogue converters. As a consequence, a more elaborate investigation will be made of this instability.

An accelerometer has been acquired to measure the acceleration imparted to vacuum tubes being tapped during short-testing operations. This device uses sensitive elements of barium titanate and is very light, weighing but 2 grams. The blows imparted to tubes being tested for tap shorts can now be measured quite accurately, since the mass of the accelerometer is so small that only minor changes in motion occur when the accelerometer is affixed to the tube. Calibrating measurements will be made for the more common tubes using this device. So far, only 7AD7 and 7AK7 tubes have been checked, for these types the acceleration has been set at about 150 g.

3.2 COMPONENT REPLACEMENTS IN WWI

Figure 3-2 lists the replacements of components other than tubes during the second quarter of 1952. A rash of intermittent troubles early in the quarter was traced to failures of the Nobleloy current-monitoring resistors in the storage-tube deflection circuits. Replacement of the failed resistors by carbon resistors has largely eliminated these troubles. Boro-carbon resistors will be used for any further replacements, as tests indicate that they can withstand heavy transient overloads without damage.

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitors	Mica 0.01 mfd	15072	1	10522	Shorted
	Bathtub 1.0 mfd	245	1	6503	Oil leakage
Crystals	D-357 (1N34a)	7700	12	8000-9000 9000-10000 10000-11000 11000-12000	2 drift; 1 low R_b; 1 oscillation 1 low R_b 2 low R_b; 1 shorted 1 intermittent open; 2 drift; 1 back resistance
	D-358 (1N38a)	3384	11	2823 5725 8000-9000 9000-10000 10000-11000	1 low R_b 1 low R_b 3 drift; 1 change in characteristics 1 drift 1 change in characteristics; 2 low R_b; 1 drift
Resistors	Carbon 2200 ohm 1 watt	285	5	9000-10000 10000-11000	3 over tolerance 2 over tolerance
	Nobleloy 5000 ohm 1 watt 1%	160	21	100-200 300-400 1000-2000 2000-3000 3000-4000 4000-5000 6000-7000	1 over tolerance 1 over tolerance 4 open; 1 over tolerance 4 over tolerance; 3 open 2 open; 3 over tolerance 1 over tolerance 1 over tolerance
	Variable 1000 ohm 2 watt	110	1	2789	Intermittent operation
	Nobleloy 1000 ohm 1 watt ± 1%	128	3	3 33 3518	Open Open Open
	1200 ohm 2 watt	567	1	11364	Over tolerance

Fig. 3-2. Component failures in WWI
April 1 - June 30, 1952

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

4. MATHEMATICS, CODING, AND APPLICATIONS

During the past quarter, the Applications Group has used the Whirlwind I computer on a number of problems arising at MIT and elsewhere in addition to working on the development of a library of subroutines and techniques for using the machine rapidly for general-purpose computation. The work accomplished during the quarter on 24 of the most interesting problems is described below.

During the next quarter, the computer will be shut down for a period of three weeks for installation of new terminal-equipment switching facilities. This new equipment will cause a considerable change in the order code of the computer and will invalidate all programs and subroutines thus far developed. This change in equipment will thus require a complete revision in the input conversion program as well as in the output subroutine now in use. Plans have been made for preparing a very versatile form of input conversion program along the line of the one described at the Pittsburgh meeting of the Association for Computing Machinery last May 2. The revised programs required by the new equipment will have the benefit of our experience with the present programs, and will represent a definite improvement.

Because of the number of problems that remain to be completed before the computer shutdown, and because of the time required to develop and test the new conversion program and to revise the subroutine library, no new problems will be accepted for computation during the coming quarter.

4.1 PROBLEMS BEING SOLVED

Problem #8. Magnetic Flux Density Study

Numerical solutions for the system of ordinary differential equations

$$\frac{\partial^2 H(x, t_j)}{\partial x^2} - \frac{B(x, t_j) - B(x, t_{j-1})}{\Delta t} = 0$$

have been obtained for the cases

(1) $H_o = 1/2 \qquad \Delta t = 0.1$

(2) $H_o = 1/4 \qquad \Delta t = 0.1$

(3) $H_o = 1/8 \qquad \Delta t = 0.1$

(4) $H_o = 1/8 \qquad \Delta t = 0.05$

by using a fourth order Runge-Kutta method; curves of flux vs time were plotted from these results. This system of equations arises in the solution of the partial differential equation

$$\frac{\partial^2 H}{\partial x^2} - \sigma \frac{\partial B}{\partial t} = 0 \qquad B = f(H)$$

by the method described in Summary Report 29. It was found, however, that the flux curves for cases 3 and 4 do not correspond within the limits of roundoff errors. It has been proposed that this is caused by the rapid changes in slope in the B-H curve being used, and a modification of the method is being programmed to test this assumption.

