Orig + 2 carbons to F.F.

2 carbons Jt
1 " Johnson
2 " extra.

July 20, 1960

Dr. Kermit Gordon
Program Director
Economic Development and Administration
The Ford Foundation
477 Madison Avenue
New York 22, N. Y.

Dear Dr. Gordon:

This is a second report ~~of~~ on work ~~done~~ under the Ford Foundation grant of May 31, 1957, to the M.I.T. School of Industrial Management for developing the field of industrial dynamics. The previous report was dated December 15, 1958.

We are nearing the end of what I consider the first phase of our program, and in the near future will be concentrating on work of a rather different character than has occupied our attention thus far. The first phase has dealt with fundamentals of methodology, the formulation of dynamic models of ~~the~~ typical industrial situations, the development of basic teaching materials, and the establishment of some introductory courses in industrial dynamics. To ~~summarize~~ conclude this first phase, I am nearing completion of a first book on the dynamics of industrial and economic systems.

The second phase, which lies immediately ahead, will deal with more advanced and complex industrial situations, with the dynamics of economic systems, with the policies and organizational forms that can improve the growth and stability characteristics of industrial activity, and with the application of industrial dynamics to actual problems of industry.

A. First Phase.

1. Educational Program.

In addition to the first-term graduate course reported last time, a second graduate term has been added to the industrial dynamics series. This makes available to our graduate students a full academic year, meeting for three class hours per week, on the dynamics of industrial systems. The subject matter covered is indicated by the accompanying ~~selection of~~ memoranda and reprints. In addition, ~~they~~ students do class projects in the formulation of models of industrial systems and the simulation of these models using the IBM 704 computer.

In the fall of 1959, industrial dynamics was offered for the first time as an elective in our undergraduate program. About twenty seniors took the subject. The material was essentially similar to that taught in the first graduate term, and qualified the students to register for the second-term graduate subject.

Industrial dynamics has been continued in the program of the Sloan Fellows for seven weeks at the beginning of the fall term. This has given the Sloan Fellows enough background so that several of them have been able to embark upon very interesting thesis studies in this area. An appendix to this report contains abstracts of a number of these master's degree theses done by the Sloans.

There are now six men on the staff of the industrial dynamics group who are working toward their doctorate degrees and who will be doing doctorate theses in this area.

We have continued in 1959 and 1960 to offer a two-weeks' special summer session in industrial dynamics for men from industrial companies, government agencies, and other academic institutions. This is part of the regular M.I.T. special summer session program. The announcement of the 1960 session and the outline of the course are included in an appendix. As given this year, the subject is similar to, but an improvement upon, that which was reported ~~to you~~ in December, 1958.

2. Methodology.

Methods have been developed which seem capable of dealing expeditiously with the flows of information, money, men, orders, capital equipment, and materials in industrial operations. Memorandum D-46 in the accompanying collection discusses this approach and some of the other memoranda illustrate its usage.

A very important part of our program has been the development of the DYNAMO II compiler. This is a system for the automatic programming of dynamic industrial models for operation on the IBM 704 computer. This makes it possibly to go automatically, and in a few minutes of computer time, from the initial equations describing an industrial situation to the final plotted results of what the policies represented by the equations imply for system behavior. This makes continuous changes and improvement of models possible. It also places the dynamic model involving as many as a few hundred variables within the reach of a student doing even a master's degree thesis. Minor improvements in the DYNAMO compiler will continue, but it is considered satisfactory for the present without further major revision. IBM is now replacing the 704 computer at M.I.T. with a 709, and the DYNAMO system has been modified to correspond.

Research has begun recently in some of the complex and subtle ramifications in the use of noise and uncertainty in dynamic models. This has not yet progressed far enough for reporting.

3. Formulation of Industrial Situations.

Much of the actual model construction thus far has been devoted to the formulation of typical industrial situations. Some of these have been done as staff research and others as theses. They have been aimed at showing the extent to which the commonly accepted organizational forms and managerial policies can lead to many of the difficulties which we observe in the management of industrial enterprises. This work has identified many of the kinds of problems which arise from the interactions within complex systems. The work during this first phase has not gone far in determining how the performance of industrial systems can be improved, except that it has been frequently demonstrated that major changes and improvements are possible.

4. Book.

The first phase of our program is being organized into a first volume on industrial dynamics, which I now have ~~now~~ near completion. It will be published by the M.I.T. Technology Press and the John Wiley Company. I hope to have this ready for the publisher in the fall of 1960, and it should become available in the summer of 1961. The accompanying collection of memoranda contain first drafts of some of this material. The material ~~It~~ is being extensively revised and extended as it is being converted to book form.

B. <u>Second Phase</u>.

During the next year, we will be reducing ~~terminating~~ the activities in the first phase, and will be re-directing our activities to the second phase of our program. The second phase will include the following:

1. Formulation of Models of More Difficult Industrial Situations.

The work thus far has dealt largely with the steady-state dynamic fluctuation of manufacturing, distribution, and inventory systems. There has been only a little treatment of corporate growth and other transient phenomena. We are now beginning to work into areas of growth, the acquisition and training of personnel, market dynamics, pricing, and competition.

2. Economic systems.

As reported previously, we have been carrying on an exploratory program in the dynamics of economic systems in which we plan to treat non-linear systems of several hundred variables and larger. Progress has been slow because most of our staff have been devoting their attention to the subjects listed under Phase One, above. We now have an expanding number of doctorate students who are interested in the dynamics of economic systems, and expect that this work will accelerate.

3. Application of Industrial Dynamics to Problems of Industry.

Most of our work thus far has been in the development of methodology and the exploration of typical industrial ~~situations~~ behavior. The time is now at hand to give increased attention to the application of these methods in actual specific, industrial situations. This calls for the tailoring of models to the particular problems of a specific company or industry. It also means the design of system improvements. The work thus far has shown how present policies and present organizational forms can lead to many of the presently observed difficulties. The next step is to show what changes in policy and organization can lead to more desirable industrial system behavior. This has been started in a modest way in connection with the research study sponsored by the Sprague Electric Company. A second company project, sponsored by the Digital Equipment Corporation of Maynard, Massachusetts, will deal with the dynamics of corporate growth. This latter study will interrelate market dynamics, the acquisition of technical personnel, financing and tax policies, capital equipment acquisition, and sales effort to show how these interact to affect growth in sales and profits.

We hope to make similar arrangements with one or two other organizations to round out the diversity of real-life problems to which we are applying the methods thus far developed.

C. <u>Staff of the Project</u>.

The total number of staff has not ~~changed appreciably~~ increased since the report a year and a half ago, although there has been considerable change and substitution. At the present time our greatest strength lies in the group of men who are full time members of our staff and at the same time are working toward the completion of their doctorate degrees and doctorate theses. We now have six such men on our industrial dynamics staff who have had prior training in the dynamics of information feedback systems and who are intensely interested in the extension of these ideas into the industrial and economic areas.

Sincerely

attached
appendix A
appendix B.

Separate
appendix C.
(collection of
memoranda)

Appendix A.

Abstracts of Selected Master of Science Theses completed in 1960-61 in Industrial Dynamics.

These ~~theses~~ illustrate some of the industrial systems ~~areas~~ that have been investigated with dynamic simulation models. Master of Science theses are completed in a period of five or six months and time does not permit the thoroughness expected in Ph.D. theses. (Thus far, none of the Ph.D. theses in Industrial Dynamics have ~~progressed to com~~ had time for completion) ~~In these~~ most of these M.Sc. theses, the system has been formulated into a complete model which has operated successfully (from the technical viewpoint) on the computer. Preliminary runs have been made, but not the next phase of successive model revisions and the interpretation of results into managerial action. Theses of this type often serve to open a new area of investigation which ~~it~~ can then be extended as Ph. D. Theses.

Appendix A.

Abstracts of Theses.

Following are several abstracts to indicate the kinds of theses in Industrial Dynamics that have been completed recently. These are all master of Science theses since none of the Industrial Dynamics Ph.D. theses have yet been completed.

As M. Sc. theses, they have been done over a period of only four to six months and serve as a preliminary exploration that opens a subject to further study. ~~Some of these some subjects are being extended~~ The theses abstracted here have been carried thru model formulation and preliminary model runs on a computer but usually time has not permitted ~~model~~ much interpretation of results nor ~~model the~~ model revision on the basis of first results.

Some of these subjects are being extended ~~as~~ on a more thorough basis as Ph.D. theses and as staff research.

Omit Thesis supervisor
and title at end of
abstract.

add the man's title &
company after his name.

Ballmer pages iv and v

add

Ray W. Ballmer,
Sloan Fellow, 1959-60
Production Superintendent
Ray Mines Division
Kennecott Copper Corporation

Hunford - abstract -

add. after name

~~Man~~ Sloan Fellow 1959-60
Manager
Nuclear Core Engineering Department
Westinghouse Electric Corporation

Beaumariage p. iv + v

Sloan Fellow 1959-60
Manager, Electronic Systems Design,
Systems Engineering
Defense Electronic Products Div.
Radio Corporation of America

Kinsley iii & iv.

Sloan Fellow 1958-59

~~Operations Manager~~,

~~Get his~~ ~~present~~ position from Ex. dev. office.

~~European~~ Geosciences Dept.

Manager,

Texas Instruments ~~Company~~ Inc.

appendix B.

1960 Special Summer Session in Industrial Dynamics.

a. Announcement

b. Outline

c. Organizations Represented.

Forrester

Massachusetts Institute of Technology Office of the Summer Session

Special Summer Program 15.21s

INDUSTRIAL DYNAMICS

June 14 through June 24, 1960

ROSTER OF REGISTRANTS

Mr. Berend M. Beins
Shell Development Company
Emeryville, California

Mr. Louis W. Brummer
The Chemstrand Corporation
350 5th Avenue
New York 1, New York

Mr. Edward J. Carstens
Industrial Engineering Division
Kodak Park Works
Eastman Kodak ~~Division~~ company
Rochester 4, New York

Mr. William C. Chester
Chicopee Manufacturing Corporation
501 George Street
New Brunswick, New Jersey

Dr. James L. Fisher
2 Shell Development Company
Emeryville 8, California

Mr. John J. Flynn
Pratt & Whitney Aircraft
400 Main Street
East Hartford 8, Connecticut

Dr. Bernard Goldberg
3 Shell Development Company
3737 Bellaire Boulevard
Houston 25, Texas

*Mr. Hans S. Gruenberg
National Biscuit Company
P. O. Box 29
New York 17, New York

Dr. Charles H. Harry
Department of Navy
Bureau of Naval Weapons
Washington 25, D. C.

Mr. James W. Hart
Westinghouse Electric Corporation
3 Gateway Center
Pittsburgh 30, Pennsylvania

Mr. Sanford M. Isaacs
Service Bureau Corporation
50 Franklin Street
Boston, Massachusetts

Mr. Arthur F. Martin
Hercules Powder Company
900 Market
Wilmington 99, Delaware

Mr. Ravi N. Mayor
Watkins Mayor & Company
Jullundur City, INDIA

(include address here)

Mr. Howard W. Megee
General Motors Corporation
12-165 General Motors Building
Detroit 2, Michigan

~~Mr. Roger A. Moser~~
~~Ethyl Corporation~~
~~P. O. Box 341~~
~~Baton Rouge, Louisiana~~

Mr. Arthur F. Pond
Monsanto Chemical Company
Plastics Division
Texas City, Texas

Mr. Max H. Post
Texas Instruments, Inc.
Exchange Bank Building
Post Office Box 7045
Dallas 9, Texas

Mr. J. Cecil Rowe
General Electric Company
Marketing Research #479
Nela Park
Cleveland 12, Ohio

~~Mr. John S. Rydz~~
~~Radio Corporation of America~~
~~Industrial Electronic Products~~
~~Building 2-7~~
~~Camden 2, New Jersey~~

Mr. Hugh W. Schwarz
Minute Maid Corporation
1200 West Colonial Drive
Orlando, Florida

Dr. Leonard V. Sloma
Roy C. Ingersoll Research Center
Borg-Warner Corporation
Wolf and Algonquin Roads
Des Plaines, Illinois

~~Mr. Norman B. Solomon~~
~~Sylvania - Data Systems Operation~~
~~183 8th Street - Department 313~~
~~Needham, Massachusetts~~

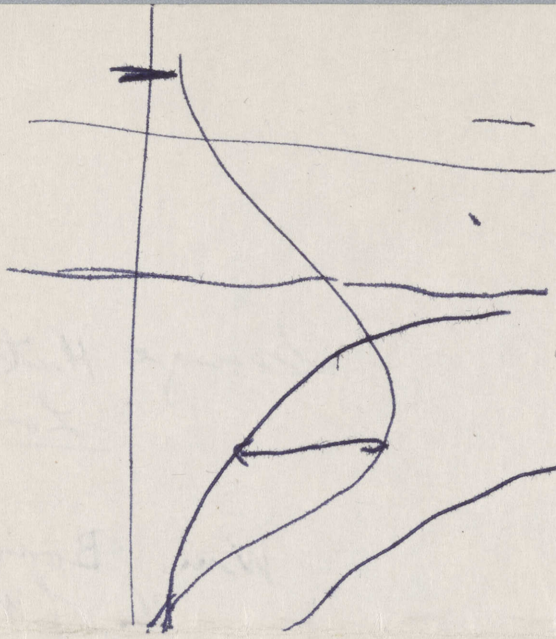
Mr. Richard W. Van Duyne
West Virginia Pulp and Paper Company
230 Park Avenue
New York 17, New York

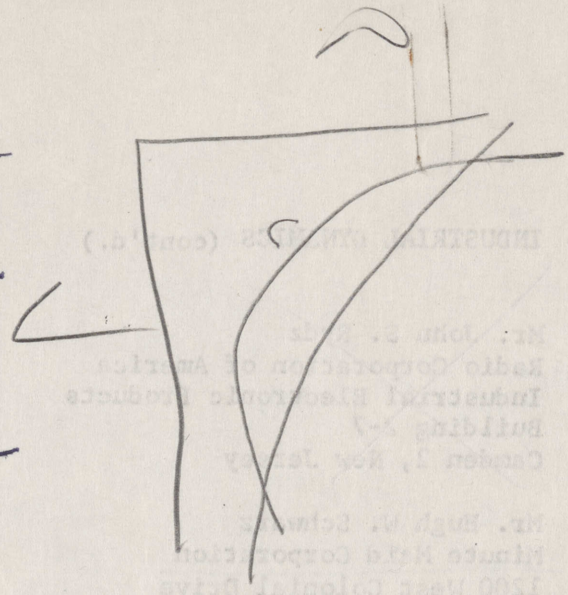
George Hutchinson
Lockheed Aircraft

Wm. Boore
U. of Washington.

Arnold Hestenes
Gen. Motors

*Accompanied by wife





Separate app. C. to F.F. report.

