PROJECT WHIRLWIND

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## FOREWORD

### Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are the development of an electronic digital computer of large capacity and very high speed, and its application to problems in mathematics, science, engineering, simulation, and control.

### The Whirlwind Computers

The Whirlwind computer is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the electrostatic storage tube, which uses an electron beam for storing digits as positive or negative charges on a storage surface.

Whirlwind I (WWI) may be regarded as a prototype from which other computers will be evolved. It is being used both for a study of circuit techniques and for the study of digital computer applications and problems.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access electrostatic storage initially had a capacity of 4096 binary digits, sufficient for some actual problems and for preliminary investigations in most fields of interest. This capacity is being gradually increased toward the design figure of 32,768 digits. Present speed of the computer is 20,000 single-address operations per second, equivalent to about 6000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

### Reports

Quarterly reports are issued to maintain a supply of up-to-date information of the status of the Project. Detailed information on technical aspects of the Whirlwind program may be found in the R-, E-, and M-series reports and memorandums that are issued to cover the work as it progresses. Of these, the R-series are the most formal, the M-series the least. A list of the publications issued during the period covered by this Summary, together with instructions for obtaining copies of them, appears in the Appendix.



# 1. Quarterly Review and Abstract

During the second quarter of 1953 the comprehensive system of service routines described in Summary Reports 31, 32, and 33 became part of the standard operational procedure of the Scientific and Engineering Computation (S&EC) Group. The group ran 27 problems on the computer, utilizing about 375 computer hours. Twenty-five of these problems originated at MIT.

A new control system for marginal checking was installed during the quarter. Rotary line-selection switches have replaced the telephone dial of the original system.

The second bank of electrostatic storage operated satisfactorily, increasing the available internal storage from 1024 to 2048 registers. The auxiliary magnetic-drum system became a reliable part of the computer. Difficulty encountered with the temperature sensitivity of the heads was eliminated by ambient-temperature regulation.

Noise still plagued the operation of the display system; tests are underway to reduce picture distortion by means of a balanced transmission system. A character generator, which generates a small figure-eight deflection pattern at a position determined by the computer, was installed. Letters and numbers are constructed from this pattern by intensification of the appropriate parts, also controlled by the computer.

No extensive analyses of life data on vacuum tubes were made during April, May, and June. Maintenance testing was intensive because of the large amount of new equipment installed. Failure rates continued to be higher than for 1952.

Seminars on Computing Machine Methods were held weekly during April and May; two seminars on Methods for the Numerical Evaluation of Integrals of the Product of Two Functions were held during April.

## 2. Mathematics, Coding, and Applications

### 2.1 INTRODUCTION

The new programming techniques described in Summary Reports 31, 32, and 33 have been integrated into the operational procedures of the Scientific and Engineering Computation (S&EC) Group. These techniques are referred to as the Comprehensive System of Service Routines (CS) and the development of these techniques is described under Problem 100 in Section 2.2. A brief description of the system is given below.

Most of the difficulties encountered in the adoption of this system have been overcome. By the end of the period the convenience and flexibility offered by the system were being realized. However, the acceptance of new problems will continue to be restricted during the next quarter because of the limited amount of staff and computer time that will be available.

Section 2.2 contains reports, arranged in numerical order, describing the progress on some 27 problems that have made use of computer time allotted to the S&EC Group during this last quarter. Tables 1 and 2 have been set up to provide the reader with a convenient reference to various interesting aspects of these problems. Table 1 lists the problems according to their fields of application. Of the 27 problems listed, 25 originated within MIT, the others having been suggested by the Retina Foundation (114) and the Educational Testing Service (112). Problem 130 has been carried out in cooperation with the Lummus Company.

Of the MIT-originated problems, the 5 marked with asterisks in Table 1 represent work being performed by members of the MIT Project on Machine Methods of Computation, another phase of the work under contract N5ori60, which is reported more extensively in a separate report ("Machine Methods of Computation and Numerical Analysis," Quarterly Progress Report No. 7, Project DIC 6915, MIT, March 15, 1953).

The mathematical problems and procedures represented by the various current problems are tabulated separately in Table 2, since different problems in the same field frequently involve different mathematical methods, while problems in different fields sometimes are solved by identical methods.

A more flexible and more effective system for getting programs into WWI and for getting their solutions out was described in Summary Report 32, Section 4.1. This system is referred to as the Comprehensive System of Service Routines (CS).

The system provides for conversion, by WWI, from Flexowriter-coded perforated tapes to tapes in pure binary-coded form. Among the many provisions included in the CS, the following two are referred to in the text. Consequently, brief descriptions of them are repeated here for the reader's convenience.



| Field | Description | Problem Number | Source |
|---|---|---|---|
| Chemical Engineering | Fractional distillation of a volatile mixture | *130 | MIT |
| Chemistry | Optical properties of thin metal films | 101 | MIT |
| Civil Engineering | Analysis of reinforced concrete walls | *113 | MIT |
| Electrical Engineering | Operation of a three-plant hydro-thermal electric generating system | 104 | MIT |
| Geology and Geophysics | Geophysical data analysis | 106 | MIT |
| | Interpretation of earth-surface resistivity measurements | *123 | MIT |
| Instrumentation Lab | System of non-linear differential equations | 109 | MIT |
| | Fourier analysis and autocorrelation calculations | 111 | MIT |
| | Atmospheric turbulence; auto-, crosscorrelation, and Fourier transform calculation | 137 | MIT |
| Mathematics | Educational testing studies; Lawley's method of factor analysis | 112 | Outside |
| | Spherical wave propagation | *119 | MIT |
| | Convolution of a transformed probability density function | 121 | MIT |
| | Matrix equations; modified conjugate-gradient method | 136 | MIT |
| Mechanical Engineering | Turbine design (Aerothermopressor) | 120 | MIT |
| Physics | Scattering of electrons from gases | *102 | MIT |
| | Design of optical instruments | 114 | Outside |
| | Neutron-proton interaction | *124 | MIT |
| | Nonlinear meson theory | 133 | MIT |
| | Eigenvalues of a real symmetric matrix | 134 | MIT |
| Psychology | Quantized group communication and learning | 118 | MIT |
| Servomechanisms Lab | Fourier transform and autocorrelation calculations | 107 | MIT |
| | Solution of the convolution integral equation | 116 | MIT |
| | Subroutines for the numerically controlled milling machine | 132 | MIT |
| | Data reduction program; polynomial fitting | 126 | MIT |
| Miscellaneous | Comprehensive system of service routines | 100 | MIT |
| | MIT Course 6.537 -- Spring 1953 | 128 | MIT |
| | MIT special summer session course, 1953 | 140 | MIT |

Table 2-1. Current Problems Arranged According to Field of Application

| Mathematical Problem | Procedure | Problem No. |
|---|---|---|
| 1. Simultaneous algebraic equations | | |
| Fifteen linear equations | Relaxation procedure | 113 |
| System of 18 non-linear equations | Iteration | 130 |
| Matrix equation | Iteration involving Hotelling's method for solving the eigenvalue problem | 112 |
| Matrix equation | Modified conjugate-gradient method | 136 |
| Eigenvalues of a real symmetric matrix | Numerical diagonalization procedure | 134 |
| 2. Transcendental equation | Iteration | 101 |
| 3. Ordinary differential equations | | |
| Eight simultaneous non-linear differential equations | 4th order Runge-Kutta | 109 |
| Solution of Schrodinger's equation in one dimension | Iterated Runge-Kutta | 124 |
| Second order non-linear | 4th order Runge-Kutta | 133 |
| Seven non-linear first order | Step-by-step Euler method | 120 |
| 4. Partial differential equations | | |
| Solution of the Mie equations | Evaluation of analytic solution involving spherical Bessel and Hankel functions | 103 |
| Calculus of variations; evaluation of the Euler expression | Three-point Lagrange derivative formulas | 104 |
| Non-linear hyperbolic | Difference equations written along the characteristics and solved simultaneously by iteration | 119 |
| (see also #113) | | |
| 5. Integration | | |
| Calculation of Fourier transform and autocorrelation functions | Simpson's rule | 107 |
| Fourier analysis and autocorrelation | Trapezoidal integration | 111 |
| Inversion of the convolution integral | Iteration using Simpson's rule integration | 116 |
| Integral evaluation | Trapezoidal integration | 123 |
| Convolution of a transformed probability density function | Trapezoidal integration | 121 |
| Auto-, crosscorrelation and Fourier transform | Simpson's rule (based on #107) | 137 |
| 6. Integro-differential equations | | |
| Coupled second-order | Iterated Heun method | 102 |
| 7. Monte Carlo technique | Random numbers generated by a recursive formula due to Wijngaarden | 118 |
| 8. Statistics | Prediction by linear operators | 106 |
| 9. Data reduction program | Polynomial fitting, etc. | 126 |

Table 2-2. Current Problems Arranged According to the Mathematics Involved

(1) Floating Address

A floating-address system enables a programmer to write his instructions so that they refer to the words of his program rather than to the location of those words in storage. The assignment of final storage locations is made by the computer as part of the conversion.

(2) Number Systems and Programmed Arithmetic

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly 7 decimal digits) with magnitudes between $2^{63}$ and $2^{-64}$.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

## 2.2 PROBLEMS BEING SOLVED

### Problem 100. Comprehensive System of Service Routines

The comprehensive system of service routines which uses two magnetic tape units as auxiliary storage has been in operation during the past quarter. A description of the system is contained in Summary Reports 31, 32, 33. Several changes have been made in the system (which are listed below) but these represent changes in the mechanics of the system rather than changes in the logical structure of the system.

(1) The recording of binary tape on a magnetic tape unit: The comprehensive system currently in use punches out a binary paper tape which can then be read into the computer at any time. In order to decrease the amount of computer time required for conversion the comprehensive system has been modified to record the converted binary program on a magnetic tape unit. The punch equipment has also been modified so that a binary paper tape can be punched from the magnetic tape recording without using the computer.

(2) Increased electrostatic storage: Two banks of electrostatic storage (2048 registers) became available during the past quarter. The comprehensive system has been modified so that the automatic subroutines contained in the system (i.e., programmed arithmetic and output subroutines) are stored in fixed locations at the end of the second bank of ES (instead of at the end of the first bank).