Problem #21. Optical Constants of Thin Metal Films

The program for the evaluation of optical constants has been recoded for floating-point, double-register computations, and the main program has been tested and found to be correct. It was found that the increased accuracy thus afforded was necessary, because slight inaccuracies appeared in results obtained previously with the program for single-register computation. The more important advantage of double-length computation in this problem is, however, the elimination of the danger of overflow, particularly since the order of magnitude of the results is not always predictable with the large range of experimental data considered. Now that the main program has been found to be correct, the following applications are under preparation for testing or are being tested:

(1) Evaluation of conductivities of metal blacks from infrared data.

(2) Automatic evaluation of optical constants without manual intervention on Whirlwind.

Problem #24. Matrices, Determinants, and Systems of Linear Equations

The following packaged programs for solving the linear system $A\bar{x} = \bar{b}$; A an n x n non-singular matrix, $\bar{b}$ and $\bar{x}$ n-dimensional vectors, in the (24, 6, 0) number system are now available:

Method	Capacity	Type
Gauss (Southwell) Relaxation	15 x 15 (non-symmetric)	Iterative
Gauss (Southwell) Relaxation	18 x 18 (symmetric)	Iterative
Gauss-Seidel	18 x 18 (symmetric)	Iterative
Jordan Elimination	16 x 16 (non-symmetric)	Direct
Shur-Shultz	11 x 11 (non-symmetric)	Iterative-Direct

In addition, the Shur-Shultz method is available for computing the inverse of an n x n matrix where $n \leq 11$.

The above methods are being re-evaluated for use with the magnetic tape and/or (39, 6, 0) arithmetic. Additional methods are also being investigated.

It is planned to use the Shur-Shultz program to invert a family of matrices arising in the construction of optimum interval tables using Chebychev polynomial approximation. Once these inverses are obtained, it will be possible to set up an automatic procedure for obtaining approximating polynomials for arbitrary functions up to order 19 over any finite interval.

Problem #28. Ambipolar Diffusion

This problem, recommended by Professor P. Allis of the MIT Physics Department, has to do with an electrical discharge in which electrons and positive ions are generated at a rate (r_η) proportional to the electron concentration (η). In the steady state, both particles must flow to the walls of the discharge tube at the same rate (Γ), which is determined by the concentration gradients of the electrons $(\nabla\eta_-)$ and positive ions $(\nabla\eta_+)$ and by the space charge field (E_s). The field is, in turn, determined by the space charge

$$E_s = \frac{e(\eta_+ - \eta_-)}{e_v}$$

When the equation of continuity $\nabla \cdot \Gamma = r_i \eta_-$ is added, the result is four first order differential equations for the four variables $\eta_+, \eta_-, \Gamma, E$. The boundary conditions are that Γ and E_s should vanish at the center of the tube and η_+ and η_- at the walls. By the introduction of dimensionless variables, the number of arbitrary parameters is reduced and the task of finding compatible values of electron and ion concentration necessitates solving two second order second degree differential equations.

Also interested in the problem, Mr. Robert Minnich of Harvard University offered his services as a numerical analyst and programmer. By the fall of 1951 a program had been written and operated using fixed-point single-length number system. Unfortunately, the solution so obtained lacked precision, and the problem was temporarily set aside. When interpretive routines for extra-precision floating-point computation became available, a new program was written, and satisfactory data were produced. However, Dr. David Rose of the Bell Laboratories was convinced that better results could be obtained. Minnich took the challenge and reprogrammed the problem using a three-point Newton Cotes integration formula and a bi-section procedure. This proved worthwhile, and during the past quarter twenty parameter values have been operated on Whirlwind satisfactorily. Five more parameters are to be operated using a slight variation in the bi-section procedure to determine if any greater accuracy of the data can be obtained.

Problem #30. Digitally-Controlled Milling Machine Program

A problem originally proposed in earlier discussions with Mr. John Runyon of the MIT Servomechanisms Laboratory has been coded and is ready for WWI. The servomechanically-controlled milling machine described in previous reports (see Summary Reports 28 and 29) will be instructed by specially coded instructions punched on paper tape to cut a straight tapered wing section. The root and tip airfoils of the wing section are determined by NACA polynomials, modified to allow for the offsets necessary because of the radius of the cutting head.

A program has also been coded for the class of problems wherein the surface to be cut by the milling machine is specified by tabulated data. The necessary offsets will be here determined by the computer. Various subroutines have been written also which

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

simplify the coding of such problems.

Problem #41. Binary Matrix Product Statistics

This problem originated with R. D. Luce of the Group Networks Laboratory of the Research Laboratory of Electronics, MIT. It concerns experiments in group communications that consist of step-by-step passage of information among five test personnel occupying positions in a "network."