~~General~~

D-48, "Management and Management Science," Jay W. Forrester, June 1, 1959.

"Industrial Dynamics, A Major Breakthrough for Decision Makers," Harvard Business Review, Jay W. Forrester, Vol. 36, July-August, 1958.

Mathematical Models

D-46, "Models of Dynamic Behavior of Industrial and Economic Systems--A Section of Industrial Dynamics Class Notes," Jay W. Forrester, August 1, 1959.

D-41, "Standard Symbols for Industrial Dynamics Flow Diagrams," Jay W. Forrester, May 15, 1959.

A Production Distribution System

D-34, "Equations for Production-Distribution System--A Section of Industrial Dynamics Class Notes," Jay W. Forrester, March 10, 1959.

D-42, "Additional Information on the Illustrations in the July 1958 Harvard Business Review Article--A Section of Industrial Dynamics Class Notes," Jay W. Forrester, October 27, 1959.

"Advertising: A Problem in Industrial Dynamics," Harvard Business Review, Jay W. Forrester, Vol. 37, March-April, 1959.

Part II

D-49, "Equations for Advertising in the Production-Distribution System in the July 1958 Harvard Business Review Article," Jay W. Forrester, August 19, 1959.

Electronic Component Supplier

D-51, "Description of the Model of Production, Scheduling and Customer Delay as Used in the 1959 Summer Session," David J. Howard, August 24, 1959.

D-62, "An Industrial Dynamics Study of an Electronic Components Manufacturer," Willard R. Fey, January 4, 1960.

D-47, "DYNAMO II, an IBM 704 Program for Generating Dynamic Models," P. Fox, July 24, 1959.

D-53, "Procedure for Constructing and Submitting a Model to be Run by DYNAMO," David J. Howard, January 11, 1960 (D-35 revised).

D-60-1, "DYNAMO IIA: Addendum to Industrial Dynamics Memo D-47," G. Duren, P. Fox, A. Pugh, May 31, 1960.

~~Appendices~~.

Industrial Dynamics Memoranda, ~~Notes~~ Material.
~~Summer Session~~

D-48

~~D-46~~

H. B. R

D-46

D-34

D-42

D-49

D-51

D-62

D-47

D-60-1

Annual Report to Ford Foundation
Industrial Dynamics Research
School of Industrial Management
Massachusetts Institute of Technology

December 15, 1958

"Industrial Dynamics"

Course Outline 15.581

1958 Summer Session
Material

Memorandum D-23

JUN 3 1957

THE FORD FOUNDATION
477 MADISON AVENUE
NEW YORK 22, N. Y.

May 31, 1957

FPP
for your
file
JJ

C
O
P
Y

Dr. James R. Killian, Jr., President
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear President Killian:

It is a pleasure to advise you that The Ford Foundation has approved a grant of $275,000 to the Massachusetts Institute of Technology for research in management problems, over a period of five years, at its School of Industrial Management. This grant is being made in response to Associate Dean Eli Shapiro's request of November 9, 1956, endorsed by your letter of May 16, 1957, and as modified by this letter.

The attached statement sets forth the general terms and conditions applicable to Foundation grants. If there are any points that need clarification, or if you wish additional information, please do not hesitate to communicate with us.

Payment of the grant will be made upon your request directed to the Treasurer of the Foundation, Mr. Richard W. Lambourne, in annual installments based on your estimate of funds required for the year, approximately in accordance with the following schedule: $40,000 in 1957; $60,000 in 1958; $80,000 in 1959; $65,000 in 1960 and $30,000 in 1961. It is expected that the School of Industrial Management will make every effort to obtain funds from other sources in order to maintain this research program at a level of approximately $80,000 after 1959.

On behalf of the Foundation, may I extend every good wish for the success of this program.

Sincerely yours,

Joseph M. McDaniel, Jr.
Secretary

cc: Dean Eli Shapiro

Attachment

THE FORD FOUNDATION
477 MADISON AVENUE
NEW YORK 22, N. Y.

TERMS OF GRANT

<u>Grantee</u>

Massachusetts Institute of Technology
Cambridge 39, Massachusetts

<u>Date</u>: May 31, 1957

<u>Amount</u>: $275,000

<u>Period</u>: Approx. five years

1. <u>Expenditure of Grant Funds</u>: The grant is for the purposes stated in the attached Letter of Grant Notification, and is subject to modification only with the Foundation's approval.

2. <u>Reversion of Grant Funds</u>: The grantee will return to the Foundation any funds:

   (a) Not expended in accordance with the conditions of the grant and within the period stated;

   (b) If the grantee loses or is denied its exemption from Federal income taxation under Section 501(c)(3), formerly 101(6), of the Internal Revenue Code.

3. <u>Reports to the Foundation</u>: The grantee will furnish the Secretary of the Foundation with a written report upon the completion of the project or program for which the grant has been made. Where the period of the grant exceeds one year, the grantee will furnish interim reports, <u>as requested by the Foundation.</u> These reports will contain such information as may be necessary for the Foundation to determine that its grant is being used for the purposes intended and for the Foundation to fulfill its own public reporting responsibilities.

4. <u>Public Reports:</u> The Foundation will announce this grant in a general press release, which will appear about two weeks after the next quarterly meeting of its Trustees. However, your public relations department is free to make an initial announcement any time at your convenience after receipt of this letter, and without prior clearance by us. The Foundation's Office of Reports would like to receive copies of the release and photographs, film scripts, or any published material about the grant or the work it has made possible.

5. <u>Commitment</u>: This grant is made with the understanding that the Foundation has no obligation to provide other or additional support for the grantee, unless otherwise stated in the accompanying Letter of Grant Notification.

FF-027-A
(Revised 3/1/57)

ANNUAL REPORT

OF

INDUSTRIAL DYNAMICS RESEARCH

Jay W. Forrester

SCHOOL OF INDUSTRIAL MANAGEMENT

ESTABLISHED UNDER A GRANT FROM
THE ALFRED P. SLOAN FOUNDATION, INC.

50 MEMORIAL DRIVE
CAMBRIDGE 39, MASSACHUSETTS

December 15, 1958

Dr. Neil Chamberlain
The Ford Foundation
477 Madison Avenue
New York 22, New York

Dear Dr. Chamberlain:

The Ford Foundation requested that we submit annual reports on work under the grant which was made a year ago to MIT for developing the field of Industrial Dynamics. This letter is intended to serve as such a report, but if you find that it is not suitable for your purposes, I would appreciate your suggestions on any other information which would be helpful to you.

For a general description of the field of Industrial Dynamics and its goals, I refer you to the enclosed article, "Industrial Dynamics--A Major Breakthrough for Decision Makers" by Jay W. Forrester, from the July-August 1958 issue of the Harvard Business Review.

In general, progress has been more rapid during the last year than we had anticipated. Our Industrial Dynamics research and educational programs have both advanced further than originally expected. Much interest among industrialists has been shown in our program and the student enthusiasm has been high. On the research side, a basis has been laid which should make this coming year's work especially effective. A number of new staff members have joined us this fall, and they should increase the stature and competence of the group.

1. <u>Educational Program</u>

(a) Industrial Dynamics Graduate Course

Last fall Professor Joseph V. Yance began teaching a graduate course in Industrial Dynamics. This was well received by the students. An outline (Subject 15.581) is attached. The course was repeated in a modified form in the spring term and will continue to be given in updated form.

(b) During the last five weeks of the spring term, I taught Industrial Dynamics to the Sloan Fellows. There were 36 men in last year's group, and they were divided into two sections of 18 each. Such a group is small enough for active discussion and individual participation. These men are between 30 and 40 years old, sent for a full calendar year by their companies, and are an intensely interested, challenging group.

For the new Sloan group this year the Industrial Dynamics series was moved to the fall schedule to avoid conflict with thesis. The work was expanded to seven weeks of three class hours a week, plus an hour and a half of laboratory.

I have been much encouraged by the active interest and participation of the Sloan group both in class and in thesis work. One of the men from last year's group (from Hughes Aircraft) reports that he has received company authorization to set up within the company a group for starting an analysis of the dynamic aspects of corporate management.

(c) Senior Executive Program

This fall, for the first time, Industrial Dynamics is being included in our program for Senior Executives as a sequence of four, two-hour lectures.

(d) Summer Program.

During the last two weeks of August we gave a special summer program in Industrial Dynamics for men from government and industry. Sixteen highly qualified participants attended. I am enclosing an announcement of the session, a list of participants, and an outline of the material covered. It was highly successful. Although it was a major effort, we probably will plan to repeat it next year with newly and better prepared background material so that the work can be carried further. In two weeks one cannot aspire to create experts in the subject, but rather to give the men sufficient understanding that they feel competent to judge the potentialities of the approach for their own organizations.

(e) Future Academic Courses

During the next year, we foresee developing additional graduate courses in the field of Industrial Dynamics.

Also, in time, we can expect a series of undergraduate courses growing out of the Industrial Dynamics research. These would be designed as a unifying framework to tie together and show the interactions between the subjects taught in other management school subjects. Such a series could be thought of as supplying to the field of management the kind of foundation that physics provides for engineering.

2. Research Activities.

(a) General

Much of our time last year was devoted to clarifying the methods which we think appropriate for the construction of dynamic models for representing industrial and economic behavior. Enclosed is a rough draft of Part I of Memorandum D-16 on "Formulating Quantitative Models of Dynamic Behavior of Industrial and Economic Systems" which I wrote last spring. This, along with Part II which should be completed in the next few months, will show our approach to the analysis of time varying economic and corporate behavior. By way of illustration and in the absence of Part II of D-16, I am attaching Memo D-23 which lists the 57 equations (without explanation) that represent the production-distribution-advertising-market system illustrated in the Harvard Business Review article. A detailed discussion of these equations constituted the central theme and illustrative example for the Summer Session course and the series taught to the Sloan Fellows.

(b) Digital Computer Programs

We have also made substantial progress in the expeditious handling of models of this type on the IBM 704 computer. For nine months we have been using a first design of an automatic computer programming system designed for our type of work. By December or January we expect to bring our automatic programming techniques to a point where the model builder can go directly from his equations, punched on cards in their original equation form, to the automatically plotted results which he desires without interposing any time delay or effort devoted to computer programming.

This should facilitate all of our work and, by making the 704 computer readily accessible without extensive digital computer training, it should enhance the scope of thesis research.

(c) Theses.

Several interesting Master's degree theses have been done in the Industrial Dynamics area during the last year. (1) One study was on the organization and fluctuating behavior of the textile industry. (2) Another by Abraham Katz, a Sloan Fellow from RCA, explored the basic underlying fundamentals which lead to success or failure in the management of military research and development contracts. This thesis, especially, has attracted much attention as a way to organize and interrelate the body of descriptive knowledge about a pressing national crisis arising from our low efficiency in using research and development manpower. Mr. Edward Roberts, on our staff, is continuing this analysis as a part of the Ford Foundation project. (3) Two of the theses explored the mathematical and methodicalogical defects which exist in some of the economic model work found in the literature.

Two men doing Master's degree theses in Industrial Dynamics last year had previously completed their doctorate programs in physics.

A selected list of thesis titles in Industrial Dynamics is appended.

(d) Dynamics of Economic Systems.

Last February John Enos, Assistant Professor of Marketing, transferred to the Industrial Dynamics program to start our work on dynamics of economic systems. This was delayed by preparations for the summer session but is now underway with the help of two full-time research assistants, Michael Spiro, a doctorate candidate in the Economics Department, and Theodore Parker, now completing a Master of Science degree in Electrical Engineering and planning to register for doctorate study in economics next fall. Here the approach will be from the direction of descriptive economics, undertaking to cast into formal, quantitative, mathematical relationships our existing descriptive and intuitive knowledge of economics. This leads one immediately into complex, non-linear, probably unstable, systems. The area is one which has received little research attention thus far and we anticipate high returns for our effort. Success is, however, less sure and will be further away than in the analysis of corporate and industry operations.

(e) Dynamic Interactions in the Consumer Market.

An offshoot of the Ford Foundation Industrial Dynamics project has taken root in the MIT School of Industrial Management Marketing Group where it is financed by other research funds. This research is to explore the dynamic, quantitative interactions between advertising, demand, consumer inventory, price, market saturation, etc. It should lead to a generalized model of market behavior covering new product introduction, sales growth, market maturity, and sales decline. Such a generalized model would contain the important intrinsic market factors. The designation of particular product and environmental characteristics would adapt it to a particular product.

(f) Special Studies.

Several studies have explored particular subjects, largely for the purposes of training and the development of techniques. Professor Yance has formulated a dynamic model of price, ordering, inventory and supply interactions in the hide and shoe industry. Also, he has added some aspects of price behavior and demand elasticity to the system described in my Harvard Business Review article. Professor Enos added some aspects of capital investment to that same system.

(g) Research Sponsored by Industrial Companies.

Part of our program is to encourage special studies directed at particular industrial problems to be sponsored by industrial organizations. One such is now active in cooperation with and financed by the Sprague Electric Company. With them we are investigating management problems connected with a line of electrical components consisting of some 10,000 catalogue items. The study is of a research nature to explore the interactions of factory lead time, inventory management, delivery service to the customer, customer inventories, price policy, employment levels, and customer ordering policies, and competitive companies. An interesting facet of this problem, which also occurs in many other industries, is the fluctuation of incoming orders which is a reflection of the actions of the company itself. As deliveries slow down, customers order further in advance creating an apparent increase in demand. This slows down deliveries still further until employment is built to levels higher than justified by the actual demand for the product. An industry-wide cyclical demand pattern thereby results from the employment and inventory policies.

3. Staff of the Project.

The staff of the project has now grown to four assistant professors, two research associates, five full-time research assistants, and a digital computer programmer. My own time for directing the program is provided from the Sloan Research Fund of the School.

Biographies of the present staff are given in the appendix.

Present staff duties are as follows:

Professor Enos, assisted by Parker and Spiro, dynamics of economic systems.

Professor Greenberger, assisting with Industrial Dynamics instruction of the Sloan Fellows, major research area not yet selected.

Professor Hadley, assisting with Industrial Dynamics instruction of the Sloan Fellows, research into the problems of smoothing, averaging, and extrapolation, and the effect of noise on the usefulness of information used for reaching management decisions.

Professor Yance, teaching the graduate subject 15.581 on Industrial Dynamics for our regular graduate students, extending his work on the hide and shoe industry to include seasonal effects.

Mr. Fey, assisted by Reis, working on the cooperative Industrial Dynamics research program sponsored by the Sprague Electric Company.

Mr. Roberts, continuing the research started last year by Abraham Katz on the underlying dynamic principles controlling the success and failure of research and development management.

A very substantial improvement has occurred within the last year in the caliber of applicants for staff positions in the Industrial Dynamics work. Interest in the subject has developed to the point where it is attracting some of the best younger staff members and graduate students at MIT.