(3) The drum input program: A new input program for binary tape was written during the past quarter and is now being used exclusively for reading binary tape into the computer. The input program proper is stored in a fixed group (group 11) of the magnetic drum. A seven-register program in test storage records the contents of electrostatic storage on a drum group (group 0) and then reads group 11 (containing the input program proper) into electrostatic storage. At this point the input program proper assumes control and records the paper tape to be read in on group 0 (i.e., it may record over part or all of the initial contents of electrostatic storage). After the paper tape has been recorded on group 0, group 0 is read into electrostatic storage and the program is ready to be operated.

A special entry point was provided in the input program proper for reading the programs in the comprehensive system from a magnetic tape unit into the computer. The comprehensive system itself has been modified to use the drum input program.

(4) Post mortems: The post mortem programs available for use with the comprehensive system have been modified in accordance with the changes in the comprehensive system discussed above. In addition several of the previously available post mortem programs have been modified to record their results on the oscilloscope rather than on the typewriter.

The present version of the comprehensive system will be considered as final. Future efforts will be directed towards developing a comprehensive system of conversion and input which automatically incorporates the magnetic drum into the conversion procedure and, ideally, makes any logical distinction between the magnetic drum and electrostatic storage unnecessary in writing programs. Several such systems are now under discussion. Future discussion of this procedure will occur under Problem 150.



### Problem 101. Optical Properties of Thin Metal Films

This problem was initiated by L. Harris and A. L. Loeb of the Chemistry Department and is being coded by Loeb. During the first quarter of 1953 a program had been coded and tested to compute the reflection and transmission of radiation by a metal film on a transparent backing as functions of the index of refraction of the backing, the wavelength of the incident radiation and the index of refraction, absorption coefficient and thickness of the film.

The aim of this problem is to compute the index of refraction and absorption coefficient, and from these the conductivity and dielectric constant, of the film in terms of reflection and transmission. For this a successive approximation scheme has been developed and coded, using the computation of reflection and transmission referred to above. During the second quarter this program has been in the testing stage.

### Problem 102. Scattering of Electrons from Gases

Most of the preceding quarter was spent in rederiving the radial scattering equations for Helium using the method of the adiabatic perturbation. Inasmuch as the algebra was quite involved it was thought necessary to take great care to attempt to keep algebraic errors to a minimum.

The derived equations are of the form

$$(L_\lambda + V_2')R_\lambda = E_\lambda R_\lambda + E'_\lambda R_\lambda \qquad (1)$$

where $L_\lambda$ is the unperturbed second order differential operator, $V'_\lambda$ is an additional term introduced by the perturbation (polarisation term), $E_\lambda$ is the integral operator (exchange integral) for the unperturbed equation, $E'_\lambda$ is an additional integral operator introduced by the perturbation, and $R_\lambda$ is the radial scattering function for which the equation is solved.

The first step in the solution of the equation (1) was to evaluate the effect of the polarization term $V'_\lambda$. Since the operator $L_\lambda$ contains the term $\frac{\lambda(\lambda+1)}{r^2}$ and $V'_\lambda$ has the asymptotic value $\frac{a}{r^4}$ where $a$ is of the order of $\frac{1}{100}$, it is found that this term can be neglected for all values of $\lambda > 0$. Further, in the case of the s-wave ($\lambda = 0$) we can compare $V_0'$ with the unperturbed potential V and the energy $k^2$ since

$$L_o' = \frac{d^2R\lambda}{dr^2} - V + k^2$$

In this case we find that $V_o'$ is one order of magnitude smaller than V out to r = 1.5. But for reasonable values of the energy (>1 e.r.) V in turn has become much smaller than $k^2$. Hence it turns out that the polarization term can be neglected for all values of $\lambda$.

The next step in the solution is the evaluation of the effect of the new exchange integrals (the $E'_\lambda$ operator).

Work on this problem has been carried out by J. L. Uretsky of the MIT Physics Department.

Problem 104. Hydro Thermal Power System; Calculus of Variations

The reprogramming of the three-plant economy loading problem using magnetic drum storage has been successfully completed by Dr. R. J. Cypser of the MIT Electrical Engineering Department. The drum has proven to be fully reliable and has resulted in a further reduction in computer time needed per iteration. Computer performance and the results obtained have been very satisfactory. In the final run of this quarter, a total of 72 iterations, at 3 1/3 minutes per iteration, were obtained without a single defect in computer operation. Each iteration resulted in a smooth reduction in the annual cost of system operation, with a total reduction in the magnitude of the surface gradient by a factor of 20.

Problems requiring further work are: a) the determination of optimum sampling spacing and approximation formulae for the ultimate system, b) the adjustment of penalty functions in accordance with both the magnitude and rate of change of operation violations, in order to allow larger steps per iteration without excessive excursions into regions of limitation violations, and c) the application of the method to the proposed twelve-plant system.



Problem 106. MIT Seismic Project

During the past quarter the Geophysical Analysis Group at MIT has written several new programs and has improved some of its older programs, with the view of making a more complete analysis of seismograms. The General Prediction program now prints, punches, or displays on an oscilloscope individual errors and/or error curves for 8 linear operators, each operator comprising up to 33 terms and utilizing up to 4 seismogram traces. The prediction distance k and the number of past lags M is arbitrary. In addition the program has provisions for forming the variances of the seismogram traces, which are necessary in determining the statistical significance of the error curves. Production runs have yielded more than 60 sets of errors and error curves and more than 40 variance curves. These results have been taken out because of (1) the necessity for good reproduction in reports and (2) the need for the numerical information for further study. This information, derived in about 4 hours of computer time, represents about 44 weeks of hand computation.

The auto-crosscorrelation programs were written in this period. One has been used to derive twenty-five 100-lag correlation curves and ninety 20-lag curves. The other, which includes a normalization, is not yet working. These curves represent about 30 weeks of hand computation or 3 hours of machine time.

An iterative method of solving simultaneous equations by least squares was programmed and was used several times. The first runs indicated the delicacy in the choice of increments. A few further runs resulted in reductions of the square errors by 62 percent after twenty minutes of straight computation. Hand computation using this method would require one year. The results of these runs will be used in obtaining solutions by a method of steepest descent.

A subroutine for calculating power frequency spectra was used, but after several trials was found to be unsatisfactory. A program more suitable to our data has been written but is not yet working.

Recently a program was written with the hope of detecting seismic reflections by a form of phase relationships, but the results have not been analyzed to see whether the method will be successful.

In investigating the nature of linear operators we have used the General Prediction program with so-called cosine operators and obtained exact verification of theoretical deductions. As the theory advances this program will be used for further checks.

Problem 107. (a) Autocorrelation and (b) Fourier Transform, Evaluate Integrals

All development work on Problem 107 has been completed during the present quarter. Programs are available for autocorrelation, using rectangular rule or Simpson's rule for the evaluation of the integral, and for Fourier sine or cosine transformation. The problem number will be kept active for the use of certain authorized groups whose calculations can be completely effected by the routines developed under this problem. Problem 107 has been or is being used by the following laboratories in this fashion.

Project Lincoln -- spectral analysis of empirical data to obtain frequency content

Hydrodynamics Laboratory -- determination of resonant frequencies of an open-ended tank

Massachusetts Memorial Hospital (AEC Sponsored) -- investigation of intestinal motility under normal and imposed conditions

In addition, the programs of Problem 107 have been used within the operation of other problem numbers as follows:

Problem 116 -- Torpedo Impulse Response -- sine and cosine transforms

Problem 106 -- MIT Seismic Project -- determination of frequency spectra of seismic traces

Problem 137 -- Atmospheric Turbulence -- determination of power spectra of measured gust loads on an aircraft probe

A final report including complete descriptions of all of the programs is being written for immediate publication.

Problem 109. Fighter Gunsight Calibration, 8th Order Differential Equation

The problem of a two-dimensional aircraft pursuit course in the horizontal plane has been successfully solved by M. H. Hellman of the MIT Instrumentation Laboratory for a particular set of initial conditions. The problem consisted essentially of solving eight simultaneous non-linear differential equations by the Runge-Kutta Method. The solution was obtained with time as the independent variable.

The principal object of this problem is to solve the three-dimensional or slant plane pursuit course. Based on the successful solution of the two-dimensional problem, the major effort is now being made to solve the three-dimensional problem.

Problem 111. Fourier Analysis -- Autocorrelation Problem

The first part of this problem, which has been solved, was to determine the Fourier coefficients of a function which is regarded as a linear combination of trigonometric functions with unknown coefficients and of a presumably stationary random process

It is now desired to subtract out the trigonometric functions from the given function and determine the autocorrelation function of the remainder, i.e., of the stationary random process. Work on this second phase is near completion. This work is being carried out in cooperation with the MIT Instrumentation Lab.

Problem 112. Lawley's Method of Factor Analysis; Characteristic Vectors (modified)

This problem, undertaken in cooperation with Dr. F. M. Lord of the Educational Testing Service, seeks a diagonal matrix J and an m x m matrix F such that $JF = FS^{-2}(R-S^2)$, where R is a positive definite symmetric n x n matrix of given data



and S is a diagonal matrix with elements $s_i = \sqrt{\sum_{r=1}^{m} f_{ri}}$

At the beginning of the current quarter one complete run was made with m=4, the iterations converging in about 40 repetitions. Following this, the program was entirely rewritten in order to

a) reduce inaccuracies due to rounding off,

b) substitute the magnetic drum for magnetic tape for storing the R matrix, and

c) utilize the newly doubled capacity of WWI electrostatic storage.

At the same time a program was added to compute

$$w_{ij} = E_{ij}^2 / s_i^2 s_j^2, \text{ where}$$

$E_{ij} = r_{ij} - \sum_r f_{ri} f_{rj}$ -- the $w_{ij}$ are needed for a test of statistical significance to determine the appropriate value of m.

The remainder of the quarter was spent in removing errors from the new program. This process is virtually completed and production runs should be resumed shortly.

Problem 113. Shear Wall Analogy, Simultaneous Linear Equations

The concept of the lattice analogy, that of relating the deformations of an elastic plate to the deformations of a pin-connected lattice network of bars, has been utilized to carry out the theoretical stress analysis of reinforced concrete shear walls.

In the quarter just ending, the program prepared for this theoretical analysis was performed using a series of parameters representing several different shear walls. In the program originally prepared for the solution of this problem, machine performance time was about 15 minutes per complete solution. Hand computations were required at intermediate stages of the solution and several performances had to be made for complete results on one wall. In the present program, no intermediate hand calculations are required and the performance time for each complete solution averages 5 minutes.