As anticipated, the equi-probable process did not account for the observed group results. This was apparently not due to insufficient statistics. Successive read-outs at 1000, 2000, and 3000 runs showed a deviation of approximately 1% from the mean at the maximum point of the calculated distributions. Indeed, an analytical calculation for one model showed that the probability of all nodes possessing complete information at 3 steps is 0.00244, while the Monte-Carlo calculations gave 0.00167. In another model the comparable probabilities were 0.0705 versus 0.0757.

A more complicated model based upon the use of conditional probabilities from act to act and trial to trial will be attempted. In this conception the passage of information will be jointly governed probabilistically by the decisions in the previous act and by the success or failure (in the sense of minimum acts or not) on the previous trials. In the one case in which an analytical solution exists for a system under these hypotheses, agreement with the data is good. The Monte-Carlo procedure will be used to test those models for which analytical solutions have not been found.

Problem #42. Hyperbolic Partial Differential Equations: Numerical Integration

For the problem of spherical wave propagation being studied, the process is considered adiabatic such that the speed of sound, c, and the density, ρ, are related by

$$C = A\rho^{\frac{\gamma - 1}{2}}$$

where γ is the adiabatic exponent.

In previous single-register computations, this function was approximated by a series valid for a certain range of ρ. However, preparatory to a (24, 6, 0) program which will be used to investigate the general wave phenomenon over much greater ranges, a new and more general method of finding the function is being tested.

During this phase, some analytical work not connected with the computer has been pursued on the problem.

Problem #45. Crystal Structure

The computation of the electron density, f(x, y), in a centrosymmetric crystal cell (see Summary Report 29) has been proceeding on a production basis during this quarter. Approximately 15 sets of data have been processed. The general 3-dimensional case is being programmed. In addition, a program is being written for the computation of the structure factors used in the calculation of f from empirical data.

Problem #46. Torpedo Depth Response

During the last quarterly period the torpedo depth response problem was completed on Whirlwind. This problem, which originated with Robert Kramer of the Servomechanisms Laboratory at MIT, was essentially the solution of four simultaneous first order differential equations plus a linear constraint relationship between certain of the variables. The method of numerical solution used was the four-point Runge-Kutta method. The (15 15, 0) floating-point interpretive subroutine was used throughout to obviate the necessity of scale factoring. Solutions with integration intervals of 0.01, 0.1, and 0.2 second were obtained. The 0.01- and 0.1-second intervals gave solutions agreeing with each other and with a 1-percent analytical solution to about three significant figures. The 0.2-second interval solution oscillated wildly.

Problem #47. Partial Differential Equations of an Internal Combustion Engine

This problem involves the solution to a hyperbolic type of second order partial differential equation with proper initial and boundary conditions, which represents the unsteady flow in an inlet pipe of an engine. The method of solution consists in constructing simultaneously networks of characteristics in the physical plane (abscissa: distance along the pipe; ordinate: time) and the state plane (abscissa: local velocity of fluid; ordinate: local velocity of sound).

The problem has been coded in the (24,6, 0) floating-point system. Since the middle of May, satisfactory results have been produced steadily. Computed pressure in the inlet pipe, pressure in the cylinder, and the amount of air entering the cylinder for each cycle of engine operation are displayed on the oscilloscope and recorded photographically. This method of obtaining results has proved to be satisfactory in regard to both speed and accuracy.

Because of the scarcity of computing time, it will not be possible to test the present program more thoroughly. It has been planned to stop the computation in the very near future. At that time, a more detailed report on what has been done will be submitted.

Problem #48. Gust Loads on Rigid Airplanes in Two Degrees of Freedom

A program for yielding the response of a rigid airplane in two degrees of freedom to a sharp-edged gust has been written and successfully tested. In addition, a second program has been written and tested for finding the response to a graded gust by integrating the sharp-edged gust results in the following manner

$$(AR)_g = \frac{1}{s_g}\int_o^s (AR)_{s.e.}(\sigma)d\sigma \qquad 0 \leq s \leq s_g$$

$$(AR)_g = \frac{1}{s_g}\left[\int_o^s (AR)_{s.e.}(\sigma)d\sigma - \int_o^{s-s_g} (AR)_{s.e.}(\sigma)d\sigma\right] \quad s_g < s$$

With these two programs, the airplane response for three gust gradient distances, S_g, may be found for a wide number of parameters. Upwards of 200 cases are to be studied, and in order to reduce print-out time, the computer has been programmed to select and print only the peak values of airplane acceleration ratio (the alleviation factor), the wing-load ratio and tail-load ratio, rather than the complete time histories of the quantities. This program yields several results per hour.

In the second quarter of 1952, Whirlwind computed 55 cases for the Aero-Elastic and Structures Research Laboratory, Department of Aeronautical Engineering, MIT, which is conducting the gust-load investigation under contract with the Bureau of Aeronautics.