The MIT Economics Department has been very helpful by referring to us top caliber graduate students who apply to the Economics Department and want full-time staff positions while carrying on their study programs.

4. Future Program.

We expect to use the Ford Foundation funds in three principal ways:

First, is the development of basic theory, principles, and techniques. In this category are such items as the automatic computer programming development and Hadley's study of smoothing and noise in data.

The second area is that of realistic but hypothetical studies of company and industry situations to show industrial executives the possibilities for analysis of industrial behavior and corporate policy. The objective of this phase is to encourage similar work on particular practical cases within the companies themselves, as cooperative company-sponsored research programs involving the company and the School of Industrial Management. An example of such exploratory studies is my article in the Harvard Business Review and the work on research and development management by Roberts. The Sprague Electric project is an example of how these studies should develop into activity within industrial companies.

The third area for using Ford Foundation funds is the dynamics of economic systems. Laying the foundation for this work under the Ford Foundation grant is of tremendous importance and is an activity which is initially less likely to attract industrial company support. Once a foundation has been established for a new approach to economic behavior, we anticipate that the financial support of the work can be broadened.

I hope that you will feel as we do that the results of the first year are encouraging and show promise for the future. Please let me know if there is any other information which would be helpful to you.

Sincerely,

Jay W. Forrester

JWF:KDB

Appendix

"Industrial Dynamics", Harvard Business Review reprint

Outline of course 15.581

1958 Summer Session, (announcement, list of participants and outline of the Summer Session program)

Memorandum D-16, Formulating Models

Memorandum D-23, Equations

List of Theses

Biographies of Staff

"Industrial Dynamics"
Harvard Business Review

Course Outline 15.581

1958 Summer Session
Material

Memorandum D-23

Industrial Management Reprint Number 62

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

School of Industrial Management

Industrial Dynamics

A Major Breakthrough for Decision Makers

by Jay W. Forrester

A reprint from the HARVARD BUSINESS REVIEW, Vol. 36, July–August 1958.

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

Industrial Dynamics

A Major Breakthrough for Decision Makers

By JAY W. FORRESTER

REPRINTED FROM
HARVARD BUSINESS REVIEW
JULY–AUGUST 1958

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

From *In This Issue*

Jay W. Forrester

One of the most exasperating problems of a top executive is to explain satisfactorily to himself or to others exactly why things happen as they do. For example, why do small changes in retail sales sometimes lead to such large swings in factory production? Why isn't the new high-speed data-processing equipment helping to improve management decisions more? When a factory manager is able at all times to produce more goods than are being sold to consumers, why does he ever find himself unable to fill orders?

The trouble, according to Jay W. Forrester, is that businessmen have too often viewed management functions as separate activities and not as part of a unified system. Drawing on research in data processing, information-feedback systems, and military decision making, Mr. Forrester proceeds to outline a way of fitting together the many management functions into a *meaningful whole.* The appropriate title of his article is *Industrial Dynamics: A Major Breakthrough for Decision Makers.*

The author is Professor of Industrial Management in the Massachusetts Institute of Technology School of Industrial Management. He was formerly the head of the Digital Computer Division at the M.I.T. Lincoln Laboratory. In that capacity he guided the technical design of the Air Force SAGE system for continental air defense. A pioneer in the development of servomechanisms, he also worked out the method of "random access magnetic storage" which is now the standard "memory" device for reliable, high-speed digital computers. A member of many professional associations and the author of a number of basic technical papers, Mr. Forrester was recently made a Fellow of the Institute of Radio Engineers, the highest form of professional recognition I.R.E. gives.

— *The Editors*

Edited under the direction of the Faculty of
THE GRADUATE SCHOOL OF BUSINESS ADMINISTRATION, HARVARD UNIVERSITY

Address all communications, including manuscripts and change of address, to *Harvard Business Review*, Soldiers Field, Boston 63, Mass. Telephone: KIrkland 7-9800.

Reprints of all articles and features in this issue, as well as most previous issues, are available. Information on request from Business Office at above address.

A Volume Index to the *Review* is published annually with the November–December issue.

The contents are currently indexed in *The Industrial Arts Index* and in the *Bulletin of the Public Affairs Information Service*.

VE RI TAS

Harvard Business Review is published bimonthly by the Graduate School of Business Administration, Harvard University. Nathan M. Pusey, President; Paul C. Cabot, Treasurer; David W. Bailey, Secretary.

$8.00 per year, $2.00 per copy — United States & Possessions, Canada.

$10.00 per year, $2.50 per copy — Pan-American Postal Union.

$15.00 per year, $3.75 per copy — elsewhere.

Make checks payable to the *Harvard Business Review*.

Entered as second-class matter at Boston, Massachusetts. Additional office of entry at Dayton, Ohio.

PRINTED IN U.S.A.

HARVARD *Business Review*

July - August 1958 Vol. 36, No. 4

ARTICLES

FEATURES

Harvard Business Review
July-August 1958

Industrial Dynamics

a major breakthrough for decision makers

By Jay W. Forrester

Management is on the verge of a major breakthrough in understanding how industrial company success depends on the interaction between the flows of information, materials, money, manpower, and capital equipment. The way these five flow systems interlock to amplify one another and to cause change and fluctuation will form a basis for anticipating the effects of decisions, policies, organizational forms, and investment choices.

My aim in this article is to look ahead at the specific kinds of progress which will be achieved and at the concepts which will make this progress possible. While I shall suggest certain ways of thinking about management that should be helpful to executives today in working on inventory control, production scheduling, advertising, sales, and other problems, my primary concern here is not with techniques and prescriptions. Rather, I am interested in the development of a professional approach to management.

Business leaders, like leaders in other areas, are influenced by an "image of the future." Their ideas about where they are going may be clear or vague, but in any event they have a subtle and far-reaching impact on administrative thinking and decisions. A look at some promising new concepts of management should, I believe, convince even the skeptical executive that his job is developing into much more than an art, that conceptual skill will play an increasingly vital role in company success, and that management is fast becoming second to none as an exciting, dynamic, and intellectually demanding profession.

Toward a Theory

To develop the status of a profession, management must discover the underlying principles which unify its separate aspects. It must develop a basic theory of behavior. It must learn how to convert experiences and particular case examples into a contribution to this general theory. And, finally, it must be able to employ the basic principles of the theory as a useful practical guide for explaining and solving new problems as they arise. By accomplishing these aims, management will become a true profession during the next generation.

Author's note: The studies on which this article is based were made possible through the support of the Sloan Research Fund of the School of Industrial Management at Massachusetts Institute of Technology, the Ford Foundation, and the use of the IBM 704 computer at the Massachusetts Institute of Technology Computation Center.

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

The task of management is to interrelate the flows of information, materials, manpower, money, and capital equipment so as to achieve a higher standard of living, stability of employment, profit to the owners, and rewards appropriate to the success of the managers. Looked at in this way, its goals are rooted as deeply in the public interest as the broad objectives of the legal, medical, and engineering professions.

In the past, with management considered more of an art than a profession, education and practice have been highly fragmentized. Manufacturing, finance, distribution, organization, advertising, and research have too often been viewed as separate skills and not as part of a unified system. Too often management education consists of gathering current industrial practice and presenting it to the student as a sequence of unrelated subjects. Similarly, in his work in industry the manager specializes within departments where his experience perpetuates the atmosphere of unrelated compartmentalization.

The next big step in management education will be the development of a basis for fitting together the many management functions into a meaningful whole. Around this central core specialized subjects and experience will take on more significance. Men can be developed more rapidly. They will be able to start from a point now accessible only through long training or fortuitous experience.

Such strides will far exceed in importance recent steps in using computing machines to execute clerical tasks or in applying operations research methods to isolated company problems. For we can expect to gain, during the next 25 years, a far better understanding of the dynamic, ever-changing forces which shape the destiny of a company. This understanding will lead to better usage of available information, to improved understanding of advertising effectiveness and the dynamic behavior of the consumer market, and to company policies that keep pace with technological change.

Beyond these achievements, there will be improvements in company organization resulting from a sounder basis for effective decentralization, from altering the relationships between line and staff tasks in the company, from the more effective utilization of scientific manpower, and from reducing the routine duties and enhancing the creativity of managers. And executives will gain in "clairvoyance." For example, they will be able to anticipate clearly (as I shall illustrate later in the article):

- How small changes in retail sales can lead to large swings in factory production.
- How reducing clerical delays may fail to improve management decisions significantly.
- How a factory manager may find himself unable to fill orders although at all times able to produce more goods than are being sold to consumers.
- How an advertising policy can have a magnifying effect on production variations.

Tools of Progress

The new management concepts will rest in part on recent advances in the data-processing industry, in part on military research (which has given us an improved understanding of decision making and experience in analyzing and simulating the characteristics of complex systems), and largely on 20 years of research in information-feedback systems.

Electronic Data Processing

The performance of electronic computers has increased annually by a factor of nearly 10 per year over the last decade; in almost every year we have seen a tenfold increase in speed, memory capacity, or reliability. This represents a technological change greater than that effected in going from chemical to atomic explosives. Society cannot absorb so big a change in a mere ten years. We therefore have a tremendous untapped backlog of potential devices and applications.

Viewing data processing in another way, we can trace the shifting frontiers of the field by dividing progress into five-year periods:

- From 1945 to 1950 was the period of electronic research and the demonstration that machines having many thousand vacuum tubes would indeed operate.
- From 1950 to 1955 was the pioneering period in applying computers to the solution of scientific and engineering problems.
- During the present period, 1955 to 1960, electronic machines are being substituted for clerical effort in commercial organizations.
- From 1960 to 1965 we can expect to see the application of digital computers to physical process control. Already there are digital machine tool

controls and the SAGE air defense system; we can look forward to dramatic improvements in processes, to reduction of capital investment requirements in the oil and chemical industries, and to civilian air traffic control systems, all based upon new kinds of information utilization for control purposes.

¶ From 1965 to 1970 we should see all these developments converging into pioneering improvements in the central management process. The routine, repetitive types of decisions will become more formalized, while management creativeness will be directed to *how* decisions and policies should be made rather than to the actual repetitive making of such decisions.

During the 1965–1970 period of widespread company testing of new management methods, electronic data processing will be an essential tool. Let me emphasize, however, that it is no more the focal point of the future management profession than a slide rule is the essence of engineering.

Decision Making

Historically, military necessity has often led not only to new devices like aircraft and digital computers but also to new organizational forms and to a new understanding of social forces. These developments have then been adapted to civilian usage.

Such new exploration is now happening in the military *command* (or *management*) function. As the pace of warfare has quickened, there has of necessity been a shift of emphasis from the tactical decision (moment-by-moment direction of the battle) to strategic planning (preparing for possible eventualities, establishing policy, and determining in advance how tactical decisions *will* be made). The battle commander can no longer plot the course of his enemy on a chart and personally calculate the aiming point. In fact, with a ballistic missile he would have no time even to select his defensive weapon.

Likewise in business: as the pace of technological change quickens, corporate management, even at the lower levels, must focus more and more on the *strategic* problems of running the business and less and less on the everyday operating problems.

In the systems development for the military, it has been amply demonstrated that carefully selected formal rules can lead to tactical decisions that excel those made by human judgment under the pressure of time and with insufficient experience and practice. Furthermore, it has been found that men are just as adaptable to the more abstract strategic planning as they are to tactical decision making, once their outlook has been lifted to the broader and longer-range picture.

Simulation

Also from military research we have available the methods of simulation for determining the behavior of complex systems. These techniques have reached the state of development where they can now be usefully applied to industrial organizations. Simulation is being used in the design of air defense systems and in engineering work. For example:

> In planning the development of a river basin, numbers in a digital computer represent water volumes, flow rates, electric demand, and rainfall. A few seconds of computer time can solve a whole day of systems operation. Dams can be located and designed for an optimum compromise between power generation, irrigation, navigation, and flood control.

In business, simulation means setting up in a digital computer the conditions which describe company operations. On the basis of the descriptions and assumptions about the company, the computer then generates the resulting charts of information concerning finance, manpower, product movement, and so on. Different management policies and market assumptions can be tested to determine their effect on company success.

To use simulation studies will not require undue mathematical ability. To be sure, details of setting up a model will need to be handled by experts because there are special skills required and pitfalls to be avoided. However, the job of directing the situations to be explored, judging the assumptions, and interpreting the results will be within the ability of the type of men we now see in management schools and executive development programs.

Feedback Control

Systems of information feedback control are fundamental to all life and human endeavor, from the slow pace of biological evolution to the launching of the latest satellite. A feedback control system exists whenever the environment causes a decision which in turn affects the original environment. To illustrate:

- A thermostat receives temperature information, decides to start the furnace, and changes the temperature.
- A person senses that he may fall, corrects his balance, and thereby is able to stand erect.
- In business, orders and inventory levels lead to manufacturing decisions which fill orders and correct inventories.
- A profitable industry attracts competitors until, to use the economist's terms, the profit margin is reduced to equilibrium with other economic forces.
- The competitive need for a new product leads to research and development expenditure that produces technological change.

All of these are information feedback control loops. The regenerative process is continuous, and new results lead to new decisions which keep the system in continuous motion.

The study of feedback systems deals with the way information is used for the purpose of control. It helps us to understand how the amount of corrective action and the time delays in interconnected systems can lead to unstable fluctuation. Driving an automobile provides a good example:

> The information and control loop extends from steering wheel, to auto, to street, to eye, to hand, and back to steering wheel. Suppose the driver were blindfolded and drove only by instructions from his front-seat companion. The resulting information delay and distortion would cause erratic driving. If the blindfolded driver could get instructions only on where he had been from a companion who could see only through the rear window, his driving would be even more erratic.

Yet this is analogous to the situation in business. Top executives do not see the salesmen calling on customers, do not see the prospective buyers watching a TV commercial. They do not attend the board meetings of competitors. They do not have a clear view of the road ahead. The only thing they can tell with reasonable certainty (and even here there is sometimes doubt) is what happened to wages, sales, material costs, interest rates, and so on last year.

Smoother Operations

The quality of management control depends upon what information executives use and for what they use it, as well as on their skills as administrators. As a system the company has certain characteristics which are completely independent of individual functions or departments, just as an electronic computer has certain characteristics as a system of parts. For instance, there is what the engineer might call "amplification," caused by inventory accumulation, filling of supply pipelines, and inept extrapolation of trends. There are delays in decisions, shipping, communications, and accounting. These all combine to cause production fluctuations, construction of excess plant capacity, creation of company-generated rather than customer-generated seasonal sales, and detrimental advertising policies.

Without an awareness of basic information-flow principles, it is only through costly errors that managers can develop an effective intuitive judgment. For example:

> In a company manufacturing consumer durables, it was discovered, after all data became available, that in a certain year retail sales had varied by 30%. In the same year the inventory and ordering practices in the distribution system and at the factory caused this small retail variation to be amplified to a four-to-one, or 400%, variation in factory production. The estimated avoidable costs of increasing and decreasing production and of carrying excess inventory were equal to the normal anticipated profit margin! (We shall see later how this could happen.)