Altogether, 27 different walls have been investigated. These results are being analyzed and a report is being prepared for the Civil Engineering Department at MIT on the progress made to date.

Problem 114. Design of Optical Systems

During this quarter two programs have been written which will perform some of the calculations involved in the design of an optical system. The first of these is a standardized ray-tracing program which determines the location and the direction of a light ray that has passed through the optical system, given the initial direction and location of the ray. The constants specifying the optical system and the data for each ray are treated as parameters, so that variations in the optical system can be handled without extensive programming.

The second program that was written is designed to compute the third order aberration terms of an optical system. This program also utilizes parameter tapes for the optical data. This is necessary since variations must be introduced several times until the aberrations are reduced to zero.

These programs are now being used in the design of ophthalmic instruments. Further programs will be written later to perform additional optical calculations.

This work is being carried out in cooperation with the Retina Foundation at the Massachusetts Eye and Ear Infirmary.

Problem 116. Torpedo Impulse Response; Convolution

The basic problem involved here is the solution of the convolution integral equation which relates the output of a linear system to its input and its impulse response. Measurements have been made of eight input-output pairs for the system under test and it is desired to determine the impulse response relating each pair. WWI is being used to convolve the measured input with the estimates of the system impulse response to determine how nearly correct the estimates are. A high-speed analog computer* is being used to determine approximate impulse responses and then corrections to these on the basis of the difference between the predicted and measured output.

During this quarterly period initial estimates of the impulse responses relating each input-output pair were obtained from the analog computer. These estimates have an accuracy of the order of 10%. One of these impulse responses has been convolved on WWI with all the input functions for two purposes: first, to assess the ability of one impulse response to predict the system output for a variety of inputs as a check on the system linearity, and secondly, to provide a set of functions which show the error of prediction for each input and a trial impulse response. These error functions can be used in conjunction with the analog computer to estimate a correction to the trial impulse response. One case has been carried through this stage and the resultant error function had an absolute area equal to 3 1/2 percent of the absolute area of the measured output. It is felt that this is a practical technique and will be applied to the remaining seven sets of input-output data.

Theoretically, the Fourier transform provides an explicit solution to the integral equation. To assess the possibility of using the Fourier transform either to determine the system impulse response directly from the input-output data or to determine the correction to an estimated impulse response from the input-error data, the transforms are being computed by a program developed under problem 107. The results of the first trial were meaningless -- probably because too large a computing interval was taken. This interval has been reduced by a factor of four and the direct transform computed. These results are now being processed for the inverse transformation. If this technique is successful, it will be used to complete the data reduction.

* The General Purpose Simulator of the MIT Aeronautical Engineering Department in the Hood Building.



Problem 118. Quantized Group Communication and Learning; Non-Markovian Stochastic Process

The Group Networks Laboratory at MIT has been making communications experiments on groups of five people. They have developed a mathematical model for communication and learning in certain cases of such groups. Evaluation of predictions from this model has not been possible explicitly, so a Monte Carlo method using WWI has been used. The method consists of actually simulating a five person communication group, under this model, on the machine. (See the March 1953 Quarterly Report.)

The programming of this problem was straightforward, consisting mainly of collating, sorting, and table consulting. Storage limitations presented no difficulty, so emphasis was placed on speed of operation. Even so, the amount of data handling was so great that the computation was fairly slow, a complete calculation for a network requiring about 45 minutes. If the new instruction (multiply digits) in the WWI operation code had been available at the time of programming, a considerable saving in storage and machine time could have been made.

Three different communication networks have been run on this problem. The first, called pinwheel, produced results which fitted very well with the observed results. The next two, called circle and barred circle, produced more anomalous results, and only a partial fit.

It is felt at present that an error has crept in at some point in the case of these last two networks; we do not know as yet just where this might have occurred, although the code seems the most probable place. A recheck is in process to locate the trouble.

When successfully completed, the results of this problem would permit accurate checking of a theoretical model for group interaction processes with experimental data, and hence would be a valuable contribution to the development of such theories.

Problem 119. Spherical Wave Propagation

An investigation of methods for integrating numerically the hyperbolic partial differential equations governing the propagation of spherical waves is being carried out by P. Fox and A. Ralston under the supervision of Professor C. C. Lin of the MIT Mathematics Department.

A working program now exists which for any mesh spacing and any initial density distribution will compute the values of density and velocity as functions of the time $t$ and of the distance $r$ from the center of the sphere. This program uses magnetic tape for secondary storage and for delayed printing.

For a mesh width of .03125 and an initial density distribution, $\rho = 1 + 2e^{-4r^2}$, a total of approximately 90 points in the $r, t$ plane have been computed on Whirlwind I in about 23 minutes.

Physical checks on these results indicate very good accuracy for the process being used. However, the computation is still in its early stages.

Due to the slowness of the computation it is planned in the future to try larger mesh widths and different initial density distributions in order to carry the computation further in a physical sense without using too much machine time.

Problem 120. Thermodynamic and Dynamic Effects of Water Injection into Gas Streams of High Temperature and High Velocity

This problem is connected with the development of a potential gas turbine component called the "Aerothermopressor." The Aerothermopressor is a device for increasing the stagnation pressure of a hot, high-velocity gas stream by evaporative cooling. Further description of this device may be found in Summary Reports No. 32 and No. 33.

The analytical aspect of this development program is concerned with obtaining a better udnerstanding of the Aerothermopressor process and with predicting its performance under various operating conditions. The Aerothermopressor process is defined by nine parameters: 1) entrance Mach number, 2) entrance stagnation pressure, 3) entrance stagnation temperature, 4) water injection rate, 5) initial droplet diameter, 6) initial droplet temperature, 7) initial droplet velocity, 8) wall friction, and 9) variation of duct cross-sectional area. For fixed values of these parameters the state at every cross-section of the Aerothermopressor is fully defined and may be calculated from a simultaneous solution of seven non-linear, first order differential equations.

The computations currently being carried out on Whirlwind I involve a step-by-step numerical solution of these equations for various combinations of the initial parameters. This analysis is accomplished by a (24,6) program occupying approximately 2000 registers with each execution of the program (one step) requiring about 9 seconds computational time. This program incorporated provisions for starting and stopping the calculations at any point in order to obtain flexibility in varying the cross-sectional area of the duct, as well as providing for the insertion of normal shocks.

Production runs have been successfully initiated, and the results now under study will be discussed in a later report. It is estimated that the information obtained thus far is equivalent to about 540 man-days of arduous computation for a computer using a desk calculator.

The Aerothermopressor development program is being carried out at MIT under sponsorship of the Office of Naval Research and under the guidance of Professor Ascher H. Shapiro. The analytical work discussed above is being done by Bruce D. Gavril as part of a Sc.D. thesis for the MIT Department of Mechanical Engineering.

Problem 121. Determination of Weak Signal plus Noise Probability Functions

During sporadic periods throughout the past six months a series of test programs has been written with the view towards finally calculating a convolution integral involving the transformation of a type of probability density function common to problems in the statistical theory of noise.



The latest modification of this series was designed to compute the convolution integral

$$W(\gamma) = \int_0^{\gamma} W(B)\, W(\gamma - B)\ dB.$$

The computation of W(B) was an integral part of the program. This function,

$$W(B) = \frac{e^{\frac{-\phi^2 + \rho}{2}}\, I_o\left(\phi \rho^{1/2}\right)}{5\phi^3} \qquad \phi = B^{1/5}, \qquad B \geq 0$$

$$= 0 \qquad , \qquad B \geq 0$$

is seen to possess an infinite singularity at the origin. For convenience of calculation, the function was defined to be zero at the origin and the range of the variable B broken into four intervals of different increments. The trapezoidal integration approximation was employed, and the program automatically computed the functions for the appropriate increment value of each interval range.

The program includes sub-programs for evaluation of the fifth root of a real number, and two other sub-programs for the evaluation of the negative exponential and the modified Bessel Function, both by means of terminated infinite series approximations.

Problem 123. Earth Resistivity Interpretation: Integration of Empirical Functions

As described in Summary Report No. 33, this problem involves the evaluation of the integral

$$\lambda \int_0^{\infty} J_o(\lambda r)\, r\phi(r)\, dr$$

where r is the radial distance from the current source, and $\phi(r)$ is the measured potential. The program as written starts with a set of measured values of $r\phi$, and fits a set of orthogonal polynomials in the least-square sense to these points. The present program requires 20 data points and fits a fifth order set. The program then picks one of a given set of $\lambda$'s and a $\Delta(\lambda r)$ which depends on the value of r. Using the difference equations

$$J_o(x+h) = h^2 \left[\frac{1}{x} J_1(x) - J_o(x)\right] + 2J_o(x) - J_o(x-h)$$

and

$$J_1(x+h) = h^2 \left[\left(\frac{2}{x^2} - 1\right) J_1(x) - \frac{1}{x} J_o(x)\right] + 2J_1(x) - J_1(x-h)$$

where $x = \lambda r$, the program stores the required quantities. For each value of x the corresponding value of $r\phi$ is computed using the stored polynomials. The integral itself is evaluated by means of the trapezoidal rule.

This procedure is continued until the asymptotic value of $r\phi$ is approached sufficiently closely. The contribution from the remaining (infinite) range of r is constant depending on the $\lambda$ and the value of r at which the asymptotic value of $r\phi$ was assumed to be reached. This constant is added manually at the end of the calculation. The program is to print out the coefficients of the orthogonal polynomials and the results of the finite integration. The entire procedure is now being tested.

Problem 124. Deuteron Binding Energy and Wave Functions

Schrodinger's equation for the relative motion of a neutron and proton reduces, for a spherically symmetric potential, to

$$\frac{d^2u}{dx^2} = \left[\epsilon - v(x)\right] u \qquad (1)$$

where, $u = \psi r$

$x = r/r_c$; $r_c$ is to be determined.

$$\epsilon = \frac{2m}{\hbar^2} r_c^2 E \qquad (2)$$

m = reduced mass of proton and neutron

E = energy of the relative motion

v(x) = potential of the interaction

The potential function studied in this problem is of the form

$$v(x) = A \frac{e^{-x} - e^{-bx}}{x}, \quad b > 1 \qquad (3)$$

where $A = \frac{2m}{\hbar^2} r_c^2 V_o$ ; $V_o$ is to be determined.