Problem #50. Lattice Analogy Applied to Shear Walls

This is the final report on this problem. The subject of primary interest in the investigation was the predication of the load-deflection curves of reinforced-concrete shear walls. This is an extremely complex analytical problem, especially after cracking of the concrete has occurred, and hence recourse was made to the approximate method of the lattice analogy.

The fundamental concept of this method is that of replacing a plate by a pin-connected lattice having the same deformations as the plate. The lattice is given the external outline and the boundary conditions of the plate prototype and is subjected to the same loads as the prototype. The problem is then, as far as Whirlwind is concerned, one of determining all the bar stresses when the data on bar areas are given to it.

The correct solution will satisfy both the equations of statics and strain compatibility. By assuming a statically sound final solution and ensuring in the program that all corrections are statically consistent, the static equilibrium of the structure is assured. Strain compatibility is checked by means of the equations of compatibility which are derived from elastic strain considerations. As there are two basic types of lattice units, the square and the diamond, an equation for each is required. These are derived from virtual work considerations.

For the squares: $4\sum D - \sum F = 0$
For the diamonds: $2\sum D - \sum F = 0$
where D = component of force in a diagonal bar
F = force in a side bar.

As set up at present, the program checks the strain compatibility of all units in a systematic manner (Seidel process), effects statically consistent corrections, and stores the bar stresses (F) for the $(n-1)^{th}$ and n^{th} cycles. It also stores $F_n - F_{n-1} = \triangle F_n$. A subroutine is utilized to find $|\triangle F_n|_{max}$, and this latter is used as a criterion for the accuracy of the solution. Actually, the best solution would be obtained for $|\triangle F_n|_{max} = 0$; because of round-off, it is in general not possible to attain this desideratum. However, the closer this optimum is approached, the better the solution.

Correlation between theory and experiment was good in the uncracked range in regard to both initial crack predictions and deflection. Results in the cracked range were fairly good for most specimens analyzed.

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

the greatest error being in the deflection prediction at ultimate load.

Problem #52. Oil Reservoir Depletion Analysis by Iteration

This problem extends a procedure previously developed for the calculation of the pressure and saturation distributions in a producing reservoir.

The problem being solved is the so-called linear case characterized by the existence of a single dominant direction of oil flow. The method being used is one that was developed by the Carter Oil Research Laboratory. It utilizes the concept of unit expansion.

The earlier procedure corrected the non-uniformity of the pressure change in the reservoir by making use of a calculated weight factor. The present analysis replaces this weight factor by an iterative procedure. Thus the high speed of WWI is being utilized to obtain a pressure and saturation distribution consistent with the prescribed withdrawal rate and resultant flow.

A set of results have been obtained and sent to the Carter Laboratory for analysis.

Problem #53. Solution of Boundary Value Problems

This problem is the Schrodinger wave equation for self-consistent field calculation of various atoms. Since six eigenvalues and six associated wave functions must be evaluated for an atom such as Cu^+, this is a lengthy computational task. For details of the problem see Section 6 of Summary Report 29.

During this quarter a detached study of how to choose automatically the best eigenvalue has been made, and plans have been made for using the new magnetic-tape units for auxiliary memory and the oscilloscope output for displaying the intermediate and final solutions.

Problem #54. Optimizing the Use of Water Storage in a Combined Hydro-Thermal Electric System

The demand for a large number of storage registers for this program has been satisfied by the use of magnetic tape as an auxiliary storage medium. The estimated system operation together with generation requirements and stream-flow data are recorded in 54 groups of 20 (24, 6, 0) words on magnetic tape. This layout is followed by a "skip-back" routine to place the reading head in a position at the beginning of the recorded data. The appropriate data for computation at each of 54 ordinates is introduced into the computer by the following routine: skip forward one group, read forward one group, skip backward over two groups, read forward one group, compute several partial derivatives, skip back one group, and then record the results of these computations in a forward direction in one group. This cycle is repeated a total of 54 times.

The program was run successfully for four complete passes, which amounts to two steps in the present scheme of eight iterations. The results of these two steps have indicated the possibility of a substantial reduction in fuel costs and in the degree of violation of system constraints that were present in the initial operating mode.

Problem #58. Determination of Energy Levels of Oxygen Molecule

The problem: A knowledge of the electronic energy levels of the lowest states of the oxygen molecule will give detailed information about the paramagnetic behavior of oxygen. In addition, the procedure here employed will bridge the gap between the molecular orbital treatment, adequate for small inter-nuclear distance, and the Heitler-London approximation, which is valid for large values of the inter-nuclear distance.