Feedback theory explains how decisions, delays, and predictions can produce either good control or dramatically unstable operation. It relates sales promotion to production swings, purchasing and pricing policies to inventory fluctuations, and typical life cycles of products to the need for research.

Production & Distribution

How do the concepts and approaches I have been discussing apply to specific business problems? Or, to be more precise, how do the delays, amplifications, and oscillations in the circuital flow of information in a company affect its operations?

Now, in order to illustrate, I must compromise one of the main ideas I have specified; namely, that the system (meaning not the paperwork forms and procedures, but the interrelationships between all the company operations) behaves according to the characteristics of the whole and not according to the characteristics of individual parts. For the sake of brevity, I shall have to limit myself to the production and distribution functions, forgetting research, engineering, sales, and, until a later point in the article, promotional effort. In a sense this is a subsystem, with its own complex of interrelationships that can be studied as such, but it is not *the* whole. Even within this production-distribution subsystem, I shall have space to include only the flows of information and materials. Omitted altogether are considerations of money, manpower, and capital equipment.

If I can make the principles of the new approach clear, however, it should not be difficult for the reader to extend his own picture to include the areas that I must omit.

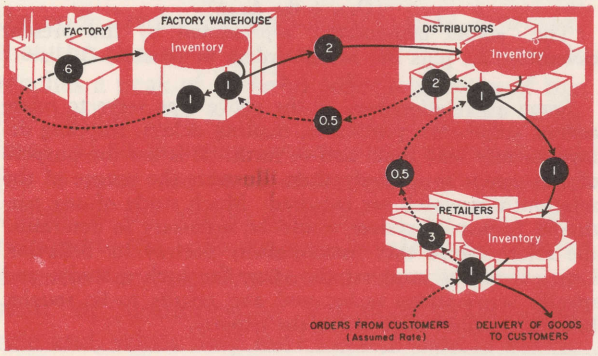
Needed Information

To begin the study of our example, we need to know three kinds of information about the system: its organizational structure, the delays in decisions and actions, and the policies governing purchases and inventories.

1. *Organizational Structure.* Exhibit i shows a typical organizational structure for the production and distribution functions in a hard goods industry like household appliances. If we examine the basic internal behavior of the system when customer orders are independently specified, we will see that, even with very simple retail sales changes and with no other external disturbances affecting the company, typical manufacturing and distribution practices can generate the types of business disturbances which are often blamed on conditions outside the company. For random, meaningless sales fluctuations can be converted into annual, seasonal production cycles. Advertising and price discount policies of an industry can create two- and three-year sales cycles. Factory capacity, even though always exceeding retail sales, can seem to fall short of meeting demand with the result that production capacity is overexpanded.

Examining Exhibit i more closely, we see that the bottom box represents the retail level. Next

Exhibit i. Organization of production-distribution system



above are the distributors, and at the left the factory and factory warehouse. The dotted lines show the information flow, here consisting of orders for goods flowing upward. The solid lines show the shipment of goods flowing downward.

2. *Delays in Decisions and Actions.* To be able to determine some of the dynamic characteristics of this system we must also know the delays in the flow of information and goods. The time delays are shown on the diagram in weeks and are reasonable values for a consumer durable product line.

Delivery of goods to the consumer averages a week after the customer places an order. At the retail level, the *accounting* and *purchasing* delays average three weeks between the time of a sale and the time when that sale is reflected in an order sent out to obtain a replacement. *Mailing* delay for the order is half a week. The distributor takes a week to *process the order*, and *shipment* of goods to the retailer takes another week. Similar delays exist between the distributor and the *factory warehouse*. The *factory lead time* averages six weeks between a decision to change production rate and the time that factory output reaches the new level. Note that three levels of inventory exist — factory, distributor, and retailer.

3. *Policy on Purchasing Orders and Inventories.* To complete the initial description of our example, we need to know the policies followed in placing orders and maintaining inventory at each distribution level. Let us consider three principal types of orders: (a) orders which directly reflect sales, (b) orders to adjust inventories with changes in business volume, and (c) orders to fill the supply lines with in-process orders and shipments. Let us suppose further that orders are treated in the following ways:

- After a sales analysis and purchasing delay (three, two, and one weeks for the three levels), orders to the next higher level of the system include the actual sales made by the ordering level.
- After a time for averaging out sales fluctuations (eight weeks), a gradual upward or downward adjustment is made in inventories as the rate of sales increases or decreases.
- The orders in process (orders in the mail, unfilled orders at the supplier, and goods in transit) are proportional to the level of business activity and to the length of time required to fill an order. Both an increased sales volume and an increased delivery lead time necessarily result in increased total orders in the supply pipeline.

The ordering rate will also depend on some presumption about future sales. Prediction methods that amount to extending forward (extrapolating) the present sales trend will in general produce a more unstable and fluctuating system. For our example, however, we will use the conservative practice of basing the ordering rate on the assumption that sales are most likely to continue at their present level.

Simulation Methods

Before we can determine how our system will function over a period of time, all of the above rather general descriptions of the system must be expressed in explicit quantitative form. Illustrations of these relationships are shown in Exhibit II.

The next step is to determine how the system as a whole behaves. To do so, we might use some pattern of consumer purchases as an input and observe the resulting inventory and production changes. A very good test would be to see what happens after a small sudden change in retail sales. The effect on the company system can be obtained by "simulation" methods — simulation being the technique of obtaining results from a model. For the industrial system, simulation consists of tracing through, step by step, the actual flow of orders, goods, and information, and observing the series of new decisions required. For example:

Our production-distribution system might be simulated by a group of men around a table, one to represent retailers, another the postal service, another transportation, another the factory, and so on. Five minutes might represent a week, and in each time interval the proper purchase orders and deliveries would be made according to the rules illustrated in Exhibit II. Alternatively, the whole exercise can be done by one person in tabular form on paper. Better still, the entire sequence can be programed on a digital computer.

Digital computer simulation was used to obtain the results in Exhibit III. A sudden 10%

Exhibit II. Formal quantitative statements of the relationships in Exhibit I

To determine the behavior of a system by simulating the performance of its parts requires that one describe exactly, and in detail, the characteristics which are to be included. The validity of the outcome of the system studies depends on the judgment of what is pertinent to include in the system description. The following examples show some of the kinds of relationships that are needed in the study of the production-distribution functions of Exhibit I:

(1) The inventory level at retail at the end of any time period is the inventory at the beginning of the period, plus goods received, minus goods delivered during the period. The relationship can be expressed as:

$$I_{r,k} = I_{r,j} + \Delta t R_{d,jk} - \Delta t S_{r,jk}$$

which says that the inventory at retail, I_r, at time k is equal to the inventory one time period earlier at time j, plus the goods received in the time interval (the length of the time interval, Δt, multiplied by the rate at which goods are received from the distributor, R_d, over the time interval j to k) minus the goods delivered (Δt multiplied by the rate of sales to customers, S_r, in the interval j to k). Δt must be a solution time interval that is short compared with any significant time delays which are to be represented in the system. (Δt is here 1/20 of a week, since third-order exponential delays as short as half a week — the mailing delays — are used in the system description.)

(2) Similarly, unfilled orders equal previous unfilled orders plus new orders minus shipments; that is:

$$U_{r,k} = U_{r,j} + \Delta t N_{r,jk} - \Delta t S_{r,jk}$$

where U_r represents unfilled orders at the retail level; N_r is the rate of receipt of new orders; and S_r is the rate of delivery of goods.

(3) Shipments by the retailers are dependent on various factors, here assumed to be the normal order-processing delay, d_{ur}, the level of unfilled orders, U_r, and the physical ability to fill orders based on a ratio of actual inventory level, I_r, to a desirable level of inventory, D_r. Expressed mathematically:

$$S_{r,kl} = \frac{U_{r,k}}{d_{ur}} \sqrt{\frac{I_{r,k}}{D_{r,k}}}$$

This is to represent the aggregate of many retailers each selling a multiplicity of catalogue items in the product line.

(4) We need to know the rate at which the retail level purchases from the distributor level. This reflects (a) the rate of customer purchases from retail plus (b) the rate necessary to adjust the retail inventories plus (c) the adjusting of orders in process in the pipeline between retailers and distributors. The mathematical statement is:

$$P_{r,kl} = P_{c,jk} + \frac{1}{d_{Ir}} \left[(D_{r,k} + d_{pr} A_{r,k}) - (I_{r,k} + G_{r,k}) \right]$$

This says that the purchasing rate by retailers, P_r, in the next time interval, kl, is equal to the purchasing rate by customers, P_c, in the previous time interval, jk, as it is thought to be after data-processing delays (provided by another equation), plus an allowance for inventory and pipeline adjustments which are made gradually over the length of time, d_{Ir}.

The "desired ownership," consisting of the desired inventory, D_r, plus the orders which need to be in process to sustain the current business level (obtained by multiplying the total delay, d_{pr}, a variable time, in filling orders by the average rate of recent sales, $A_{r,k}$), is:

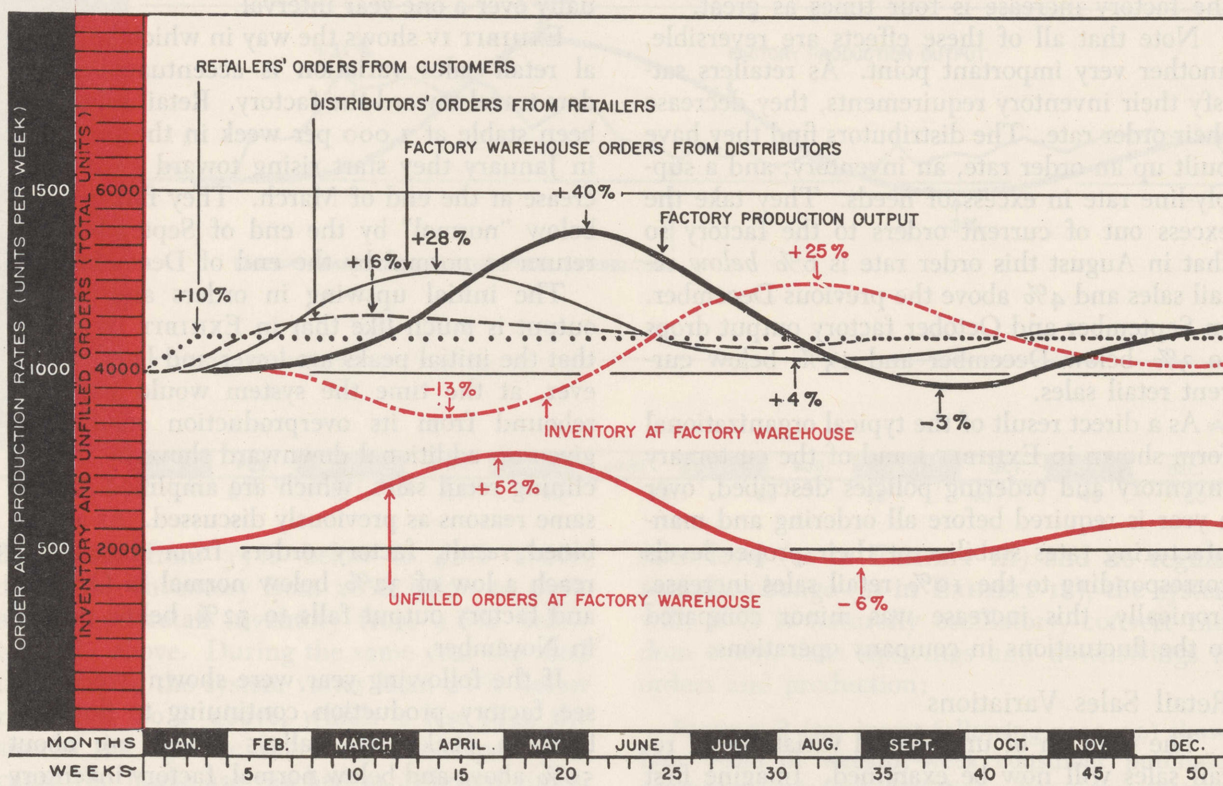
$$(D_{r,k} + d_{pr} A_{r,k})$$

The "actual ownership," represented by the present inventory, I_r, plus the actual orders in process, G_r, is:

$$I_{r,k} + G_{r,k}$$

Some 40 relationships of the above type serve to relate orders, shipments, purchases, mailing delays, shipping delays, factory lead time, and so forth. It is not the purpose of this article to do more than illustrate the nature of the relationships which are required. Much study of any particular industrial system would be necessary to determine the relationships that meaningfully portray its behavior. Line executives must set the stage for such a study, and specify assumptions to be used and results to be sought, but the detailed work is for the experts.

EXHIBIT III. RESPONSE OF PRODUCTION-DISTRIBUTION SYSTEM TO A SUDDEN 10% INCREASE IN RETAIL SALES



increase in retail sales was introduced in January. The resulting fluctuations are shown in order rates, factory output, factory warehouse inventory, and unfilled orders. (Retail orders are here given as an input which is independent of what happens in the production-distribution system itself. Retail orders are, of course, not actually independent since they are affected by availability of the product and by advertising, a point to be discussed later.)

Fluctuating Behavior

Because of accounting, purchasing, and mailing delays, the increase in distributors' orders from retailers lags about a month in reaching the 10% level. It is important to note, however, that the rise does not stop at 10%. Instead it reaches a peak of 16% in March because of the new orders that have been added at the retail level (a) to increase inventories somewhat and (b) to raise the level of orders and goods in transit in the supply pipeline by 10% to correspond to the 10% increase in sales rate. These inventory and pipeline increments occur as "transient" or "nonrepeating" additions to the order rate and, when they have been satisfied, the retailers' orders to the distributors drop back to the enduring 10% increase.

The factory warehouse orders from distributors show an even greater swing. The upturn in orders at the factory warehouse lags the upturn at the distributor level. The incoming order level for distributors is above retail sales for four months, reaching a 16% increase in March, which is readily mistaken for a corresponding true and enduring increase in business volume of the same amount. The distributors' orders to the factory, therefore, include not only the 16% increase in orders they themselves receive but also a corresponding increase for distributor inventories and for orders and goods in transit between distributor and factory. As a result, orders at the factory warehouse reach in April a peak of 28% above the previous December.

Let us turn now to manufacturing orders. They are placed on the basis of the increasing factory warehouse orders and the falling warehouse inventory, which drops 13%. As a result, the factory output, delayed by a factory lead time of six weeks in this example and then a week of data-processing delays, reaches a peak in

June which is 40% above December. While retail sales are still at 10% above December, the factory increase is four times as great.

Note that all of these effects are reversible, another very important point. As retailers satisfy their inventory requirements, they decrease their order rate. The distributors find they have built up an order rate, an inventory, and a supply-line rate in excess of needs. They take the excess out of current orders to the factory so that in August this order rate is 6% *below* retail sales and 4% above the previous December. In September and October factory output drops to 3% below December and 13% below current retail sales.