This potential goes over to the Yukawa potential, $e^{-x}/x$, as $b \to \infty$. However, unlike the Yukama potential which is infinite for $x = 0$, the function (3) has the value $A(b-1)$ at $x = 0$. Thus the parameters A and b together determine the depth of the potential at the origin; the parameter b also determines the "range" of the potential.

The main task is to determine those combinations of A and b which lead to solutions of equation (1) consistent with experimental data. First we impose the boundary conditions $u(0) = (0)$, $u(\infty) = 0$, on equation (1); the quantity $\epsilon$ thus becomes an eigenvalue.

The boundary conditions on u are those of a bound state, so that $\epsilon$ is proportional to the binding energy $E_B$ of the deuteron (proton and neutron bound by nuclear forces). Since $E_B$ is known from experiments, equation (2) can be solved for $r_c$ in terms of known quantities, where $\epsilon$ is the eigenvalue of (1) for a particular A and b.

Next we set $\epsilon = 0$, and solve for u. If $x_a$ is the value of x where u crosses the x-axis then $x_a = a/r_c$, where a is the so-called Permi scattering length, a quantity



which can be deduced from neutron-proton scattering experiments. Since $r_c = a/x_a$, we can again solve for $r_c$ in terms of known quantities.

For a given value of b, two curves of $r_c$ as a function of A are plotted. The first is based on $r_c = a/x_a$, where $x_a$ is a function of A; the second curve is $r_c = \hbar\sqrt{\epsilon/2mE_B}$, where $\epsilon$ is a function of A. The most probable value of A, for a given b, is determined from the intersection of the two curves. Due to experimental errors in a and $E_B$ the two curves are broadened, so that a unique value of A, for a given b, is not determined.

The results are helpful, however, in giving an idea of the approximate form of the nuclear potential. Further studies of this two-parameter potential may include tensor forces; the deuteron quadripole moment then enters as a third experimental quantity which can also be calculated.

The computational part of this problem has been carried out on the Whirlwind I computer by D. Combelic of the MIT Physics Department.

Problem 125. Analytical Differentiation

The problem is concerned with the representation and manipulation of functions in a coded analytical form by WWI. In particular, a program was written which carried out the process of analytical differentiation. The program receives as input on tape the function expressed in symbolic code and from this derives the analytical forms of the derivatives. The derivatives are then typed out in the same code. The program is designed to handle all functions of one variable which can be formed of finite combinations of rational, trigonometric, logarithmic, and exponential forms.

The problem is related to the future use of compilation routines in problem solving. Implicit in the concept of a highly efficient compilation system is the necessity for the manipulation by the computer of analytical relations independent of the values of the variables involved.

The completed program was incorporated into a thesis entitled "Analytical Differentiation on a Digital Computer" which was submitted by John Nolan to the Department of Mathematics at MIT in partial fulfillment of the requirements for the S. M. degree.

Problem 126. Data Reduction

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory, MIT. The over-all problem is composed of many component sections which will be developed separately and then combined at a later date. The first stage of this development is to devise a program to automatically fit polynomials to arbitrary empirical functions over suitable ranges of the independent variable. The procedure to be used utilizes the minimization of the integral of the square of the error between the polynomial and the given function, and is implemented through the use of Legendre polynomials and matrix multiplication. A further requirement is that all errors must be less than some specified tolerance. Then rather than store the many separate values of the given functions in the over-all program and use interpolation between these given values, the coefficients of the fitted polynomials will be stored; and the polynomial will be evaluated for the desired values.

The program now being developed will do the following operations:

(1) Fit by least squares a polynomial of specified degree to the given function.

(2) Calculate the error between this polynomial and the given function and plot the errors on the scope to be photographed.

(3) Calculate the mean and variance of the errors and plot a histogram on the scope.

(4) Type out the coefficients of the polynomial and the statistical information.

(5) If the error tolerance has been satisfied, do step 6. If the error tolerance has not been satisfied, fit a polynomial of the next higher degree to the errors found in step 2 and repeat steps 2 through 5.

(6) Add the corresponding coefficients of all polynomials and do steps 2 through 5, using the given function, to give the final fit information.

It is intended that the program be made available to other users of WWI in library form. Probably six out of seven places of a reasonably smooth function can be fitted in this manner. The program is 100 percent complete.

### Problem 128. MIT Subject 6.537 Digital Computer Applications Practice -- Spring 1953

Eight students were registered for the second semester MIT subject 6.537, Digital Computer Applications Practice, which was run as a seminar and laboratory course. Each student selected a problem to program by himself for solution on the Whirlwind I computer. The problems chosen were as follows: a second order non-linear ordinary differential equation solved by the Runge-Kutta method, simultaneous linear algebraic equations (arising from a wave equation) solved by a new (Crout) elimination procedure, simultaneous linear equations solved by a new (Craig) descent method, a mistake-diagnosis routine, a floating-address conversion program, heat balance in a chemical reactor (polymerization of tetrafluoroethylene), optimization of an assembly-line problem, and the game of Nim. Abstracts from the students' term reports are included below.

#### The Solution of Simultaneous Linear Equations

A program for a system of ten equations was written using an analysis adapted from "The Solution of Simultaneous Linear Equations with the Aid of the 602 Calculating Punch" by Frank Verzuh. The method involves the use of a Gauss-Jordan reduction. It was planned that the program would be rewritten in general parametric form for an arbitrary system of equations, but there has not yet been time. The solution for ten equations requires approximately two minutes using (24, 6) floating decimal numbers.



#### An Error-Diagnosis Program

An error-diagnosis program has been written that enables one to obtain via printer a predetermined amount of useful information to perform error diagnosis. Single or consecutive registers in the program may be selected for error diagnosis information print-out by presetting a flip-flop register. The program is being altered to allow one to specify the number of times a register is to be passed before diagnosis.

#### Scope Presentation of Solutions of Second Order Differential Equations by the Runge-Kutta Method

The essence of the problem is to develop a program to solve and plot the solution of any second order differential equation in which the highest derivative may be expressed as a function of the other variables and constants. The solution is to be plotted using the full face of the scope over the area of interest, the axes having been plotted at the optimum locations. The results obtained were quite satisfactory.

#### An Iterative Procedure for Solving Systems of Simultaneous Linear Equations

An iterative procedure suggested by Mr. E. Craig of the MIT Electrical Engineering Department has been programmed to solve sets of up to thirteen simultaneous linear equations. The method theoretically requires no restriction on the coefficient matrix. In practice, however, the method either failed to converge, or converged very slowly for some of the sets that were tested.

#### Man-hour Determination and Scheduling for an Assembly Line by Use of a WWI Program

Given a one-dimensional assembly line, a means of determining the least number of men to feed it and of scheduling their time has been devised. With certain adjustments and assumptions the program can be used for more general cases. The present program is prohibitively long timewise, but can be used with certain changes to make it shorter.

#### A Floating-Address Conversion Program

Tape 2589 is a conversion program which will convert programs punched in standard Flexowriter form, store the program in electrostatic storage, and start at the address designated. Floating addresses, WWI orders, decimal numbers, and starting addresses are converted by the program. If the number of floating-address assignments and/or references to floating addresses is above limits, appropriate information is entered into flip-flop registers and the program will cease converting. A 'bouncing-ball plot' (2589 pl) was converted successfully.

#### The Application of a Large-Scale Digital Computer to the Game of Nim

The problem of mechanizing the playing of the game of Nim using WWI was deemed interesting and has been programmed for the case of six piles containing six units each. With the present program, the human player has the first move -- the machine wins only if the human player makes a mistake. A check register alarm signifies the end of the game. It was hoped that a scope display of results would be ready, but due to unexpected delays, this was not completed.

A WWI Program to Schedule the Control of the Temperature in a Polymerization Reaction

A program has been devised to describe temperature in a polymerization reactor as a function of time and conversion, and, by varying external reactor temperature, to arrive at an optimum control schedule for the reactor. A test run was made using a reasonable set of reactor and reaction constants, and graphical results have been prepared.

Problem 130. Six-Component Distillation, Variable Enthalpy and Equilibrium Data; Simultaneous Non-Linear Equations

A continuous fractional distillation tower is designed to receive a volatile mixture of fixed composition at a fixed rate and to effect a specified separation of the "light key" component from the "heavy key" component by means of a number of distillation stages. The present problem is to calculate the number of stages, or "theoretical plates" required. (See Quarterly Report for March, 1953.)

When the composition, temperature, and overflow rate of the liquid on a given plate of the tower are known, the composition and flow rate of the vapor over that plate, and the corresponding liquid and vapor data for an adjacent plate, can be calculated. The calculations involve three types of equations.

(1) Material Balances. These are linear equations connecting the compositions of adjacent plates with the composition of the product withdrawn from the tower.

(2) Enthalpy Balances. These equations relate the liquid rates, the vapor rates, and the compositions and temperatures of adjacent plates with those of the upper or lower product stream from the tower. They are non-linear, and involve two thermodynamic functions, the liquid and vapor enthalpy functions, which are known only empirically.

(3) Vapor-Liquid Equilibrium Relations. These equations relate the liquid composition to the vapor composition at each plate, and determine the temperature. They are non-linear, and involve a thermodynamic function, the vapor-liquid equilibrium function, which is known only empirically.

The behavior of the tower thus is represented by a set of simultaneous equations, involving three tables of thermodynamic data, which must be solved by non-linear iteration. Simplified versions of these equations, ignoring the enthalpy balances, have been set up for punched card or manual calculation. More rigorous calculations, however, are demanded by current engineering practice.

A Whirlwind program for performing such calculations, including enthalpy balances on each plate, has now been written and tested. The program was written by Mr. J.F. O'Donnell, Jr. of the MIT Chemical Engineering Department. It represents the empirical thermodynamic functions by means of abridged numerical tables, with routines for linear interpolation. The vapor-liquid equilibrium data is given for each component as a function of temperature at the operating pressure, by means of the MIT tables of K values. The liquid and vapor enthalpy data are given in the Scheibel and Jenny form, as functions of temperatures and average molecular weight of the vapor or liquid mixture.



Checking of the program against results previously calculated by hand was initiated by Mr. O'Donnell and completed June 25 by Mr. J. Horowitz of The Lummus Company, New York, N.Y.

Work accomplished in this quarter permits the following conclusions to be drawn:

(1) Calculations of this type and to this degree of rigor are well within the capacity of a modern digital computer. There seems to be no problem of convergence in the iterative solution.