Briefly, from an appropriately-chosen complete set of one-electron wave functions, $\{u_i\}$, a finite set of $N \geq 16$ functions are chosen. From this set, determinental wave functions, ψ_i, satisfying the exclusion principle can be formed. The two s and three p states provide, with spin included, ten such orbitals for each atom, leading to a value of N = 20. This leads to about 5000 different wave functions, which is prohibitively large. Hence it is assumed that the 1s and 2s states are permanently filled, leaving 8 electrons and 12 spin orbitals to be filled. The number of functions is thus reduced to 495. Making use of the symmetry properties of the molecule and assuming no magnetic interaction, it is possible to select from this number 9 and 12 linear combinations which are $^3\Sigma_g^-$ and $^1\Sigma_g^+$ states, respectively. These functions then provide the secular equations of order 9 and 12 whose solutions provide the desired lower electronic energy levels and the orthonormalized transformed wave functions. The $\{u_i\}$ selected enabled the elements of the secular equations to be obtained analytically. The calculations were carried through for 7 values of the internuclear distance. The minimum value of the energy curve obtained gave the binding energy of the molecule.

Mathematical technique employed: The two previously described methods (Summary Report 29) were discarded as being too inaccurate in one case and too time-consuming in the other. Instead, a direct diagonalization scheme dating back to Jacobi and Runge was employed. A (24, 6, 0) program was written capable of treating a secular system up to order 12. The results were gratifying: each 9 x 9 required 2-1/2 minutes and each 12 x 12 required 6 minutes computing time, respectively. The eigenvalues were correct to 6 decimal digits; the eigenvectors were accurate to about 4.

Problem #59. AEC Positron-Electron Calculation

This problem was originated by M.E. Rose and P.R. Bell of the physics division of Oak Ridge National Laboratory. The purpose of the computation is to provide physicists and chemists with tabulated values of the field factors for use in determining the maximum energy of beta rays leaving the nucleus in forbidden transitions. (cf. M.E. Rose, "Relativistic Wave Functions in the Continuous Spectrum for the Coulomb Field," Physical Reviews, v 51, p 484, 1937; E. Grueling, "Theoretical Halflife of Forbidden Beta Transition," Physical Reviews, v 61, p 568, 1942; D.L. Pursey, "The Interaction in the Theory of β-decay," Philosophical Magazine, v 22, p 1191, 1951).

The momentum spectrum for forbidden transitions can be represented by the momentum spectrum for allowed transitions multiplied by a correction factor $C_{nx}(p, z)$ (n, x, p, z denote respectively the order of forbiddenness, the type of interaction, the electron momentum, and the nuclear charge number). The factor $C_{nx}(p, z)$ can be evaluated as a linear combination of the field factors with coefficients that depend only on the neutrino energy, the spin number, and the order of forbiddenness.

Whirlwind I will be used to evaluate the solution to the Dirac electron wave equation which has been found by M.E. Rose in a closed form. The solution, which is a combination of hypergeometric, gamma, and elementary functions of a complex argument, is readily evaluated by iterative methods on an automatic computing machine. The (39, 6, 0) floating-point interpretive arithmetic will be used to maintain a relative precision of 0.1 percent in the final computed values.

The results of the Whirlwind computations will be punched on cards, sorted, multiplied by a factor, and tabulated using a sorter and card-programmed calculator (CPC). These values will be included in a report published by Oak Ridge National Laboratory and will receive wide distribution. It was necessary to do part of the computation on the CPC, as the program for the entire computation would not fit in the present Whirlwind I internal memory. The Fermi functions for allowed transitions and the corrections for finite size of the nucleus have already been tabulated and published by ORNL.

The code for Whirlwind I has been developed. The computations should be completed during the next quarter. The computed values will be recorded as a scope display on film after the code has been tested.

Problem #60. Determination of the Energy Levels of the Deuteron

For this problem Schrodinger's equation reduces to

$$\frac{d^2y}{dx^2} = \left[E - AV(x)\right] y$$

where

$$V(x) = \frac{e^{-x}}{x}\left[1 - e^{-cx}\right]$$

E is proportional to the binding energy of the deuteron,

A and C are parameters which determine the form and depth of the nuclear potential,

and the boundary conditions require that y, the probability density, be zero at x = 0 and at x = ∞.

A completely automatic solution to this boundary value problem has been obtained. For each set of the parameters, successive trial values of E converged to the true eigenvalue, the number of trials depending on how good was the first trial value.

The solution time varied from 30 to 60 minutes for one set of parameters. Since it is planned to obtain solutions for approximately 100 sets of parameters, the program must be modified so as to decrease the solution time.

Two major modifications are being made. (1) Auxiliary storage (magnetic tape) will be

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

used to store the potential function, so that it must be calculated only once for each of the ten values of the parameter C. (2) When $AV(v) \ll E$, which it is for large values of x, the equation has an exponential solution, and $y' = -\sqrt{E}\, y$. Thus for some large x, the value of $(y' + \sqrt{E}\ y)/y' = B$ is approximately proportional to the error in E. The next trial value of E can be estimated from the two previous values of B. This method promises to give much faster convergence of the trial values of E to the true eigenvalue. It is expected that using this scheme for finding the eigenvalue, and using auxiliary storage for the potential function, will result in reducing the solution time to a matter of 2 or 3 minutes.