As a direct result of the typical organizational form shown in Exhibit I and of the customary inventory and ordering policies described, over a year is required before all ordering and manufacturing rates stabilize at their proper levels corresponding to the 10% retail sales increase. Ironically, this increase was minor compared to the fluctuations in company operations.

Retail Sales Variations

The problem of unexpected variations in retail sales will now be examined. Imagine first that our system has operated in the past with constant sales, and then sales rise and fall gradually over a one-year interval.

Exhibit IV shows the way in which an annual retail sales variation is accentuated as orders travel toward the factory. Retail sales have been stable at 1,000 per week in the past, and in January they start rising toward a 10% increase at the end of March. They fall to 10% below "normal" by the end of September and return to normal by the end of December.

The initial upswing in orders and factory output is much like that in Exhibit III except that the initial peaks are lower and later. However, at the time the system would naturally rebound from its overproduction state, it is given an additional downward shove by the declining retail sales, which are amplified for the same reasons as previously discussed. As a combined result, factory orders from distributors reach a low of 32% below normal in October, and factory output falls to 52% below normal in November.

If the following year were shown, we would see factory production continuing to fluctuate between peaks and valleys which are about 50% above and below normal, factory inventory

Exhibit IV. Response of production-distribution system to a 10% annual rise and fall in retail sales

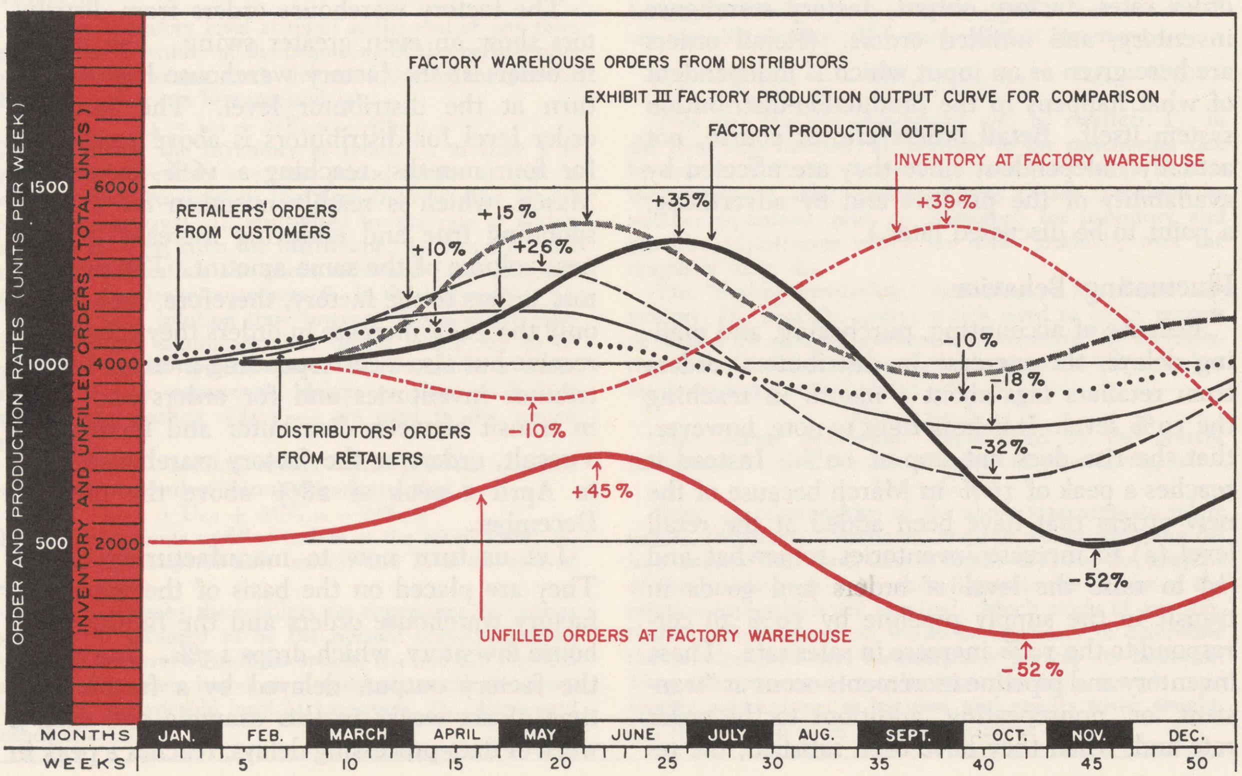
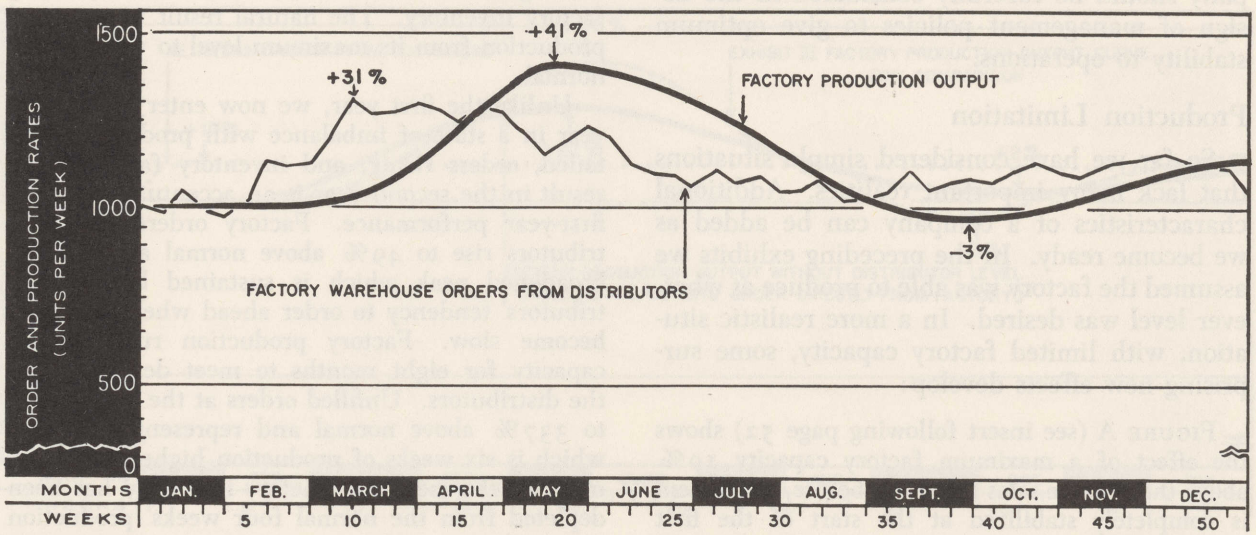


Exhibit v. Random short-run uncertainty in distributor purchases



fluctuating from 35% below to 42% above, distributor inventory from 28% below to 25% above, and retail inventory from 9.6% below to 9.3% above. During the same year the total inventory in the system varies from 21% below normal to 20% above, which is less than the sum of the separate distribution level peaks because the peaks are reached at different times.

Random Fluctuations

Note that the curves of Exhibits iii and iv are smooth and free of the short-term random fluctuations which are seen in most charts of actual industrial orders. Uncertainty and random behavior can be inserted into systems studies of this type, as shown in Exhibit v. Here the size of orders placed by distributors has been made subject to a week-by-week random uncertainty. Because of the tendency of factory scheduling to smooth out short-run order fluctuations, and because little amplification exists between distributor orders and the factory, the factory output is almost identical to that of Exhibit iii.

However, if the random-buying fluctuation is introduced in *consumer* purchases, more amplification develops. An oscillatory system like this will respond to the random disturbances by fluctuating at a speed determined by the characteristics of the system itself, rather than in a pattern that is easily traceable to the external disturbance (in this case, a tendency toward peak-to-peak production fluctuations some seven to ten months apart).

Even if there is no average change in retail sales level (as in Exhibit iii) and no regular periodic change (as in Exhibit iv), the system will, if it is naturally oscillatory, convert random events into upswings and downswings in orders and production:

Figure B (see insert following page 52) shows four years of week-by-week consumer purchases containing random deviations from the average, but no meaningful changes in sales rate. Also shown is the resulting factory production output for the production-distribution system described earlier in the article.

The system, by virtue of its policies, organization, and delays, tends to amplify those retail sales changes to which the system is sensitive. Retail sales fluctuate from week to week over a range of 10% or 15% with one week falling 31%. Factory output rises and falls over periods of several months with amplitudes of 20%, 30%, and 40% away from the average.

Just as waves in a bathtub build up slowly when disturbed by a small movement of the hand, so it takes most of the first year to build up serious amplitudes in the factory production swings. These waves then feed on the sales fluctuations. By the time of the third or fourth year, examination of the factory production curve could easily lead the casual observer who lacked the retail sales information to conclude that a seasonal sales pattern was present.

I know of company situations in which such an erroneous conclusion about seasonal sales has led to the establishment of employment, inventory, and advertising policies which in succeeding years *caused* a seasonal manufacturing pattern and thereby confirmed the original error.

The possibility of this happening in any company should be carefully considered in the design of management policies to give optimum stability to operations.

Production Limitation

So far we have considered simple situations that lack many important realities. Additional characteristics of a company can be added as we become ready. In the preceding exhibits we assumed the factory was able to produce at whatever level was desired. In a more realistic situation, with limited factory capacity, some surprising new effects develop:

Figure A (see insert following page 52) shows the effect of a maximum factory capacity 10% above the average sales level. As before, the system is completely stabilized at the start of the first year; then it is assumed that retail sales rise and fall 10% during each year.

Retail sales never exceed production capacity. Yet, because of the inventory and pipeline effects, distributor orders to the factory *do* exceed its capacity. Furthermore, as factory deliveries become slower, distributors begin to order further in advance of needs and still more orders are put into the system. As a result, the factory operates at full capacity for four and one-half months during the first year.

Then, inventory demands are filled at a time that coincides with falling retail sales. In the last half of the year, falling retail sales and inventory liquidation combine with the reduction of in-process orders which results from improving delivery. In September and October we see a combination of rapidly falling factory orders, a swiftly declining backlog of unfilled orders, and a suddenly rising factory inventory. The natural result is to curtail production from its maximum level to 52% *below* normal.

Unlike the first year, we now enter the second year in a state of imbalance with production curtailed, orders rising, and inventory falling. The result in the second year is an accentuation of the first-year performance. Factory orders from distributors rise to 49% above normal and show a broadened peak which is sustained by the distributors' tendency to order ahead when deliveries become slow. Factory production runs at full capacity for eight months to meet demand from the distributors. Unfilled orders at the factory rise to 337% above normal and represent a backlog which is six weeks of production higher than normal. In the meantime factory inventory has been depleted from the normal four weeks' production to less than a week.

In spite of low factory inventory and slow deliveries, note that retail inventories deviate only 10% and that actually there is little likelihood of losing retail sales.

Note that at point A in the 84th week shipments equal incoming orders; inventory is constant. This is therefore the point at which unfilled orders reach a peak and start down. Again, since the factory has produced in excess of sales for eight months and because the long-run average production must equal retail sales, manufacturing must be sharply curtailed in the remaining four months. It drops to a minimum of 54% below normal and produces at a level below average retail sales for a period of 14 weeks.

In succeeding years the system would act much

Exhibit vi. Effect of reducing clerical delays

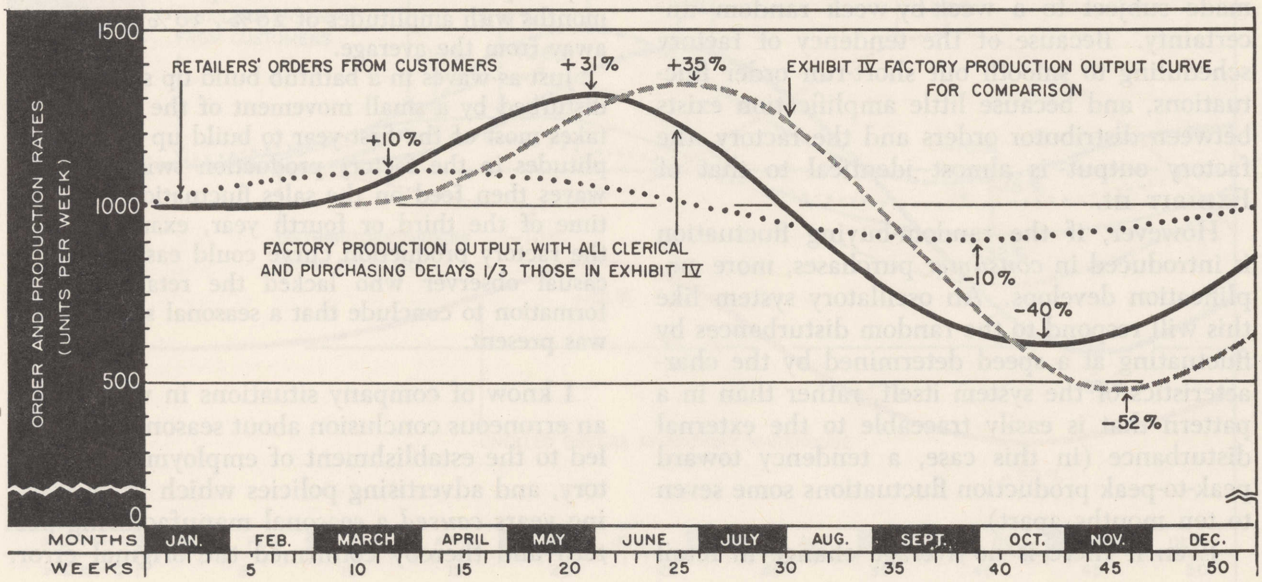
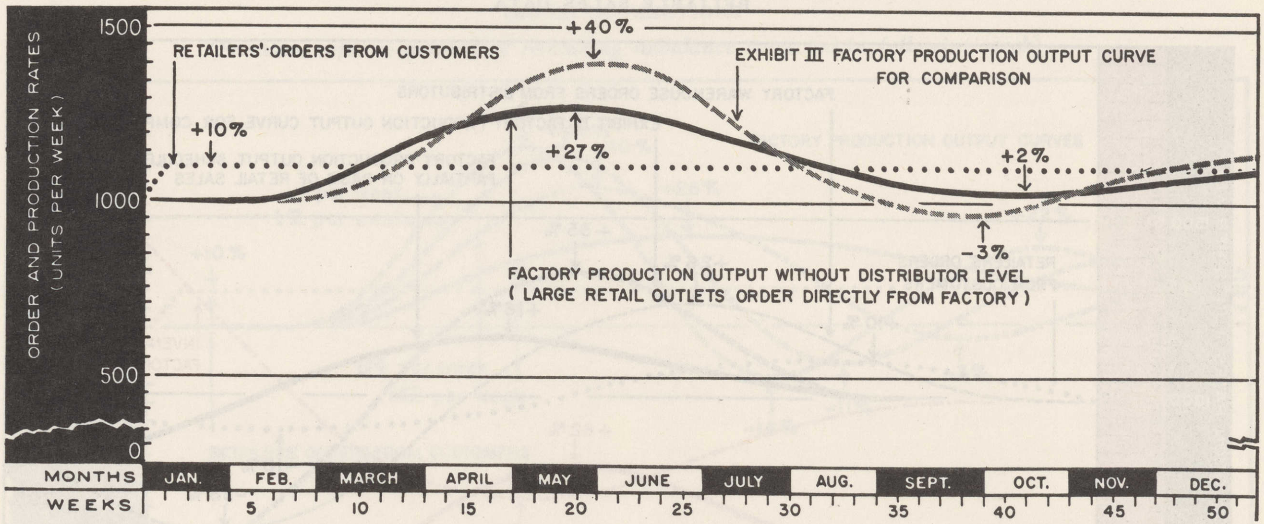


Exhibit vii. Distributor level eliminated



as it did in the second year, though unfilled orders at the factory would go to somewhat higher peaks.