(2) A large part of the necessary thermodynamic data can be supplied by the computer itself, utilizing a minimum amount of stored data, without excessive loss of accuracy.

Before terminating this problem, it would be desirable to explore the accuracy and the speed of convergence of the present program over a wide range of design parameters. Problems of the same type, but of greater complexity remain to be attacked. In systems operating near their critical conditions, for example, the thermodynamic data must be represented as functions not only of temperature and average molecular weight, but also of the composition, which is itself unknown. This imposes another level of iteration on the calculations.

Problem 132. Subroutines for the Numerically Controlled Milling Machine

This problem consists of the writing and testing of a group of library type subroutines for performing computations associated with data preparation for the Numerically Controlled Milling Machine (NCMM) at the MIT Servomechanisms Laboratory. The immediate objective is to allow data for plane curves such as those defining the contours of cams or templates to be processed automatically, assuming points on the curve are given or may be found by a subroutine. Going from given points to NCMM tape requires the following steps:

(1) Compute the tool center offsets for the point under consideration.

(2) Compute the tool center coordinates for the point and find the increments necessary to cause the tool center to move to this point from the tool center for the preceding given point.

(3) To obtain an acceptable feed rate, select a suitable block length and, if necessary, subdivide the increments.

(4) Convert the floating point numbers used up to now to fixed point integers, convert these numbers to NCMM code and record them.

Routines have been written to perform these steps. Satisfactory operation has been obtained for three of the seven routines that have been written. Three more are now being "debugged" so the programming necessary for the problem is almost complete while the testing of the routines is about fifty percent complete.

Problem 133. Non-Linear Meson Equation

Solutions of the radial equations of a class of non-linear meson equations are being sought by D. Finkelstein of the Laboratory of Nuclear Science and Engineering at MIT. These equations were described in Summary Report No. 33. A program has been written that obtains solutions of these differential equations one at a time. It only remains to adapt it to cyclic operation to obtain the desired family of solutions.

Problem 134. Numerical Diagonalization Procedure

A program has been written by A. Meckler of the MIT Solid State and Molecular Theory Group to find the eigenvectors and eigenvalues of a real symmetric matrix A. The method proceeds by determining an orthogonal matrix $U_i$ so that if

$$A_{i+1} = U_i A_i U_i^t \quad (A_o = A)$$

the element of $A_{i+1}$ in the same position as the off-diagonal element of $A_i$ of largest absolute value is zero. It can be shown that under very general conditions

$$\lim_{i\to\infty} A_{i+1} = \lim_{i\to\infty} U_i \cdots U_o A_o U_o^t \cdots U_i^t = A_\infty$$

is a diagonal matrix whose diagonal elements are the eigenvalues of A and that the columns of $\lim_{i\to\infty} U_i \ldots U_o$ are the eigenvectors of A.

The program at present will handle a matrix of order $\leq 31$ and involves the use of only one drum channel. It should be pointed out that complex Hermitian matrices can be treated by the following trick. Let the complex Hermitian matrix be H

$$H = A + i B$$

where A is real symmetric and B is real antisymmetric. The eigenvalue-eigenvector condition is

$$HZ = \lambda Z$$

or

$$(A+iB)(x+iy) = \lambda (x+iy)$$

which on separation of real and imaginary parts yields

$$Ax - By = \lambda x$$
$$Ay + Bx = \lambda y$$

which can be represented as a super-matrix equation

$$\begin{pmatrix} A & -B \\ B & A \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$



In this way the problem of diagonalizing a complex Hermitian matrix can be reduced (or expanded) to that of a real symmetric matrix of double the order.

Testing of the program has been delayed by difficulties that arose in the use of preset parameters and floating addresses. All this has been corrected and the program itself is now to be tested.

Problem 136. Matrix Equations

A program has been written by D. Arden of the MIT Digital Computer Lab for the solution of a set of linear algebraic equations by a variation of the Hestenes-Stiefel method devised by E. Craig of MIT.

If x = h is the solution of the matrix equation Ax = b, where A is a non-singular square matrix of order n, the method consists of taking an arbitrary initial vector $x_o$ and calculating n+1 vectors $x_i$, $0 \leq i \leq n$, by the following method:

$$x_{i+1} = x_i - \frac{p_i^T \; r_i}{q_i^T \; q_i} \; q_i$$

where

$$r_i = Ax_i - b$$

$$p_i = r_i + \frac{r_i^T \; r_i}{r_{i-1}^T \; r_{i-1}} \; p_{i-1}$$

$$q_i = A^T p_i \quad \text{and} \quad p_o = r_o$$

This amounts to proceeding in the direction $q_i$ to a point as close as possible to h. Since it can be shown that $q_i^T q_j = 0$, $j<i$, $x_n$ should be the correct solution h. If this is not the case in practice (due to numerical approximation), the result can generally be improved by starting over using $x_n$ as the initial approximation.

Several programming errors have been corrected and the program is currently undergoing further testing and modification.

Problem 137. Investigation of Atmospheric Turbulence; Autocorrelation, Crosscorrelation and Fourier Transforms

Atmospheric turbulence, acting directly on the aircraft, is a significant noise input to airborne control systems such as those now under study by the Instrumentation Laboratory and Meteor Engineering Division. A quantitative description of this random input is necessary before the design study of an airborne control system can be completed. If the turbulence is assumed to be a stationary random process, the methods of generalized harmonic analysis may be used to obtain the required description in terms of the power spectral density of the random input.

In 1950, some data was obtained for the power spectrum of vertical gust velocity which is the principal noise input for the longitudinal motion of an aircraft. However, no satisfactory data was obtained for those gust inputs which excite the aircraft laterally. A simplified model of the lateral gust structure has now been postulated. In order to determine the validity of this model (and properly modify it, if necessary), the power spectra of each of three assumed lateral gust components are now being determined simultaneously by flight measurements using an automatically controlled aircraft as a "probe." Three autocorrelations and three crosscorrelations must be calculated in order to determine, by Fourier transformation, the power spectrum matrix for three selected outputs of the aircraft control system. By an inversion of the aircraft control system performance matrix, the three desired input power spectra are then calculated. A suitable cross power analysis has been developed for this calculation.

The successful analysis of this data should provide a complete, quantitative description of the gust structure in the atmosphere in a form suitable for analysis and synthesis of linear control systems.

Both the autocorrelation and the Fourier transform (sine and cosine) programs written by D.T. Ross will be used (see Problem 107). A crosscorrelation program, based on a modification of Ross' autocorrelation program, is being prepared by N. Zierler and C. Block of the Instrumentation Laboratory. Testing of this crosscorrelation program should take place very shortly.

For the autocorrelation program, 2150 data points were supplied at equally spaced time intervals (in this case, 0.1 second) for each of three functions. The first attempt to autocorrelate these functions was not successful due to a misunderstanding regarding the recording of the WWI output on magnetic tape. These difficulties have been remedied, and further computations will be carried out in the near future.

Problem 140. Summer Session System

Development of an instruction code, conversion program, interpretive routine, post mortem and mistake-diagnosis routines, to be used by students participating in the MIT 1953 summer session course on "Digital Computers and Their Applications," has been begun.

A complete, but tentative, instruction code (Digital Computer Laboratory report, M-2227) and the description of a proposed conversion routine (Digital Computer Laboratory report, M-2235) have been written.



# 3. Operation of Whirlwind I

During the period of 1 March - 1 June, 715 hours were assigned to the applications groups. Of this time 83 percent was useful. Figure 3-1 shows how computer time was allocated during the period.

## 3.1 MARGINAL CHECKING

A long-desired improvement in our maintenance facilities was achieved with the design and installation of the new marginal-checking control system. The original system had been put together somewhat haphazardly, as the need for it developed, and because of this was difficult to operate and repair. In addition, various components of the system had proved to be of questionable reliability.

| Activity | Hours Per Week | | | | | | | | | | | | | Total Hours |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mar 2-8 | Mar 9-15 | Mar 16-22 | Mar 23-29 | Mar-Apr 30-5 | Apr 6-12 | Apr 13-19 | Apr 20-26 | Apr-May 27-3 | May 4-10 | May 11-17 | May 18-24 | May 25-31 | |
| ES Installation, Maintenance, & Special Test | 19 | 19 | 27 | 20 | 22 | 18 | 29 | 15 | 17 | 20 | 12 | 20 | 20 | 258 |
| Marginal Checking | 6 | 8 | 6 | 6 | 4 | 7 | 19 | 18 | 6 | 6 | 7 | 6 | 6 | 105 |
| Installation | 13 | 5 | 12 | 12 | 12 | 16 | 47 | 9 | 14 | 13 | 11 | 6 | 6 | 176 |
| Maintenance | 21 | 29 | 22 | 20 | 38 | 21 | 10 | 14 | 22 | 25 | 21 | 20 | 6 | 269 |
| Term. Equip. Testing | 24 | 21 | 23 | 28 | 12 | 24 | 12 | 19 | 26 | 20 | 26 | 45 | 39 | 319 |
| Development of Trouble Location Techniques | 4 | 4 | 5 | 4 | 4 | 6 | 0 | 4 | 3 | 4 | 5 | 4 | 4 | 51 |
| Engineering & Scientific Computation | 29 | 24 | 30 | 29 | 24 | 26 | 23 | 30 | 35 | 32 | 34 | 28 | 23 | 367 |
| Other Applications | 29 | 31 | 26 | 27 | 35 | 30 | 23 | 20 | 24 | 32 | 28 | 17 | 26 | 348 |
| Total Hours | 145 | 141 | 151 | 146 | 151 | 148 | 163 | 129 | 147 | 152 | 144 | 146 | 130 | 1893 |

Fig. 3-1. Allocation of Computer Time during Period 1 March - 1 June 1953

The new control panel is illustrated in Fig. 3-2. Rotary line-selection switches have replaced the telephone dial of the original system. The mode-selection switch is now rotary instead of a ganged push-button switch. Selecting a marginal-checking line by dialing was a time-consuming and often inaccurate procedure; now it is merely necessary to turn the three line-selection switches to the correct line number and push the "Select" button. Changing the position of any of the line-selection switches or the mode-selection switch will automatically deselect any selected marginal-checking line. This is a safety precaution against accidental momentary selection of more than one marginal-checking line or mode. It also conveniently saves many operations of the "Release" button when manually selecting a series of marginal-checking lines.