The method can be easily extended to higher order wave functions. Further it appears that the completely automatic solution of this type of boundary value problem will have wide application not only in physics, but in other fields as well.

Problem #70. Correlation of Solvolysis Rates

This problem was originally introduced by the Chemistry Department of MIT and is now being pursued by Robert Mosely, using the following method.

An equation of the form

$$\log k/k^{o} = sn + s'e$$

has been found to correlate the rates of solvolysis of twenty organic compounds in eighteen solvents, where k and k^{o} are the rate constants for solvolysis of an organic compound in any solvent and in a standard solvent, respectively, n and e represent the nucleophilicity and electrophilicity of the solvent, and s and s' represent the susceptibility of the compound to nucleophilic and electrophilic attack, respectively. The compounds range from methyl bromide to triphenylmethyl fluoxide, and the solvents include ethanol-water, methanol-water, acetone-water, acetic acid-acetic anhydride, and formic acid-acetone mixtures.

A set of n, e, s and s' obtained by a subjective process of trial and error gives remarkably good agreement between calculated and observed rate constants. However, an objective procedure for obtaining such values is greatly desired. An iterative method of least squares appears to be the most suitable.

If the quantity

$$\left(\sum_{ij} Z_{ij} - \sum_{ij} s_i n_j - \sum_{ij} s_i' e_j\right)^2$$

where

$$Z_{ij} = \log \frac{k_{ij}}{k_i^{o}}, \sum_i$$

refers to all compounds, and $\sum_j$ to all solvents, is minimized, then the least squares criterion is satisfied. This may be achieved by an iterative procedure.

Differentiation with respect to each n leads to a set of equations of the form

$$\sum_i Z_{ij} s_i = M_j \sum_i s_i^2 + e_j \sum_i s_i s_i' .$$

Similarly, differentiation with respect to e leads to a set of equations

$$\sum_i Z_{ij} s_i^1 = n_j \sum_i s_i s_i^1 + e_j \sum_i s_i'^2$$

If a set of s_i and s_i' are first assumed, these two sets of equations may be solved in pairs to give a set of n and a set of e. Differentiation with respect to s and s' leads to

$$\sum_j Z_{ij} M_j = s_i \sum_i M_j^2 + s_i' \sum_j M_j e_j$$

$$\sum_j Z_{ij} e_j = s_i \sum_i M_j e_j + s_i' \sum_j e_j^2$$

Using the values of n and e obtained above, these two sets of equations may be solved in pairs for s and s'. These new s and s' values may then be used to obtain another set of n and e. Repetition of the procedure leads to the best set of n, e, s, and s', but only very slow convergence appears obtainable by this method.

Problem #71. Optimum Operation of a Chemical Reactor

A (24, 6, 0) program for solving the set of three simultaneous non-linear differential equations describing the operation of a chemical reactor has been written and tested. The program gives satisfactory results; however, in order to solve the equations for the large number of parameters requested it is necessary to speed up the program by a factor of at least ten. By increasing the length of the integration steps and by using the (15, 15, 0) interpretive routine, the desired speed-up can be obtained. This modification is now being tested. The final form of the problem is expected to be run during the next two weeks.

Problem #74. Optimization of Strip Mining Techniques

The specific problem to be solved is that of determining how the mining rate of an excavator varies with the system operating factors affecting this rate.

In the mining fields, the ore to be removed is found in a matrix of earth 10 to 30 feet below the surface. The excavator removes the earth which covers the ore matrix and piles it on worked-over land. Then it removes the matrix and deposits it on the side of the strip. There, the ore is transported to a processing plant and subsequently readied for shipment.

Operational data pertaining to the system factors were collected to establish performance criteria for the operating variables. Next, a mathematical model was established analytically to represent the performance of the system as a function of all operating variables. Letting the mathematical model represent the system, the problem becomes that of modifying the design or magnitude of those factors which determine the operating variables to obtain expressions of the performance in terms of mining rate under all desired conditions. Eleven parameters have been identified which are both necessary and sufficient for the description of the system operation.

The large number of results that must be obtained necessitates the use of the magnetic-tape typewriter. The results will be recorded on magnetic tape with positive solutions representing mining rate and negative solutions impractical parameter combinations. The initial trials will be for verification purposes, so these results will be typed directly from the computer.

Problem #78. The Solution of Algebraic Equations Using Graeff's Method with Ratio Test

The program is based on Graeff's method and operates on the coefficients of a given algebraic equation with real coefficients to form the coefficients of a new equation whose roots are some high power of the roots of the original equation. If no two roots are equal, the roots of the new equation are widely separated and can easily be evaluated. Repeated or complex roots can be detected by imputation of the computed coefficients, which is facilitated by a ratio test incorporated in the program. They can then be formed by making use of simple relationships which exist between the coefficients.

