Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, LI

To: Group 63 Staff

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In the previous note the magnetic characteristics of a ferrite at absolute zero temperature were discussed. In this lecture the magnetic characteristics of ferrites in the neighborhood of the Curie temperature will be investigated and compared with the previous results.

At a temperature slightly below the Curie point, the spontaneous magnetization can be approximated by the equation*

$$I_s = \lambda I_a - \mu I_b = FM\sqrt{\frac{\Theta_p - T}{\Theta_p}}\left(\lambda\sqrt{k} - \frac{\mu}{\sqrt{k}}\right)\sqrt{\frac{\lambda k + \frac{\mu}{k}}{\lambda k^2 + \frac{\mu}{k^2}}} \qquad \text{LI-1}$$

wherein k is the positive root of the equation

$$\lambda k^2 + (\beta\mu - \alpha\lambda)\, k - \mu = 0 \qquad \text{LI-2}$$

and F is a function of J, equal to 1.486 for $J = \frac{5}{2}$.

If we define the term $\Delta = \lambda\sqrt{k} - \frac{\mu}{\sqrt{k}}$, then $\vec{I}_s$ is in the direction of $\vec{I}_{as}$ when Δ is positive and it is in the same direction as $\vec{I}_{bs}$ when Δ is negative.

* Derivation given in Appendix VII

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This factor Δ changes sign when $k = \frac{\mu}{\lambda}$. Substituting this value of k into equation LI-2, we find that Δ changes sign along the line given by the equation

$$\lambda(\alpha + 1) - \mu(\beta + 1) = 0 \qquad \text{LI-3}$$

This is the same line as that drawn in figure 107 (meeting 49). At that meeting it was found that equation XLIX-19, which is identical with equation LI-3, was the equation of the line along which the Curie-Weiss law of paramagnetism holds. We now see this line has a double significance, since it also separates the region in which $\vec{I}_s$ is directed parallel to $\vec{I}_{as}$ from the region in which $\vec{I}_s$ is parallel to $\vec{I}_{bs}$ at the Curie temperature.

The next problem is to determine in which of these two regions $\vec{I}_{as}$ predominates and in which region $\vec{I}_{bs}$ predominates.

If we start at a point on the line $\Delta = 0$ and vary β positively while keeping α constant, we go into the region above the line. Similarly, if we start at the same point and vary α positively while keeping β constant, we enter the region below the line. Therefore, let us determine the signs of

$$\left(\frac{\partial\Delta}{\partial\alpha}\right)_\beta \quad \text{and} \quad \left(\frac{\partial\Delta}{\partial\beta}\right)_\alpha$$

at the line $\Delta = 0$, where the subscript represents the parameter which is kept constant.

$$\left(\frac{\partial\Delta}{\partial\alpha}\right)_\beta = \frac{\partial\Delta}{\partial k}\frac{\partial k}{\partial\alpha} = \frac{1}{2}\left\{\frac{\lambda}{\sqrt{k}} + \frac{\mu}{k\sqrt{k}}\right\}\frac{\partial k}{\partial\alpha}$$

$$\left(\frac{\partial\Delta}{\partial\beta}\right)_\alpha = \frac{\partial\Delta}{\partial k}\frac{\partial k}{\partial\beta} = \frac{1}{2}\left\{\frac{\lambda}{\sqrt{k}} + \frac{\mu}{k\sqrt{k}}\right\}\frac{\partial k}{\partial\beta}$$

Since $\lambda = \frac{\mu}{k}$ at the line $\Delta = 0$

$$\left(\frac{\partial\Delta}{\partial\alpha}\right)_{\beta,\Delta=0} = \frac{\mu}{k\sqrt{k}}\frac{\partial k}{\partial\alpha}$$

$$\left(\frac{\partial\Delta}{\partial\beta}\right)_{\alpha,\Delta=0} = \frac{\mu}{k\sqrt{k}}\frac{\partial k}{\partial\beta}$$

The positive root of k, obtained from equation LI-2, is

$$k = \frac{1}{2}\left(\alpha - \frac{\beta\mu}{\lambda}\right) + \sqrt{\frac{1}{4}\left(\alpha - \frac{\beta\mu}{\lambda}\right)^2 + \frac{\mu}{\lambda}}$$

Thus

$$\frac{\partial k}{\partial \alpha} = \frac{1}{2} + \frac{\alpha - \frac{\beta\mu}{\lambda}}{4\sqrt{\frac{1}{4}\left(\alpha - \frac{\beta\mu}{\lambda}\right)^2 + \frac{\mu}{\lambda}}}$$

$$\frac{\partial k}{\partial \beta} = -\frac{\mu}{\lambda}\left(\frac{1}{2} + \frac{\alpha - \frac{\beta\mu}{\lambda}}{4\sqrt{\frac{1}{4}\left(\alpha - \frac{\beta\mu}{\lambda}\right)^2 + \frac{\mu}{\lambda}}}\right)$$

Therefore

$$\frac{\partial k}{\partial \alpha} > 0 \text{ and } \frac{\partial k}{\partial \beta} < 0.$$

Since

$$\frac{\mu}{k\sqrt{k}} > 0,$$

$$\left(\frac{\partial \Delta}{\partial \alpha}\right)_\beta > 0 \qquad \text{LI-7}$$

and

$$\left(\frac{\partial \Delta}{\partial \beta}\right)_\alpha < 0. \qquad \text{LI-8}$$

$\vec{I}_s$ is parallel to $\vec{I}_{as}$ in the region below the line $\Delta = 0$ and $\vec{I}_s$ is parallel to $\vec{I}_{bs}$ in the region above the line $\Delta = 0$.