The system is interlocked so that it is impossible to select a new line in any mode of operation when the marginal-checking generator is producing a voltage (excursion). This precaution is necessary because the generator is connected across shorted relay contacts at the moment when a line is selected. After the line is properly selected, the short is removed and the voltage from the marginal-checking generator is then in series with the d-c supply voltage to the circuit being marginal-checked. It may be added to or subtracted from the supply voltage as required. When the output of the marginal-checking generator is not zero, the "Pot Off Zero" pilot light is turned on.

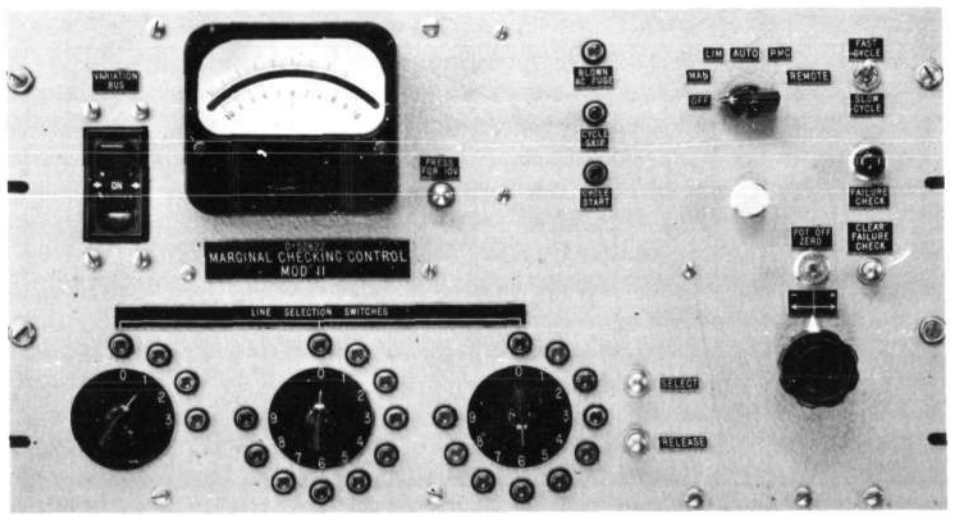



Fig. 3-2. New Marginal-Checking Control Panel



In the Manual mode of operation a voltage swing of $\pm 90$ volts may be added to the d-c supply voltages. In the other three modes of operation, the positive and negative excursions which may be used are limited by potentiometers on the marginal-checking switching panels for the individual marginal-checking lines. These excursions are generally set to 10 percent less than the voltage (margin) which will cause failure on that line with the appropriate test program.

The Limited Manual mode of operation is the same as the Manual mode except for the limited voltage excursions just described. In the Automatic mode, the marginal-checking equipment will select lines in numerical sequence, taking about 2.3 seconds from one line to the next, including applying the positive and negative excursions for each line. This is considerably faster than the old system, due to the use of a gas-tube counter in place of the unreliable stepping switches used in the old system. Also available by using the switch in the upper-righthand corner of the panel is a "Slow Cycle" taking about 10 seconds from line to line.

In the Automatic mode the marginal-checking equipment automatically starts the computer as each line is selected and stops it as soon as the excursions have been applied. If a computer failure is induced by the applied excursion, the excursion will be removed and the marginal-checking equipment will stop in a "Failure Check" condition, with the line on which the failure occurred still selected. The actual margin which caused failure may then be measured by using the manual potentiometer and reading the meter. After this is done, pushing the "Clear Failure Check" button will allow the automatic marginal checking to resume where it left off and select the next line in sequence.

The most important change in the new control system is represented by the Programmed Marginal Checking (PMC) mode. The marginal-checking lines are selected by a gas-tube decoder in this mode. Marginal-checking line numbers (in a binary-coded decimal form) may be read into the decoder from the computer. Thus the computer may be programmed to select its own marginal-checking lines according to the test program being used. These marginal-checking line numbers are read in from a tape in the mechanical paper-tape reader, as called for by the computer.

The PMC mode is economical of time because only the marginal-checking lines needed for any given test program are selected. Since the marginal-checking lines are not grouped in any particular numerical order but may be widely scattered, this represents a great saving over the Automatic mode where it would be necessary to index through many unwanted lines.

### 3.2 MAINTENANCE DIFFICULTIES

During this period about twenty hours of computer time were spent in tracking down a trouble brought to light by a complicated applications program. The only symptom originally was a failure to get identical results each time the program was run.

The program was rewritten to be somewhat self-checking; however, it was necessary to resort to drastic measures in order to isolate the trouble completely. First it was determined by experiment that the self-checking program was most sensitive to margins on one particular marginal-checking line. This line varied the gate-tube screens in four digit columns. Then these gate-tube screen circuits were progressively rewired so that they were supplied with a fixed voltage, until only one of these circuits was being varied by the marginal-checking line. The trouble was finally found to be caused by a cathode follower driving the suppressor grid of this gate tube. The cathode follower was peculiarly PRF-sensitive because of cathode interface and had caused no trouble at all with any of our usual test programs, or in fact with any other program except this particular one.

Another case of PRF-sensitivity was brought to light when a series of highly intermittent errors during a maintenance period was traced to a 3E29 driving one of our operation-matrix lines (the operation matrix determines what order the computer is to perform). This tube had also failed to cause any low margins with our usual test programs. Because of this a new test program was devised which did show up this tube (as well as several other 3E29's in the operation-matrix driver panels) as deteriorating. The test program consists essentially in filling up all available ES registers with one particular order so that the computer performs only this particular order for long periods of time. With this program deteriorating tubes which fail to keep an operation-matrix line properly selected may be isolated. Periodic checks with this program will become part of our routine maintenance procedure.

## 3.3 ELECTROSTATIC STORAGE

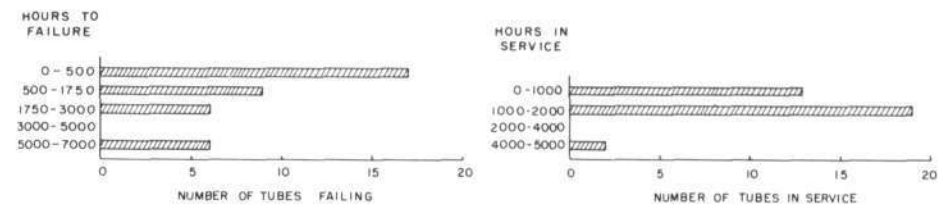
During this quarter a second bank of tubes was added to increase the total of available electrostatic storage from 1024 registers to 2048 registers. This results in much greater flexibility for computer programs. Associated with this installation are two spare tubes which are now being installed with facilities for replacing inoperative tubes in a bank within a few minutes. A double parity check system has also been added. This feature allows the computer to ignore a false readout signal if a repeat of the readout yields the correct information from storage. Work is continuing toward improving storage-tube operation.

There are two principal problems in maintaining the electrostatic storage system. One is that the ES deflection voltages have been found to drift with time, particularly if the deflection has been left set in one position for a long period of time (one or two seconds or more). The deflection-voltage drift has been traced to cathode-current drift in the 715 tubes driving the ES deflection lines. When the tubes are pulsed on, the cathode current may not stay at its initial value but may droop off slowly, depending on the condition of the tube. This is a different problem from cathode interface which is relatively a short time-constant affair. The cause of this cathode-current droop is not now known; it is being investigated as a doctorate thesis by H. B. Frost. In the meantime, to combat ES deflection shift, we are pretesting our 715's to select those which do not exhibit this drift in cathode current.



The other principal difficulty in maintenance of electrostatic storage is positive switching of ES tube surfaces. Changes in the design of storage tubes have achieved a reduction in the frequency of positive switching but have not eliminated the problem.

Fig. 3-3 gives information on the life of storage tubes used in the computer during the quarter. Information on failures includes those tubes removed to make way for newer design tubes.




(a) FAILURES, APRIL 1, 1953 - JUNE 30, 1953 (b) TUBES IN SERVICE, JUNE 30, 1953

Fig. 3-3. Life of Storage Tubes in WWI

## 3.4 MAGNETIC DRUM

The auxiliary drum system is now operating reliably as a part of the computer. Difficulty encountered with the temperature sensitivity of the heads has been eliminated by ambient-temperature regulation. The manufacturer has corrected the head defect, but we do not as yet have any of the corrected heads on the auxiliary drum.

## 3.5 DISPLAY

The most important difficulty with the display system still appears to be noise on the deflection system. It causes undesired motion of spots on scopes with consequent picture distortion. Work is underway to reduce the noise by means of a balanced transmission system, but no results have been obtained as yet. A character generator has been added to the display system. The character generator generates a small figure-eight deflection pattern at a position on the display scope determined by the computer. The parts of this figure eight which are intensified are also under the control of the computer. Thus the letters and numbers which may be constructed out of a figure-eight pattern may be displayed more quickly and with simpler programming than by the conventional method of intensifying groups of spots.

# 4. Circuits and Components

## 4.1 VACUUM TUBES

### 4.11 Vacuum-Tube Life

In the second quarter of 1953 the WWI computer has continued to operate at about the same rate as in the previous quarter, with a total period of operation of 1700 hours. This figure, which includes all the time power was turned on for maintenance testing as well as computation time, is about 85 percent of the maximum possible time in the quarter. Such utility is ideal for obtaining life data on vacuum tubes.

No extensive analyses of life data have been made during the past quarter; consequently, none are reported. However, the records of the three most numerous types in the computer have been calculated from the data of Fig. 4-1, and these records are shown below in comparison with the previous data.

| Tube Type | FAILURE RATE, PERCENT PER 1000 HOURS<br>1952 | First Quarter 1953 | Second Quarter 1953 |
|---|---|---|---|
| 7AD7, SR1407*, 6145* | 2.00 | 4.5 | 3.1 |
| 7AK7 | 0.26 | 0.7 | 0.4 |
| 6SN7GT | 1.07 | 1.5 | 2.0 |

* Improved 7AD7 designs

During the period a considerable amount of new equipment has been installed to operate in conjunction with the computer. As a consequence maintenance testing has continued to be rather intensive, a circumstance which accounts in part for the continued high failure rates. However, the failure rate for 7AD7 tubes is rather high. Of the three 7AD7 designs, the original type tubes constitute about 1/2 of the total of 2766 in service, but account for about 85 to 90 percent of the total failures. Thus the true failure rate for 7AD7 tubes is about 6 percent per 1000 hours. This rate is high because many 7AD7 tubes operate in flip-flop circuits which do not have adequate margins for poor tubes, so that any interface impedance developed by the 7AD7 soon makes the flip-flop circuit marginal.