The accuracy with which the roots can be obtained is limited only by the number of significant figures and magnitude of the power index which can be handled by the programmed arithmetic subroutine. Using the (15, 15, 0) number system in a particular example, it was found that the real and complex roots of a seventh degree polynomial could be obtained with errors of less than 2 percent when three of the roots differed by less than 4 percent in magnitude. This result was obtained when the roots of the original equation had been raised to the 128^{th} power. By using a number system with more significant figures and increasing the power to 256 or 512, a considerable improvement in accuracy could be obtained.

A detailed account of the method used for finding the actual roots from the results of the program will be found in Whittaker and Robinson's book, "The Calculus of Observations."

Problem #79. Tracing Rays Through a Spherical Lens

This problem originated at the Laboratory for Electronics, Inc., in Boston. The original engineering problem called for the design of a microwave lens which would satisfy three requirements:

1. The lens had to be sufficiently thin to meet certain maximum-weight restrictions.

2. A path equality condition for the rays (normals to wavefronts) going through the lens from a point source had to be met.

3. The final refracted rays (normals) had to be incident on a certain reflecting plane surface, the angle of incidence being approximately 90 degrees.

The first attempt at solution consisted of setting up a system of differential equations. But this was abandoned in favor of a method which would be more easily adaptable to quick computation.

As a result, a vector method was decided upon (which could handle skew rays as well as plane rays) which reduced all computation to the four basic arithmetic operations and a square root operation. Specifically, the following plan of attack was then de-

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

cided upon: for speed and simplicity in calculation, a good sampling of only spherical surfaces (circles for generatrices) would be considered. Then as time permitted, Feder's method (resembling the "Newton Approximation Method") could be used to improve on the spherical design if necessary and produce an aspheric.

Even with this simple approach, it was estimated that the computation would take several man-years of steady work. Therefore a request was made and granted to use WWI. Three successive programs have been devised, the last two being refinements of the first program. The first program exploded the conjecture that if the angle criterion were satisfied, then the path equality criterion would also be satisfied. Thirty-one lenses were printed out which satisfied the angle criterion, but the spread of path lengths was too great in each case. It was felt that perhaps some "good" lenses were being overlooked. The second program imposed both restrictions, and after it had run for over one hour and produced no suitable lenses, a change in program was felt to be desirable. Certain alterations in the original specifications would have to be made.

After consultation with the engineer in charge of the project, it was therefore decided:

1. To rotate the characterizing line (plane) through 2^{o}

2. To try a variety of source positions given by the position vector $\vec{T}(T_x, T_y)$, allowing T_x to vary between -36 and -24 (in steps of 3 units) and T_y, between 9 and 12 (in steps of 1 unit)

3. To incorporate in the program instructions to print out for each lens design (over 100 designs were tried) the best specifications of source position (a) with respect to the angle criterion, (b) with respect to the path equality criterion.

A successful run was made on WWI and some very useful results were obtained.

Problem #84. Departure Curves for Various Types of Resistivity Logs in Oil Wells

Electrical surveys in oil wells measure potentials on the axis of the drill hole. From the combination of such measurements for various electrode arrangements, the actual resistivities of the formations traversed by the drill hole and of the zones invaded by drilling fluids, immediately around the hole, have to be determined. This is done graphically by the use of "Departure Curves" which give the relation between the measured potentials and the resistivity distribution in the formations. The departure curves are computed from solutions of Laplace's equation in cylindrical coordinates expressed in terms of modified Bessel functions and cosine terms.

This work is being undertaken in cooperation with the Continental Oil Company.

4.2 SUBROUTINES COMPLETED

Library Subroutine Number	Tape Number	Title
EX 0.1t	T-1048-1	e^{-x}, $0<x<1$
OT 1.7t	T-1060-2	Print C(v3) Through C(v4) as Octal Number, Sign Digit and Complement, Point, Page Layout
OT 109.10t	T-930-2	15n, 0, 0 MRA Print and/or Punch, Decimal Fraction, Sign, Number of Digits Arbitrary, No Carriage Return, Sign Agreement (Interpreted)
TF 7.2t	T-1172	Sine-Cos x, x in radians, x in MRA (24, 6, 0)
OT 2.4	T-936-3	Print C(AC) as Decimal Fraction, Round-off sign and magnitude, point, single column layout
MT 1.1	T-988-8	Record Block of N + 1 words on Magnetic Tapc. In Forward Mode.