We see in Figure A an occurrence which may lead a company to overexpand production capacity. For 12 months in the second year, inventory was below the desired level; for 9 months from the first of November until July the order backlog was rising. Viewed from the circumstances at the factory, this condition might easily lead management to undertake expansion plans — even though retail sales never exceed production capacity.

Improving Control

To explain the behavior of an industrial organization is only the first step. After adequately representing the current operations of a particular company or industry, the next step is to determine ways to improve management control for company success.

Faster Order Handling

To improve industrial stability and to make the company less vulnerable to external influences, executives may consider various alternatives. For instance, they might wish to explore the effect of reduced clerical and data-processing delays, which is often suggested as a quick and easy step toward better management control. However, companies often find the results disappointing, a point illustrated in Exhibit vi. Here, the clerical delays assumed in Exhibit i (the three, two, and one weeks of sales analysis and purchasing time) have been reduced to one-third their previous values. But the effect is only slight improvement: the production peaks are reduced from five times to four times the retail sales changes. Because clerical delays are such a small factor in the operation of the system as a whole, no amount of speed-up can radically change over-all performance.

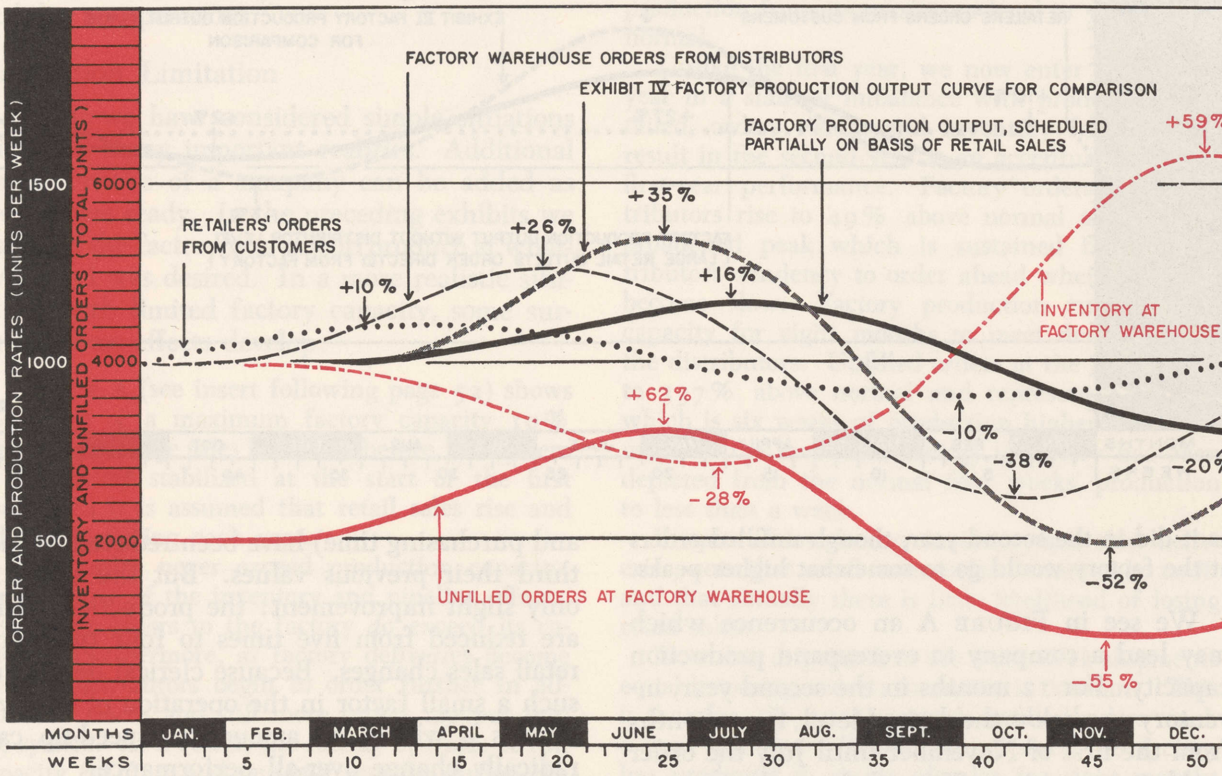
What about changes in ordering procedures? One possibility would be to place retail orders directly with the factory. The result is shown in Exhibit vii. When the inventory accumulation, fluctuation of in-process orders, and delays of one of the three stages are removed, production overshoot is 27% instead of the 40% in Exhibit iii.

The foregoing raises an interesting question about industries having more than three distribution levels. For example, in the textile industry, which shows marked instability, there are often four or five distribution levels from yarn manufacture to the final consumer. May not a good deal of instability be caused by the existence of so many levels?

Better Sales Data

Another way to improve factory production stability is to have reliable retail sales information on which to base factory production schedules. Exhibit viii assumes that we have "reasonably" reliable retail sales data that are eight weeks late in becoming available to management. (This lag allows for time to collect the data and average out short-run random fluctuations.) By reliable data we mean here that the retail sales

EXHIBIT VIII. EFFECT ON FACTORY SCHEDULES OF USING REASONABLY RELIABLE SALES DATA



information is good enough so that we are willing to give it six times as much weight in scheduling production as we do to the actual orders received at the factory. Here are the results:

The first production peak is 16% above the start, instead of 35% as in EXHIBIT IV. Later peaks are two times the retail sales changes, instead of five times as in EXHIBIT IV. The supply pipeline and inventory changes at the retailer and distributor levels are being met more from factory inventory and less from peaks in production, which necessitates larger swings in factory inventory, in the unfilled order backlog, or in both.

The need for confidence in the retail sales information is apparent in October, when the factory output is being reduced only gradually in spite of incoming orders being down nearly 40% from normal, and in spite of a rapidly falling backlog of orders and a rapidly rising inventory. To achieve this degree of stabilization of production, one must be able to let factory inventory fluctuate from 35% below normal to 60% above.

Inventory Adjustment

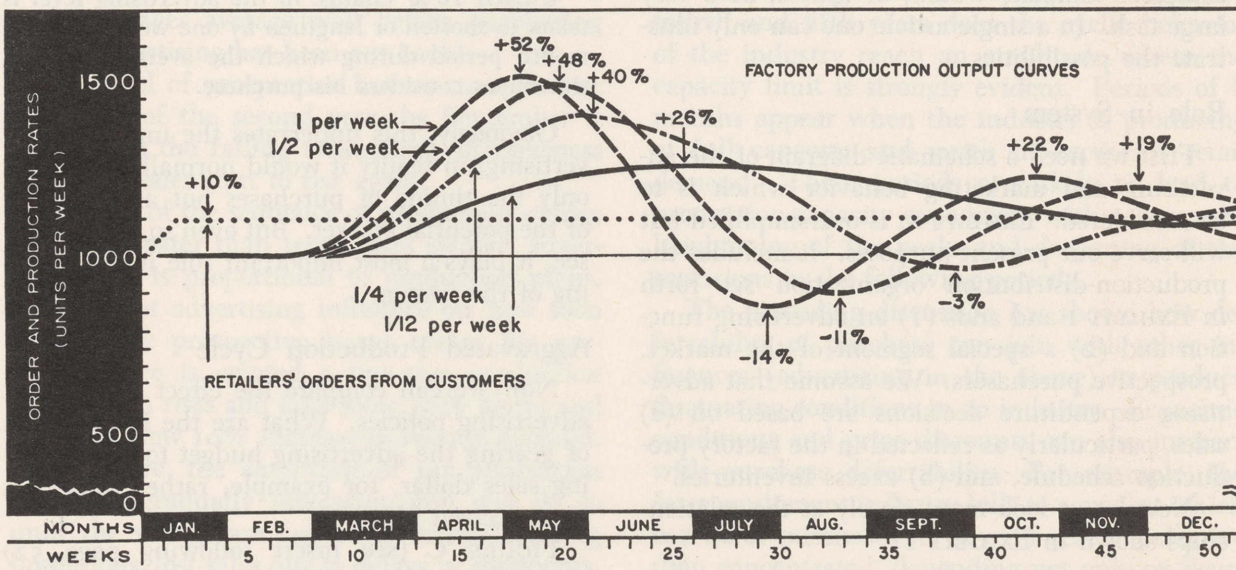
The behavior of a simple production-distribution system like that shown in the exhibits is probably more affected by the practices followed in adjusting inventories and in-process orders than by any other single characteristic. The amount of inventory change and its timing are both important. These points deserve special emphasis, and should be considered with especial care by decision-making executives. To illustrate what can be accomplished:

In EXHIBIT IX we take the same production-distribution system as before but assume that management has changed the rapidity with which the inventory corrections are made (but not the total amount of adjustments made). Management has specified that an inventory correction should be based on sales smoothed over the previous eight weeks (exponential smoothing with eight weeks time constant) so that in no case can the adjustment follow a sales change more rapidly than the delay introduced by the smoothing. The question is what time interval to specify for making inventory correction. When inventories are lower than desired for the current level of business activity, how much of the difference or imbalance should be added to the orders sent to the factory in the next week?

The factory output curves portray the data management needs for a decision. Each curve is based on a different correction rate — a different frac-

Exhibit ix. Changing the time to make inventory and in-process order corrections

(Fractions refer to amount of remaining imbalance corrected in following week)



tion per week of any inventory discrepancy which is to be corrected through adjustment of the ordering rate to the next higher supply level. (This is the d_{Ir} term defined in the fourth point in Exhibit ii.) This correction rate varies from the top curve, in which orders for any imbalance in inventory and in-process orders are fully placed in the following week, to the lowest curve, in which only 1/12 of any *remaining* imbalance is corrected in the following week. The lowest curve leads to about 60% of an initial imbalance being corrected in 12 weeks and 85% corrected in 24 weeks.

We see that the production fluctuations for the 1-week correction (the top curve) reach 52% above, 14% below, and 22% above the initial value as a result of a sudden 10% rise in retail sales. There is an interval of 24 weeks between successive peaks. At the other extreme, the 12-week correction time (the lowest curve) leads to peaks of only 25% above, 3% below, and 14% above the initial value as the production rate approaches the 10% higher level. The interval between peaks is longer — 34 weeks. As for the curve for the 4-week correction, it is the same as that used in Exhibit iii and is the basic system used in the other exhibits.

We see from Exhibit ix that the more gradual inventory correction to business level changes leads to improved stability. Further, the reduction of manufacturing fluctuation is achieved without increasing the inventory extremes.

Exhibit x presents, for various correction times, peak percentage changes in inventory at each level in the system and the total change for the sum of all inventories in the system. The total inventory changes are less than the sum of the separate levels because the extremes do not occur at the same time.

Under many forecasting methods, the kind of gradual adjustment of inventories I have described is impossible. The unstabilizing effects are accentuated by superimposing changes in sales, in-process orders, and inventories.

Effect of Advertising

To show how we would begin introducing into our model additional realism which would be a part of actual company operations, we will now look at *one* of the many effects of adver-

Exhibit x. Effect of correction time on inventories

Adjustment time	*Retail*	*Distributor*	*Factory*	*Total*
1 week	− 3.8% to + 5.6%	− 10.9% to + 12.9%	− 14.3% to + 32.3%	− 7.1% to + 13.6%
2 weeks	− 4.0 to + 5.4	− 11.3 to + 13.5	− 13.9 to + 30.2	− 7.3 to + 13.6
4 weeks	− 4.3 to + 5.5	− 11.5 to + 12.9	− 12.7 to + 24.6	− 7.4 to + 12.2
12 weeks	− 4.8 to + 5.3	− 11.3 to + 8.9	− 10.2 to + 13.2	− 7.4 to + 8.2

tising. Incorporating this aspect illustrates the step-by-step approach of adding new features to our analysis. An adequate representation of a complete company would, of course, be a very large task. In a single article one can only illustrate the possibilities.

Role in System

First we need a schematic diagram of the advertising and marketing behavior which is to be considered. Exhibit xi is oversimplified but will serve our present purposes. It includes the production-distribution organization set forth in Exhibit i and adds (1) an advertising function and (2) a special segment of the market, prospective purchasers. We assume that advertising expenditure decisions are based on (a) sales, particularly as reflected in the factory production schedule, and (b) excess inventories.

Now let us look more closely at the relationships shown in Exhibit xi:

¶ A time interval of six weeks is allowed for analysis of sales conditions, waiting to be sure of business trends, and the reaching of a decision to change the advertising expenditure rate.

¶ Acting on advertising decisions by agencies and media is assumed to require four weeks.

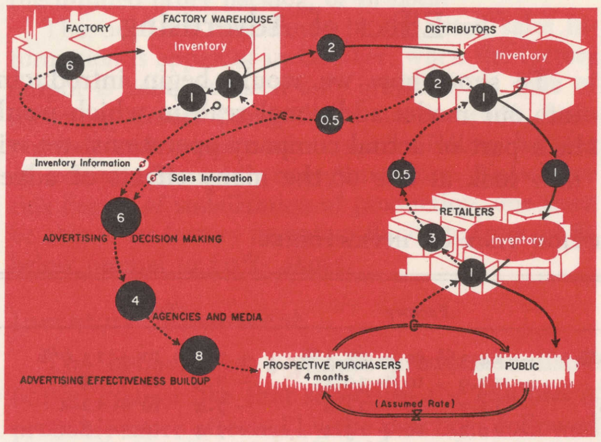
¶ Full buyer awareness of the advertising is assumed to develop over a period of eight weeks after the appearance of a new level of advertising.

¶ The rate at which consumers become prospective buyers is determined by such factors as the rate of wear-out of their existing household appliances and new housing construction.

¶ The average time that prospective buyers contemplate purchase and during which they are subject to advertising influence is assumed to be four months in the presence of an "average" level of advertising.