New short-detecting equipment has now been in service in the vacuum-tube-testing shop for about 7 months. It is now used almost exclusively for all retests of vacuum tubes for WWI. This new equipment, which has been described in Whirlwind I Summary Report 32, page 14, gives the tubes a test with more complete indications of shorts, but uses a lower level of acceleration, than previous techniques. The consequences of this sort of test are apparent in the small number of shorts and opens reported in Fig. 4-1 for the past quarter. For the 7AK7 as an example, there are no



| Type | Total in Service | Hours at Failure | Reason for failure, number failed: Change in Characteristics | Shorts Opens | Breakage | Burn-out | Gassy |
|---|---|---|---|---|---|---|---|
| 7AD7/6145/SR-1407 | 2766 | | | | | | |
| 7AD7 | | 0-1000 | 1 | 4 | | | |
| | | 1000-2000 | 1 | | | | |
| | | 3000-4000 | 1 | 2 | | | |
| | | 4000-5000 | | 1 | | | |
| | | 5000-6000 | 5 | | | | |
| | | 7000-8000 | 2 | 1 | | | |
| | | 8000-9000 | | 1 | | | |
| | | 9000-10000 | 3 | 3 | | | |
| | | 10000-11000 | 4 | 2 | | | |
| | | 11000-12000 | 5 | 6 | | | 1 |
| | | 12000-13000 | 5 | 3 | | | |
| | | 13000-14000 | 6 | 3 | | | |
| | | 14000-15000 | 1 | 3 | | | |
| | | 15000-16000 | | 3 | | | |
| | | 16000-17000 | 13 | 29 | | | |
| | | 17000-18000 | 5 | 3 | 1 | | |
| | | 18000-19000 | 2 | 7 | | | |
| SR-1407 | | 0-1000 | 1 | | | | |
| | | 1000-2000 | | 1 | | | |
| | | 3000-4000 | | 1 | | | |
| | | 4000-5000 | | 1 | | | |
| | | 7000-8000 | 1 | | | | |
| | | 10000-11000 | | 1 | | | |
| 6145 | | 0-1000 | 1 | 9 | 1 | | |
| 7AK7 | 2112 | 0-1000 | 3 | 1 | | | |
| | | 1000-2000 | | 1 | | | |
| | | 3000-4000 | | 2 | | | |
| | | 4000-5000 | | 1 | | | |
| | | 7000-8000 | 1 | | | | |
| | | 10000-11000 | | 1 | | | |
| | | 16000-17000 | 1 | 2 | | | |
| | | 17000-18000 | 1 | 1 | | | |
| 6SN7GT | 435 | 3000-4000 | 1 | | | | |
| | | 13000-14000 | 2 | 1 | | | |
| | | 15000-16000 | 1 | | | | |
| | | 16000-17000 | | 1 | 1 | 1 | |
| | | 17000-18000 | | 1 | | | |
| | | 18000-19000 | 1 | 1 | | | |
| OA2 | 19 | 0-1000 | | | 2 | | |
| C62 | 4 | 1000-2000 | 1 | | | | |
| | | 5000-6000 | 1 | | | | |
| C1A2 | 12 | 17000-18000 | | | 1 | | |
| | | 18000-19000 | 1 | | | | |
| 3E29/829B | 207 | | | | | | |
| 3E29 | | 0-1000 | | | 1 | | |
| | | 8000-9000 | 1 | | | | |
| | | 11000-12000 | | 1 | | | |
| | | 16000-17000 | 1 | | | | |
| 5U4G | 22 | 0-1000 | | 1 | | | |
| | | 2000-3000 | | | | 1 | |
| | | 5000-6000 | | 1 | | | |
| | | 8000-9000 | 1 | | | | |
| 6AG7 | 126 | 1000-2000 | 1 | | | | |
| | | 9000-10000 | 1 | | | | |
| | | 11000-12000 | 1 | | | | |
| 6AH6 | 8 | 9000-10000 | | | 1 | | |
| 6AL5 | 249 | 1000-2000 | 1 | | | | |
| | | 5000-6000 | 2 | | | | |
| | | 8000-9000 | | 1 | | | |
| | | 9000-10000 | 5 | | | | |
| 6AS7G | 197 | 0-1000 | 1 | 7 | | | |
| | | 2000-3000 | | 2 | 4 | | |
| | | 4000-5000 | 5 | | | | |
| | | 7000-8000 | 1 | 2 | 1 | | |
| | | 10000-11000 | | 1 | | | |
| | | 13000-14000 | | 1 | | 1 | 1 |
| | | 14000-15000 | | 8 | | | |
| | | 15000-16000 | | 4 | | | |
| | | 17000-18000 | | | 1 | | |
| 6AU6 | 80 | 0-1000 | | 1 | | | |
| | | 10000-11000 | | 1 | 1 | | |
| 6J5 | 6 | 8000-9000 | | 2 | | | |
| 6L6G/6L6GA/5881 | 121 | | | | | | |
| 6L6G | | 3000-4000 | 5 | | | | |
| | | 6000-7000 | 1 | | | | |
| | | 7000-8000 | 1 | | | | |
| | | 9000-10000 | | 1 | | | |
| | | 14000-15000 | | 1 | | | |
| | | 16000-17000 | 1 | | | | |
| | | 18000-19000 | 4 | | 1 | | |
| 5881 | | 1000-2000 | 2 | | | | |
| 6V6 | 1 | 4000-5000 | 1 | | | | |
| 6X5GT | 13 | 1000-2000 | 2 | | | | |
| | | 9000-10000 | | 1 | | | |
| | | 10000-11000 | | 1 | | | |
| | | 17000-18000 | 1 | | | | |
| 6Y6G | 325 | 0-1000 | | 1 | | | |
| | | 1000-2000 | | 2 | | | |
| | | 3000-4000 | | 1 | | | |
| | | 13000-14000 | | 1 | | | |
| | | 16000-17000 | 1 | | | | |
| | | 17000-18000 | 1 | | | | |
| 7X6 | 12 | 10000-11000 | 4 | | | | |
| 715B/715C | 36 | | | | | | |
| 715B | | 0-1000 | | 2 | | | |
| | | 4000-5000 | 3 | | | | |
| | | 7000-8000 | 1 | | | | |
| | | 10000-11000 | 1 | | | | |
| 715C | | 0-1000 | | | | 1 | |
| 12AU7(5963) | 133 | 3000-4000 | 1 | | | | |
| | | 4000-5000 | | 2 | | | |
| 5687 | 90 | 4000-5000 | | 1 | | | |

Fig. 4-1. WWI tube failures April 1 - June 30, 1953

shorts reported, instead of a majority of shorts as in the past. This lack of shorts on retest does not indicate that shorts are causing less trouble than previously; they never have been a major reason for the removal of tubes from the computer, except for some special types such as the 6AS7G. The tests now give much more realistic indication of what actually is wrong with the tubes, instead of the former indications of a large number of shorts, which followed as a consequence of the over-rigorous testing for shorts with the older equipment.

### 4.12 Vacuum-Tube Research

During this period a thesis study of cathode stability has continued. The work is now on a rather intensive level. Test equipment has been constructed, test procedures formulated and tried, and special tubes have been constructed by the Storage Tube Group. At this time it appears that this study will continue through this year, with some results becoming available during the next quarter.

Important life tests of types 6145 and 7AK7 are now in progress. Both of these types appear to be generally satisfactory, with good plate current maintenance and no interface impedance. However, both types have an annoying problem of grid emission. This grid emission develops on life, particularly in those tubes operated with no cathode current. Fortunately, most of the computer circuits in which these tubes are used do not have high resistance grid circuits, so that little trouble is caused in applications. However, the grid emission can mask rather high values of gas content, and some trouble has been experienced in balanced circuits by the shift in contact potential of grid 1 which accompanies the grid emission.

An analysis has been made of the duration and resistance of short circuits in vacuum tubes when they are tapped. This analysis was made by observing and photographing waveforms of stray current through the tubes from the grid to other electrodes while the tube was being tapped. The tests indicate that many flicker shorts occur with resistances of 100,000 ohms or more and with time durations between 10 and 100 microseconds. Flicker shorts with these parameters cannot be found on standard polyphase testing equipment, but they can be located on special equipment which uses thyratron-controlled indicators. The impulse source for these tests was hand-tapping with a tapper approximating the one described in MIL-E-1B, Amendment 1, 29 September 1952; the impulse was about 500 microseconds in length with an acceleration of about 100 g for tubes with T6 1/2 envelops, but only 50 g for tubes with T9 envelops. The material described in this paragraph is being written for publication in the Digital Computer Laboratory M-note series during the third quarter, 1953.



## 4.2 COMPONENT REPLACEMENTS IN WWI

Fig. 4-2 lists the replacements of components other than tubes during the first quarter of 1953.