Library Subroutine Number	Tape Number	Title
MT 2.1	T-989-1	Transfer Block of N + 1 words from Magnetic Tape (in Reverse Mode) to Storage, storing $N \times 2^{-15}$ in First Reg. of Transferred Block.
MT 2.2	T-1161-2	Transfer Block of N + 1 words from Magnetic Tape (in Reverse Mode) to Storage, storing final address of Block in First Reg. of Transferred Block.
OC 2.2	T-788-5	Display C(ES) as octal fraction, sign octal point, D or F scope, Layout
OC 2.4	T-863-5	Display C(ES) as decimal fractions, no round-off decimal point sign, D or F scope, Layout
EX 200.1	T-1216-1	(24, 6, 0) e^{x} subroutine (slow)
HF 207.1	T-1217-1	(24, 6, 0) sinh x-cosh x subroutine (slow)
OC 2.2	T-788-6	Display C(ES) as Octal Fraction Sign, Octal Point, D- or F-scope Layout
OC 2.4	T-863-6	Display C(ES) as Decimal Fraction, No Round-off, Decimal Point, Sign, D- or F-scope, Layout
PA 4.1	T-1180-1	(39, 6, 0) Interpretive Subroutine
OC 2.3t	T-850-6	Octal Instruction Scope Display
OC 2.5	T-891-2	Octal Integers Display

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

5. ACADEMIC PROGRAM IN AUTOMATIC COMPUTATION AND NUMERICAL ANALYSIS

The seminars on Computing Machine Methods are arranged jointly by representatives of the MIT Committee on Machine Methods of Computation and the MIT Digital Computer Laboratory. Various speakers from other MIT activities and elsewhere, as well as members of the two sponsoring groups, participate in these weekly seminars, which are held in a lecture room at the Institute. The program during the past quarter was as follows.

Date	Title	Speaker
April 3	The Method of Kernel Functions for Solving Boundary Value Problems in the Theory of Linear Elliptic Partial Differential Equations	Dr. Stefan Bergman
April 10	Numerical Solution of Boundary Value Problems by Kernel Function Methods	Dr. Franz L. Alt
April 17	Sampling Method in Analog Computation	Dr. F.M. Young
April 24	Report on the paper, "On the numerical solution of the Dirichlet problem for Laplace's difference equation," by J. B. Diaz and R. C. Roberts, Quart. of Appl. Math., January, 1952	Mr. Noel J. Hicks
May 8	The Solution of Boundary Value Problem with Automatic Computing Equipment	Dr. Frank M. Verzuh
June 12	Code Checking Routines	Dr. R. Clippinger
June 26	Numerical Integration of the Orr-Sommerfield Equation	Dr. L.H. Thomas

6. APPENDIX

6.1 REPORTS AND PUBLICATIONS

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group who are known to have a particular interest in the Project. Other people who need information on specific phases of the work may obtain copies of individual reports by making requests to John C. Proctor, Digital Computer Laboratory, 211 Massachusetts Avenue, Cambridge 39, Massachusetts.

The following reports and memorandums were among those issued during the second quarter of 1952.

No.	Title	No. of Pages	Date	Author
SR-29	Summary Report No. 29, First Quarter 1952	36		
R-211	A Magnetic Matrix Switch and its Incorporation into a Coincident-Current Memory (M.S. Thesis)	97	6-6-52	K.H. Olsen
R-212	Ferroelectrics for Digital Information Storage and Switching (M.S. Thesis)	62	6-5-52	D.A. Buck
E-452	Relay and Contact Life Test	3	3-3-52	R.E. Hunt
E-456	Block Diagram of Control for Printers and Paper Tape Units for the Final In-Out System	3	4-2-52	P.W. Stephan
E-457	Current Density Distribution in the Fringe Region of a Focused Electron Beam	5	4-8-52	C.L. Corderman
E-460	The Ferroelectric Switch	3	4-16-52	D.A. Buck
M-1499	The Interim Magnetic Tape System	25	5-23-52	S.B. Ginsburg K.E. McVicar
M-1513	Utility Programs Available to WWI Operators	7	6-17-52	J.T. Gilmore
M-1514	Outline of Present Interim Operation of WWI Input-Output Equipment	10	6-10-52	F.E. Heart
M-1516	Use of the Interim Magnetic Tape Print-Out Equipment	4	6-6-52	E.P. Farnsworth

6.2 PROFESSIONAL SOCIETY PAPERS

Two members of the Laboratory staff presented papers at the meeting of the Association for Computing Machinery held at Mellon Institute, Pittsburgh, May 2 and 3. C.W. Adams spoke on "The Operation of Small Problems on Large Computers." J. W.

APPROVED FOR PUBLIC RELEASE. CASE 06-1104.

Carr's subject was "The Automatic Assembly of Programs by Variable Preset Parameters."

At the MIT Physical Electronics Conference on March 27, H.B. Frost spoke on "The Measurement of Cathode Interface Impedance."

W.N. Papian's paper "A Coincident-Current Magnetic Memory Cell for the Storage of Digital Information" was published in the Proceedings of the IRE for April, 1952.