These results are indicated in figure 109, along with the results previously obtained for 0°K.

This is in flat contradiction with the statement made by Néel on page 151 of his original article. On that page he states that Δ is positive above the line and negative below the line. However, Néel essentially contradicts his own statement in the next paragraph, stating that below the line $\vec{I}_s$ is parallel to $\vec{I}_{bs}$, which is consistent with the development presented above.

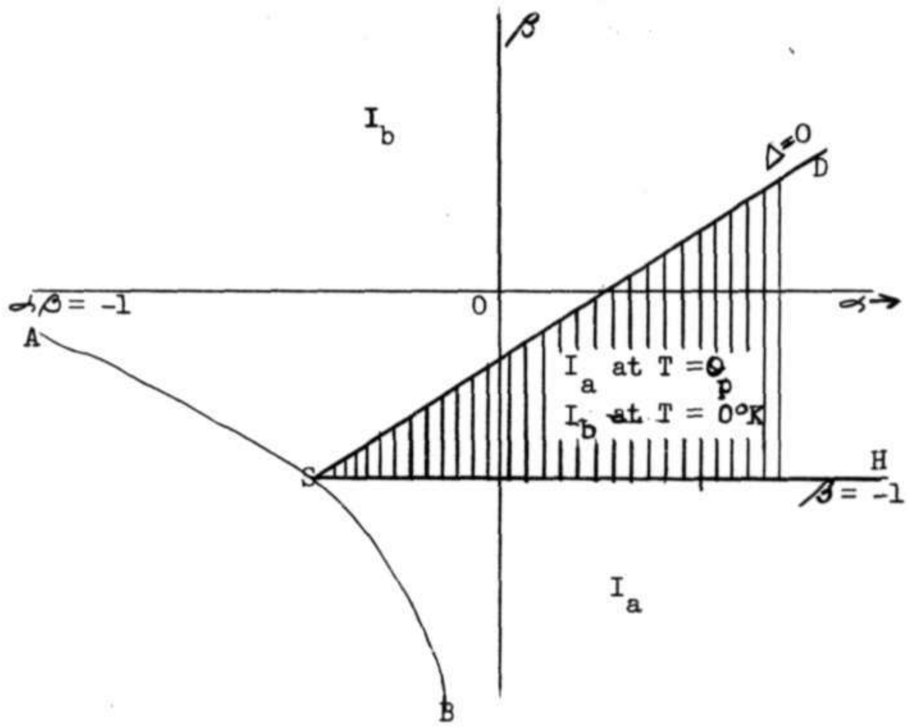

FIGURE 109

In the region ASD of figure 109 the net magnetization $\vec{I}_s$ is always parallel to $\vec{I}_b$. In the region HSB, the net magnetization is always parallel to $\vec{I}_a$. However, in the region DSH, indicated by vertical lines, the direction of net magnetization changes as the temperature is brought from absolute zero to the Curie temperature. At the absolute zero, the net magnetization is parallel to $\vec{I}_b$, while at the Curie point it is parallel to $\vec{I}_a$. In this region, there must be therefore some temperature between 0°K and the Curie temperature at which the magnetization due to the A sites and the B sites cancel each other out. This temperature, θ_c, is called the compensating temperature. The thermal variation of the magnetizations due to the A and B sites, along with the resultant magnetization is shown in figure 110 for this case.

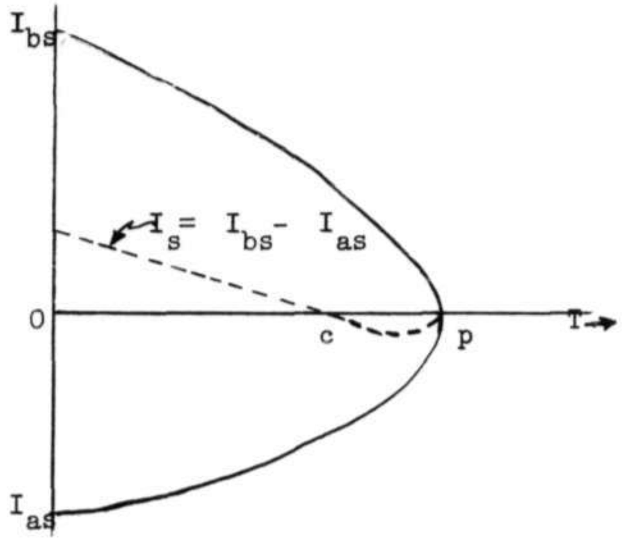

FIGURE 110

Since the measured value of I_s is always positive, the experimental curve obtained for the case depicted in figure 110 would appear as shown in figure 111.

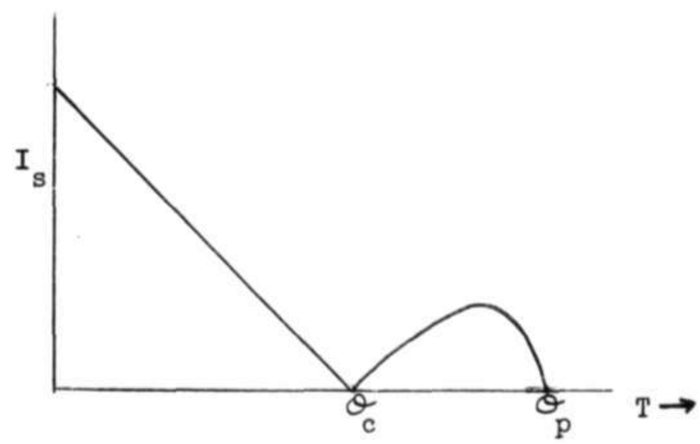

FIGURE 111

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