| Component | Type | Total in Service | No. of Failures | Hours of Operation | Comments |
|---|---|---|---|---|---|
| Capacitors | 10 mfd 600 volt oil-filled Rectangular can | 24 | 1 | 14000-15000 | Arc over on capacitor |
| | 4-30 mmfd Variable Trimmer Ceramic | 87 | 1<br>1<br>2 | 6000-7000<br>7000-8000<br>11000-12000 | Open<br>Open<br>Open |
| | 1.0 mfd 600 volt oil-filled Bath tub | 232 | 1<br>1 | 7000-8000<br>9000-10000 | Leaking oil<br>Leaking oil |
| | 0.01 mfd Fixed Mica (moulded) 10% | 1852 | 1 | 15000-16000 | Shorted |
| | 7-45 mmfd Variable Trimmer Ceramic | 122 | 1 | 6000-7000 | High resistance |
| | [illegible] mfd Fixed Mica (moulded) 5% | 3580 | 1 | 7000-8000 | Open |
| Crystals | D-357, 1N34 & 1N34A | 11697 | | | |
| | D-357 | | 3<br>2<br><br>1<br>4<br>2 | 16000-17000<br>16000-17000<br><br>2000-3000<br>17000-18000<br>13000-14000 | 1 Low $R_b$, 2 drift to Low $R_b$<br>Excessive drift and high forward resistance<br>Low $R_b$<br>Low $R_b$ & drifting<br>Low $R_b$ & drift |
| | 1N34A | | 1<br>1<br>1 | 0-1000<br>7000-8000<br>16000-17000 | Low $R_b$<br>Low $R_b$<br>Low $R_b$ |
| | D-358, 1N38 & 1N38A | 1382 | | | |
| | D-358 | | 1<br>1<br>4 | 15000-16000<br>17000-18000<br>18000-19000 | Low $R_b$<br>Low $R_b$ & drifting<br>Low $R_b$ & drifting |
| | 1N38A<br>" | | 1<br>1<br>1 | 0-1000<br>4000-5000<br>7000-8000 | Low $R_b$<br>Drift to Low $R_b$<br>Low $R_b$ |
| Potentiometers | 10000 ohm 2 watt var. comp. | 97 | 1<br>2<br>1 | 8000-9000<br>9000-10000<br>12000-13000 | Noisy<br>Noisy |
| | 2500 ohm 2 watt var. comp. | 129 | 1<br>1<br>1<br>1 | 8000-9000<br>7000-8000<br>11000-12000<br>10000-11000 | Noisy<br>Noisy<br>Noisy<br>Noisy |
| Resistors | 220 ohm 1 watt ±5% carbon | 890 | 1<br>2<br>2<br>1<br>1<br>2<br>1<br>1 | 6000-7000<br>9000-10000<br>8000-9000<br>9000-10000<br>7000-8000<br>8000-9000<br>11000-12000<br>12000-13000 | Below tolerance<br>1 above tolerance;1 burnout<br>1 below tol.;1 above tol.<br>1 above tolerance<br>1 above tolerance<br>Overheated<br>Below tolerance<br>Above tolerance |
| | 5000 ohm 1 watt ±5% deposited carbon | 136 | 1 | 6000-7000 | Above tolerance |
| | 10 ohm 1 watt ±5% fixed composition | 140 | 1 | 14000-15000 | Overheated |
| | 1000 ohm 1 watt ±5% fixed comp. | 1864 | 1 | 8000-9000 | Above tolerance |
| | 125 K 1/2 watt ±1% dep. carbon | 170 | 1 | 1000-2000 | Above tolerance |
| | 5000 ohm 1 watt ±1% dep. carbon | 360 | 1<br>2<br>1 | 7000-8000<br>9000-10000<br>6000-7000 | Above tolerance<br>Above tolerance<br>Above tolerance |
| | 1000 ohm 22 watt fixed w/w flat oval power type | 48 | 1 | 16000-17000 | Open |
| Switch | Cutler Hammer Type 7501K12 SPST Toggle | 614 | 1 | 16000-17000 | Intermittent |

Fig. 4-2. WWI component failures April 1 - June 30, 1953

# 5. Academic Program

## 5.1 SEMINARS ON COMPUTING MACHINE METHODS

The seminars on Computing Machine Methods are arranged jointly by representatives of the MIT Committee on Machine Methods of Computation and the MIT Digital Computer Laboratory. Various speakers from other MIT activities and elsewhere, as well as members of the two sponsoring groups, participate in these weekly seminars, which are held in a lecture room at the Institute. The program during the past quarter was as follows.

| Date | Subject | Speaker |
|---|---|---|
| April 7 | Solution of a Set of Partial Differential Equations on MIT Rockefeller Differential Analyzer (The solution of a problem is discussed which contains a number of features illustrating the versatility of the RDA. A method of handling boundary values for both linear and non-linear differential equations is shown. A description is given of the simplifying approximations which are required in the treatment of a transformed differential equation and the actual method of solution on the RDA.) | Mr. William F. O'Connell, Analyzer Staff |
| April 14 | Iteration Procedures for Simultaneous Equations (A geometric interpretation of iteration procedures for linear simultaneous equations will be discussed. In particular, the n-step procedure of Hestenes and Stiefel, and an n-step procedure based on minimized error developed by the author will be described. The talk will include a generalization of the ideas of descent and minimized error, and an application of the latter to non-linear simultaneous equations.) | E.J. Craig, Department of Electrical Engineering |
| April 21 | An Economic Digital Computer Proposal (This proposal is mainly in the philosophical stage. It has been reviewed by a dozen or more people throughout the country during the last fourteen years with the feeling being that it is "too perfect," especially for existing means. During the last two years, the author has been exposed to Whirlwind's mathematical capabilities and has revised the proposal into a digital form. This is the first presentation before a technical-computer group; there has been no hard thinking on the digital part of this proposal. This proposal is a suggestion for a series of simple, fundamental econmoic calculators which should give our private initiative type economic life a definite stability and improvement without radically changing any of our basic freedoms.) | Paul H. Baldwin, Project Lincoln |
| April 28 | Autocorrelation and Fourier Transform Programs for Whirlwind I (General purpose programs for the calculation of power density spectra have been written and are now available for use. A descriptive discussion of the operations and requirements of the programs will be given, including techniques for handling large amounts of data in a machine with limited storage capacity. Although the Fourier Transform Program has been designed to be used in tandem with the Autocorrelation Program, it may be easily modified for use as a separate computational tool, giving both sine and cosine transforms.) | D.T. Ross, Servomechanisms Laboratory |
| May 5 | Mechanization of Two Analog Computers (The Ham computer is a small unit which uses digital inputs and outputs, in the form of paper tapes, punched in binary code. The unit is partially analog, partially digital, and accepts input data of maximum modulus 255. The Fourier transform unit consists of a series of harmonically related sinusoidal generators. The magnitudes of the sinusoids are proportional to ordinates of the function whose transform is desired.) | S. Fine, Research Laboratory of Electronics |



## 5.2 SEMINARS ON EVALUATION OF INTEGRALS OF THE PRODUCT OF TWO FUNCTIONS

A series of seminars open to all interested persons is being sponsored by the MIT Digital Computer Laboratory to discuss methods for the numerical evaluation of finite or infinite integrals of the product of two functions. These integrals include those arising from problems involving auto- and cross-correlations, convolutions, and transforms.

Although the initial impetus for these seminars arose from problems to be programmed for Whirlwind I, other allied problems as well as theoretical discussions are to be included according to the desires of those participating in the seminars.

Meetings held during the second quarter of 1953 are tabulated below.

| Date | Subject | Speaker |
|---|---|---|
| April 9 | Connections between Time and Frequency Domain (What errors can be tolerated in an Autocorrelation Function and still give a power spectrum estimate of a specified degree of accuracy? While no pat answers to this question are available, certain recent developments in the study of the connection between time and frequency domains as used in Electrical Engineering provide tools and viewpoints to aid in the solution of this and related problems.) | Prof. E. A. Guillemin, Department of Electrical Engineering |
| April 30 | Applications of Optimum Filtering Techniques to Sample Data Systems (Mathematical procedures for finding filters to provide minimum mean square error become very simple for the sample data case. For this case time variable and certain classes of non-linear filters are readily handled. The question of how often to sample data will be discussed.) | Asst. Prof. William Linvill, Department of Electrical Engineering |



# 6. Appendix

## 6.1 REPORTS AND PUBLICATIONS

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Regular requests for copies of individual reports should be made to ASTIA; emergency requests, to Robert R. Rathbone, Digital Computer Laboratory, 68 Albany Street, Cambridge 39, Mass. Att: Code DCL-6.1.

The following reports and memorandums were among those issued during the second quarter of 1953.

| No. | Title | No. of Pages | Date | Author |
|---|---|---|---|---|
| SR-33 | Summary Report No. 33, First Quarter 1953 | 43 | | |
| R-220 | Analysis and Design of a Digital-to-Analog Decoder | 166 | 12-1-52 | R. L. Walquist |
| R-224 | Rudiments of Good Circuit Design | 13 | 5-19-53 | N. H. Taylor |
| E-536 | Diagnostic Programs and Marginal Checking in the Whirlwind I Computer | 9 | 3-26-53 | N. L. Daggett<br>E. S. Rich |
| E-537 | 1953 Test Storage Input Program | 4 | 4-7-53 | J. Frankovich |
| E-558 | 556 Drum Input Program, May 1953 | 3 | 5-26-53 | F. C. Helwig |
| E-565 | Differential Video Probe | 2 | 6-19-53 | H. E. Zieman |
| M-1985 | Operation of Indicator Lights and Intervention Register | 11 | 5-1-53 | B. E. Morriss<br>G. A. Young |
| M-1986 | "L" Cathodes in Electron Guns | 7 | 4-21-53 | T. S. Greenwood |
| M-1987 | First Note on Pulse Transformers for Memory Drivers | 4 | 5-27-53 | F. Durgin<br>E. K. Gates |
| M-2135 | Some Notes on Current Tube Types | 3 | 5-4-53 | H. B. Frost |

| No. | Title | No. of Pages | Date | Author |
|---|---|---|---|---|
| M-2205 | Some Properties of Cathode Interface Impedance | 6 | 5-29-53 | H. B. Frost |
| M-2229 | N-Step Procedures for Simultaneous Linear Equations | 15 | 6-11-53 | E. J. Craig |

## 6.2 PROFESSIONAL SOCIETY PAPERS

At the 11th Annual Conference on Electron Tube Research held at Stanford University, Stanford, California, June 17-20, C. L. Corderman presented a paper entitled "Some Effects of Gas in the MIT Electrostatic Storage Tubes."

At this same conference P. Youtz presented a paper on "L-Cathodes in Electron Guns."

D.A. Buck gave the keynote address for the Dielectric Amplifier Circuitry Symposium held at Carnegie Institute of Technology, Pittsburgh, Pa., April 16-17. His talk was entitled "Dielectric Amplifier Circuitry." This meeting was sponsored by the ONR and Carnegie Tech.

D. A. Buck also spoke on "Magnetic Amplifiers in Digital Computers" at the Northeastern district AIEE meeting, April 29-30 at Boston.

At the EE Staff Colloquium held on May 6, D. A. Buck and W. Frank presented a paper on "Nondestructive Sensing of Magnetic Cores."

C. W. Adams spoke to the AIEE MIT Student Branch on "The MIT Whirlwind I Computer and Its Uses" on May 12.

The following three addressed the American Physical Society Meeting in Washington, D. C., April 30:

N. Menyuk - "Magnetization Reversal of Square-Loop Polycrystalline Materials by Domain Growth."

A. L. Loeb - "A Free Energy Model for the Hysteresis Loop."

J. B. Goodenough - "Nucleation Centers for Domain of Reverse Magnetization."

Goodenough and Menyuk gave speeches with the same titles as listed above at RCA Laboratories, Princeton, New Jersey on June 22.