¶ Each 10% change in the advertising level is taken to shorten or lengthen by one week the four-month period during which the average prospective buyer considers his purchase.

Exhibit xi. Advertising and consumer market



Obviously, this underrates the impact of advertising; in reality it would normally affect not only the timing of purchases but also the size of the potential market. But even so, as we shall see, it plays a most important role in the working of the system.

Aggravated Production Cycle

Now we can compare the effect of different advertising policies. What are the implications of gearing the advertising budget to the incoming sales dollar, for example, rather than to a preconceived plan?

Figure C (see insert following page 52) shows the behavior of the system illustrated in Exhibit xi, when management's policy is to let advertising be proportional to expected sales, as represented by scheduled factory production. To see how the system responds to a market change, we here let the rate of appearance of *prospective* purchasers suddenly increase 10%. This builds up the pool of prospective purchasers, and gradually over 20 weeks the rate of consumer buying increases. Beginning with the twelfth week, factory production starts to rise; because of the factors earlier described, it exceeds the increase in retail sales.

The decision that leads to increased factory output also leads to a higher advertising commitment. This influences the pool of prospective buyers and further increases the retail sales between the eighteenth and thirty-sixth weeks. However, this retail sales rate, which rises in 43 weeks to 16% above the start, is 6% above the new rate of generation of prospective buyers. After the twenty-third week, when retail sales surpass the 10% increase in new prospective purchasers, the pool of prospective purchasers starts to decrease. The decreasing pool of prospective buyers becomes the controlling factor after 43 weeks, and retail sales start to fall despite a still rising advertising effectiveness.

We then see falling retail sales and a more rapidly falling production rate. The consequent fall in advertising commitments and effectiveness lets retail sales sink to a still lower level. Retail sales fall for nearly a full year, from October of the first year to September of the second year. In the meantime, retail sales have been below average demand since March of the second year because buyers are reverting to a slightly longer average wait before purchasing (since advertising has been cut back).

The pool of prospective buyers starts to rise in March of the second year; by September it overpowers the falling advertising effectiveness and retail sales start to rise again.

Because of the sequence of production swings that are greater than retail sales swings, advertising that is proportional to production plans, and a slight advertising influence on how soon the average prospective buyer makes his purchase, there is created a two-year production cycle which rises and falls some 30% above and below the new 10% increase in average demand. Furthermore, the amplitude of the production swings is gradually increasing and will do so until the swings are curbed by manufacturing limitations, lost sales due to delays in shipments, and other factors.

This pattern, with nearly two years elapsing between production peaks, reminds us of recent occurrences in household appliances and some other industries. It indicates that advertising is a *possible* cause of long cyclic disturbances. It would be premature to assume that advertising is *the* cause, however, until many other factors (e.g., consumer credit, sales forecasting methods, price discounting, and growth and saturation characteristics of the market) are also studied.

Longer-Run Effects

To make our study still more realistic, let us introduce a few more factors and project the behavior of the system over a longer period.

Figure D (see insert following page 52) portrays the results obtained from a more elaborate model of the company. It is based on the same kind of production-distribution-advertising system as that used in Figure C, and on the added assumptions that:

(1) Consumer purchases follow a random pattern which, while not affecting the long-run average of sales, does affect week-by-week sales.

(2) Factory production capacity is limited to 40% above the value of the initial sales level.

As in Figure C, a sudden, permanent 10% increase in the rate of appearance of prospective customers is followed by long, approximately two-year, fluctuations. Superimposed on this broad pattern are shorter disturbances which are caused by the system's response to random fluctuations in retail sales.

During the third year, production just reaches the maximum production capacity; not until the fourth and fifth years does the cyclic behavior of the industry reach an amplitude where the capacity limit is strongly evident. Periods of 8 months appear when the industry is producing at full capacity and much in excess of retail demand. These periods are likely to lead to production-capacity expansion, with consequent heightening of the peaks and deepening of the recessions in the following periods.

The preceding discussion has shown how deferrability of purchase can join with other influences (advertising in the above) to produce fluctuating conditions in an industry. Economic conditions and price discounts can also interact with purchase deferrability. For example, the farm equipment industry sells to a market which can defer purchases for several years. Sales are then concentrated, depending not only on years of farm prosperity but also on the history of machine usage and the existing inventory in the hands of the customer.

How can this "roller-coaster" effect be reduced? What can flatten out the cycles? Management policies themselves are an important influence. As it is, the production and inventory policies in this example are helping to accentuate the detrimental effect from the advertising policy. Yet advertising policy might have been most effective in smoothing out the peaks and valleys in Figures C and D if the effort had been consciously planned to stabilize sales. Another management tool for stabilizing production and sales is pricing policy.

Complete Analysis

I have indicated only in a crude way what can be learned from the study of a company's system of operations. The examples used have included simplified flows of information and materials but not capital equipment, manpower, or money. In actual practice, management would want to have all the latter elements included in the model.

For instance, by adding money flow, including the factors determining the cost of carrying inventory, of changing production levels, and of losses from special inventory clearance sales, executives would be able to get direct profit comparisons between various methods of operation. The model could include the restrictions imposed by available labor supply, maximum hiring and training rate, overtime, and policies regarding work-force reduction.

To set up a dynamic model for simulating company or industry behavior, one must adequately describe the real system which it represents. Getting the data will often be difficult. However, the kinds of information needed involve the basic characteristics of the company. We need factory lead time, inventory policies, construction time for new plants, union contract provisions, and advertising policy. These are the same items that one considers and tries to interrelate in the everyday management of a business.

In a complete company model more intangible items must also be specified, such as the likelihood of results from research expenditures, consumer response to advertising, and market behavior. Many of these quantities will be unknown at first; a few can be measured; some can be closely estimated; others will be guessed at. Today these factors are combined intuitively, and sometimes quite effectively, but the systems-analysis approach should help reduce the margin of error.

When the exact value of an operating characteristic cannot be established, one can test the system with a range of possible values. Sometimes the system will be found insensitive to most differences in value of a factor; in one of the cases earlier described, for example, a speed-up in data processing was found to have little effect on the behavior of the total operation. In such cases it does not make too much difference what assumption management directs the researchers to make about the activity so long as the quantity or relationship assigned is "in the right ball park." By contrast, if it is found that the system is highly sensitive, additional effort can be put into determining the correct value. At the very least, the risks of not knowing it become apparent.

Shape of the Future

Industrial dynamics is in an early stage of development. At a few isolated places, in both companies and universities, research is under way as an outgrowth of military or operations research projects, but the full impact of the new approach will not be felt for one or two decades.

What kinds of advances are in prospect? When are they likely to be made? Obviously, any predictions that we make are subject to the usual "if's" (business conditions, military pressures, and so on), but we can now foresee what is *probable.*

Two-Stage Progress

Progress will come in two stages. It is likely that the next five years will be devoted to exploratory research, development of basic analytical techniques, handling of enough specific industrial situations to demonstrate success, and establishment of new academic programs for training future managers.

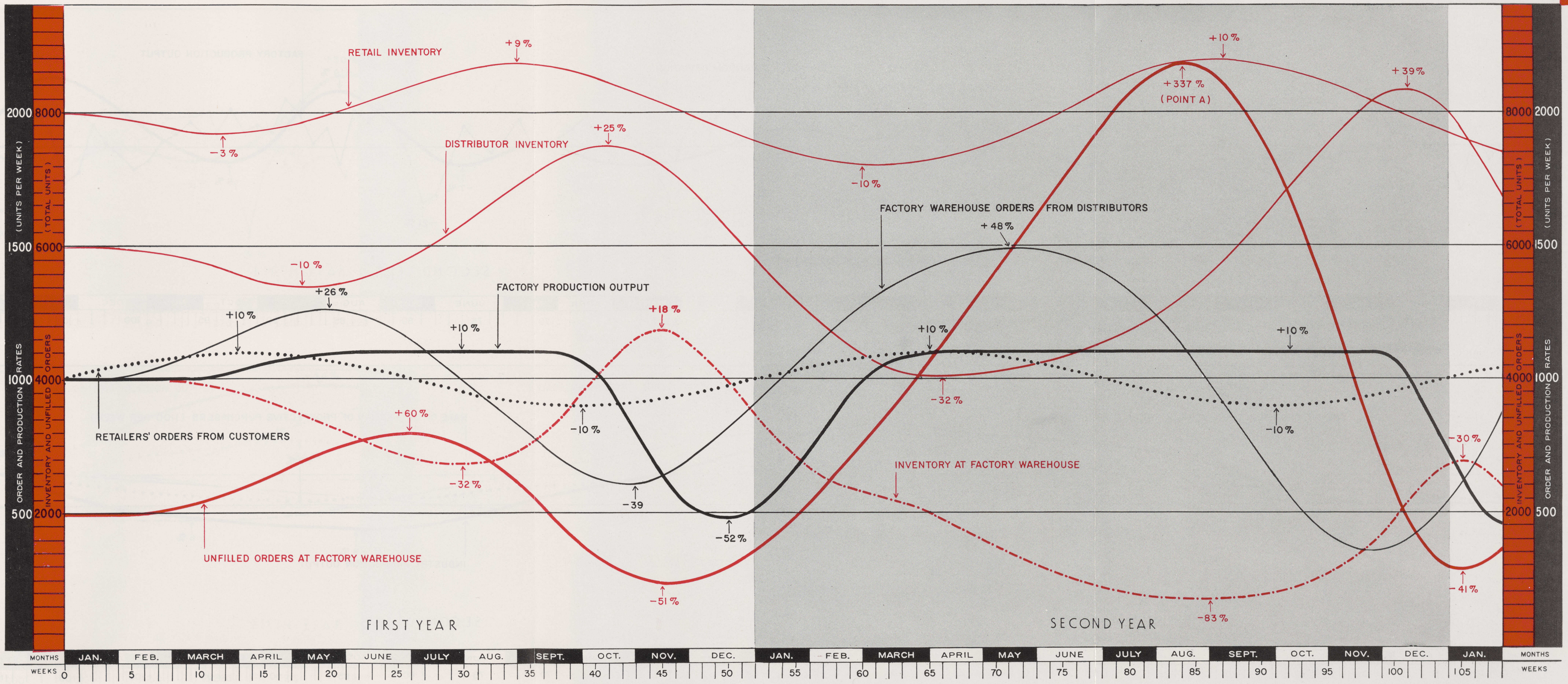
After this period there will come general recognition of the advantage enjoyed by the pioneering managements who have been the first to improve their understanding of the interrelationships between separate company functions and between the company and its markets, its industry, and the national economy. Competitive pressures will then lead other managements to seek the same advantage.

In other words, the company will come to be recognized not as a collection of separate functions but as a system in which the *flows* of information, materials, manpower, capital equipment, and money set up forces that determine the basic tendencies toward growth, fluctuation, and decline. I want to emphasize the idea of movement here because it is not just the simple three-dimensional relationships of functions that counts, but the constant ebb and flow of change in these functions — their relationships as *dynamic* activities. Such a concept is complicated and not easy to grasp. And, as with any major social change, a new approach to the management process will take time — time to develop ideas, time to test them, time to train a new generation of managers, and time for these men to reach positions of responsibility.

Many companies are already developing the background skills that will be needed. Computers that are now being acquired for other purposes can be used as systems simulators. Operations research departments can expand into the broader activity of studying management systems. Simulation techniques, feedback-control system theory, and analysis of decision processes are being developed in the engineering and research departments of many firms.

It is already clear that some companies will lead, rather than be overtaken by, these changes.

Figure A. Effect of fluctuating retail sales on factory with manufacturing capacity limited to 10% above average sales level



PULL OUT TO READ

Figure B. Effects of random deviations in retail sales on factory production

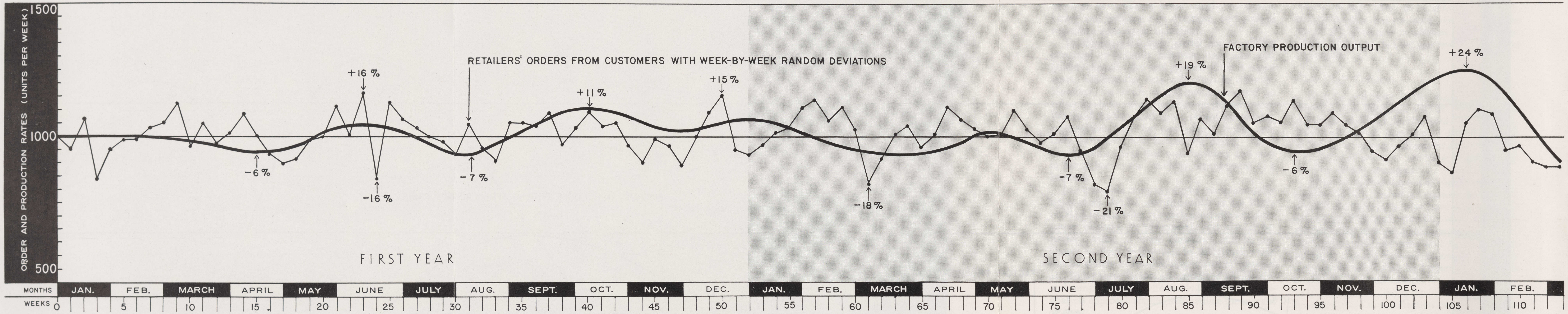
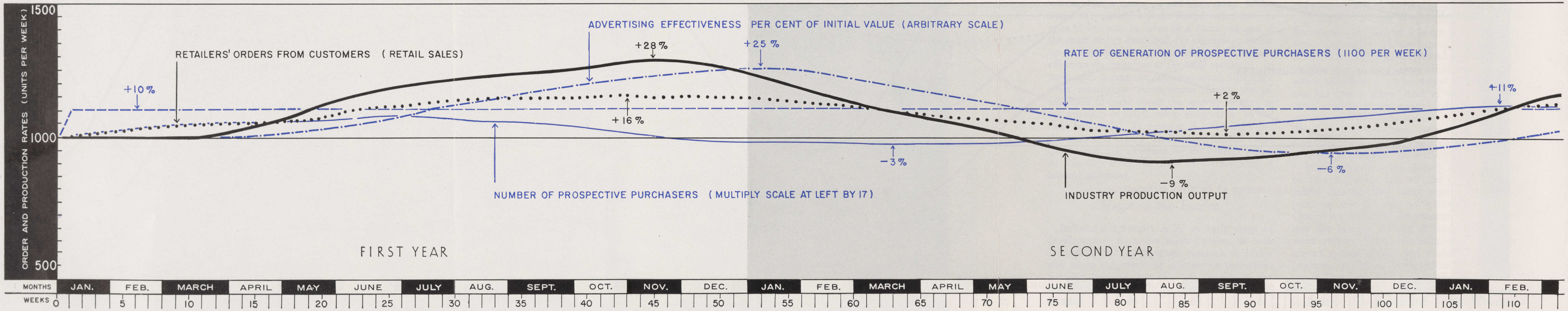
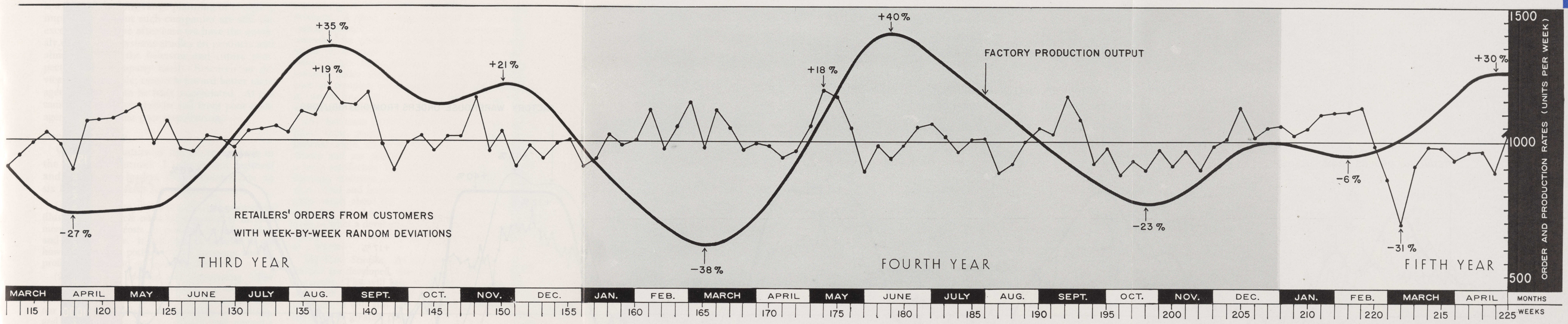
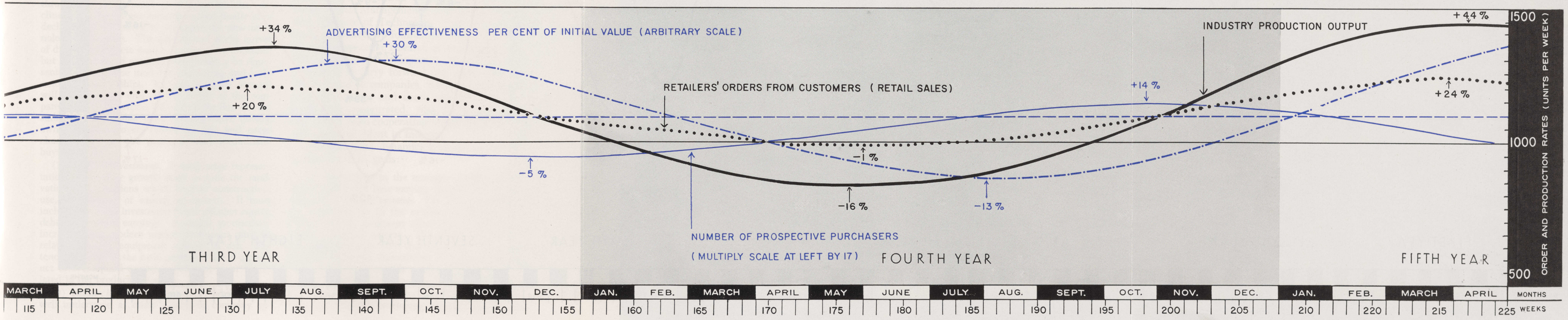


Figure C. Effect of advertising when geared to sales

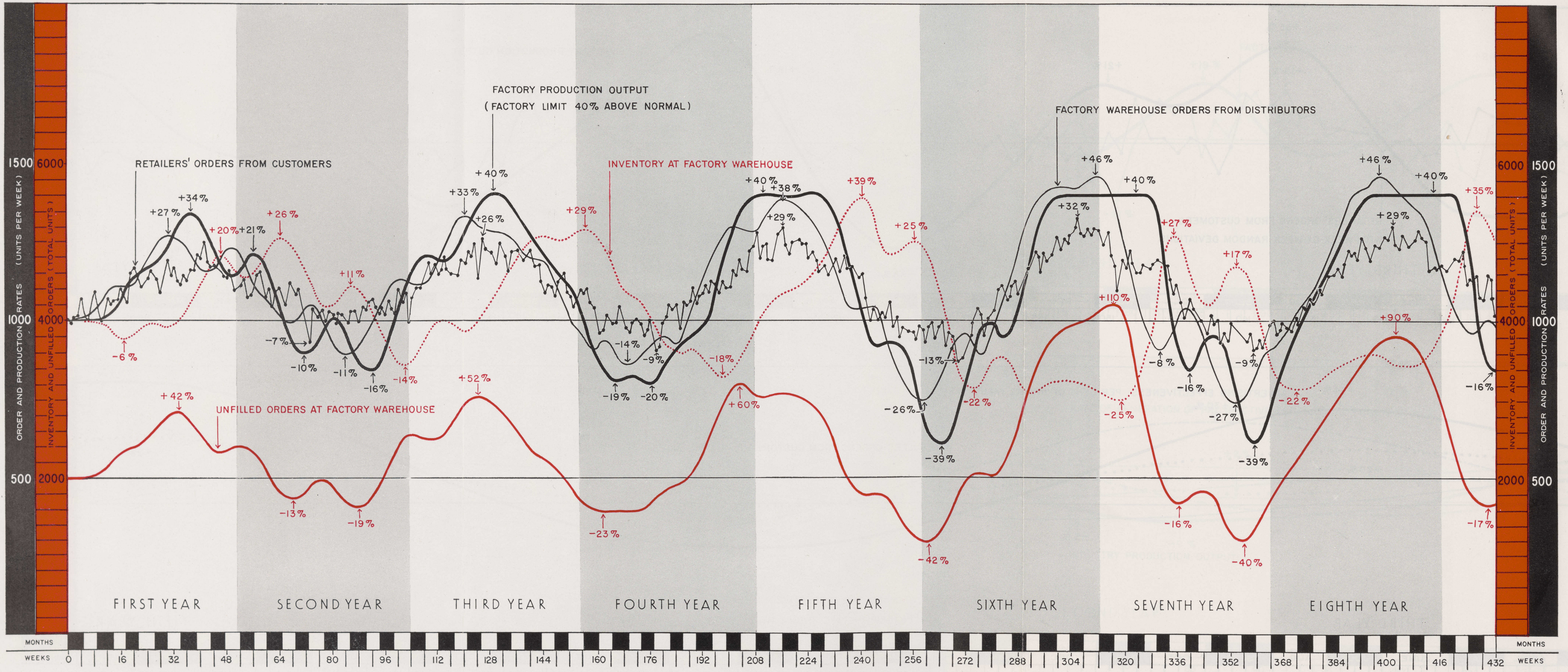






PULL OUT TO READ

Figure D. Composite model with advertising, a factory limit 40% above normal, and randomized retail sales



For them, research on the management process is a part of the long-range planning of company improvement. But such companies are still the exception. In case after case we have the anomaly of thorough systems studies on products and almost none on the behavior and future prospects of the company itself. Research on devices is fashionable; research toward better management has yet to be fully appreciated. At the same time more companies fail from poor management than from poor engineering.

Areas for Study

What is the nature of the job to be done in the immediate future? I believe that business and educational leaders should concentrate on six high-priority areas for research:

(1) *Money Flow.* Models such as those used to illustrate this article must be extended to include money flow and costs in addition to information and materials flow. It will then be possible to see how the financial position of the company and its profitability depend on operating decisions.

(2) *Consumer Market and Advertising.* The way now seems clear to establish an orderly behavioral structure to explain our experiences and our empirical knowledge of marketing, advertising, and consumer behavior. We should work toward a single unified market model to cover the entire life cycle of a product from the period of launching through sales and profit growth, profitability decline and sales volume saturation, and finally sales decline as the product is overtaken by new technological advances. The growth and decay curves of different products would, of course, be different; but large classes of products can probably be represented by the same model of marketing and technological relationships. In such a model account will be taken not only of the action of the company and its competitors but also of the flow of consumers from possible users to prospective buyers, owners, repeat buyers (based on life and usage characteristics of the product), and secondhand buyers (if any).

(3) *Capital Investment.* A long-range representation of company growth must include the motivations and decisions which lead to construction, use, and discard of capital equipment. It must include sources of investment funds whether equity, debt, or retained earnings. Potential productivity increases from modern capital equipment must be related to older equipment already in use. The tendencies during the early growth phase of a product to overexpand capital investment must be related to the product life cycle.

(4) *Research and Development.* While not so well understood as other aspects of company operations, research and development are crucial to company growth and to a succession of profitable products. A clear understanding of the relationship of research to profit and capital investment is necessary for proper representation of long-term forces on company growth. The very long lead times, longer than generally supposed, between research policy decisions and the resulting influence on company profit must be properly related to current financial resources, production capacity, and market development.

(5) *Economic Conditions.* Although, as the exhibits show, major company crises are often of internal origin, others are initiated by outside conditions and are amplified internally. Ultimately, a truly satisfactory explanation and prediction of company evolution must incorporate the influences of national and international economics. We now know much about the separate parts of the economy, but we should not trust our intuitive estimate of what a development here or a change there implies for the behavior of the economic system as a whole.

(6) *Case Studies.* As the methods earlier described are developed, they must be tested in real situations. This will help us develop the means to cope with the complexities of actual management problems; it will also help us to guide research to the areas of greatest payoff.

The six preceding areas are being incorporated in the Industrial Dynamics Program at the Industrial Management School of Massachusetts Institute of Technology. The research is being sponsored by foundation funds and also through cooperative research arrangements with interested industrial companies. To provide men competent to carry forward this work, both in research and in companies, industrial dynamics graduate courses and thesis research have already been inaugurated. Also, direct participation of men from cooperating companies will provide nuclei of talent in industry.

The industrial dynamics program at M.I.T. is aimed at four goals:

(1) To develop in the manager a better intuitive feel for the time-varying behavior of industrial and economic systems. The study of particular situations, as in the exhibits in this article, improves one's judgment about the factors influencing company success. The results are beneficial even before one feels that he has reached an accurate quantitative formulation of company behavior that will have reliable predictive value.

(2) To provide a background showing how the major aspects of a company are related to one another, so that the developing manager can derive

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

the greatest benefit from his work experience. His operating experience should be more meaningful if he has a better understanding of how his immediate environment is related to the other company functions.

(3) To help predict the future course of an existing organization. Very often present decisions determine future company welfare five years or more ahead. We have seen in the exhibits how underlying, unchanging characteristics of a company can cause erratic manifestations in operations. We can hope, eventually, for better ability to see where present company practices will lead.

(4) To improve the future prospects of a company. Beyond prediction lies the ability to redesign an organization and its policies so that it stands a better chance of success. We have seen in the exhibits a preview of how one might explore and evaluate proposed changes in the company.

Effect on Manager

What do these new developments mean to the manager? How will they affect his position in the company?

Just as automation requires new skills at the worker level, so will improved methods require new abilities at the management level. The executive of the future will be concerned not so much with actual operating decisions as with the *basis* for wise operating decisions. He will be concerned not so much with day-to-day crises as with the establishment of policies and plans that minimize emergencies.

He will be able to do these jobs with a considerable amount of sophistication and skill. He will have a basis for understanding the implications of market trends and the probable behavior of consumer demand. He will recognize the ebb and flow of forces which interact to generate fluctuating economic conditions. He will be able to relate the changing factors in research, investment, and marketing to the life cycle of a product. He will be able to relate different financing methods with their relative advantages and risks to the uncertainties of market and economic conditions. He will know how to devise organizational forms that are efficient and encourage the creativity of people.

We can expect changes in various management responsibilities. There will be a merging of many line-and-staff functions. Today, the kind of systems studies discussed in this article would be the province of staff specialists. Yet the results will determine the future success of the company; the conclusions will be only as valid as the assumptions on which the studies are based, the answers no more pertinent than the questions asked. Therefore, systems planning must, in time, be the tool of the responsible manager. It is by nature a thoughtful process of weighing the past and present, not to reach the immediate decision of the moment but to derive guiding principles for the future.

All of this will lead to a more decisive separation of policy making from operations, with the dividing line much lower in the organization than at present. New ways to predict the interaction between company functions will speed up the movement toward decentralized management. In the past power has been concentrated at the top in order to improve over-all integration and control. With increased understanding of the company operation, we can expect that improved definition of objectives and more pertinent standards for the measurement of managerial success will permit managers at the lower levels to take on more operating responsibility. Senior executives will then be free to give more attention to product innovation, economic conditions, and the organizational changes that will enhance man's creativity.

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

Course Outline 15.581

1958 Summer Session
Material

Memorandum D-23

Course Outline 15.581

1958 Summer Session Material

Memorandum D-23

23 September 1958

Course 15.581 General Description

The behavior of firms or industries can be represented by feed-back type models representing the basic decisions of a firm, the information and material channels and the delays in the channels. The Industrial Dynamics Research group is constructing models of this type and is developing techniques for analyzing the models by computer simulation. The course covers the work of this group as well as related work in economics and operations research.

We will take up models constructed by the SIM group having to do with ordering, production, prediction, and pricing and will also indicate areas for future research by covering related work in the areas of investment, cash flows, employment, and consumer behavior.

Models of this type promise to be useful both in company policy-making and forecasting through improving our understanding of the dynamic behavior of industry. The purpose of the course is to contribute to this understanding as well as to give the student experience in setting up and simulating such models.

The course will include class simulation of multi-industry systems, study of existing models and simulation technique, simulation of individual models and study of related work that points the way to future research.

23 September 1958

15.581 Course Outline and Reading List

1. Introduction to Course (1 period)

   Forrester, "Industrial Dynamics," Harvard Business Review, July-August 1958 (reprint)

2. Introduction to Simulation and Formulation (2 weeks)

   A. Class simulation--no specific rules

   B. Individual simulation

      (1) with own rules

      (2) with agreed upon rules, mathematically stated

3. Study of Existing Models (4 weeks)

   A. Accounting relations--Forrester, "Formulating...Models..." I.D. Memo D-16

   B. Delays--Pugh, "N-th Order Delays," I.D. Memo D-7

   C. Ordering--"Model 9 Equations," I.D. ditto

   D. Prediction--*Holt, "Forecasting Requirements from the Business Standpoint," Carnegie Institute of Technology, dittoed, 1957

      Yance--"Shoe Model 2," I.D. Memo D-20, dittoed

      Holt--"Forecasting Seasonals and Trends by Exponentially Weighted Moving Averages," C.I.T., dittoed, 1957

   E. Production--Holt, et al, "...A Linear Decision Rule for Production and Employment...", Management Science, October 1955, December 1955

   F. Price behavior--Allen, Mathematical Economics, Chapter 1

4. Simulation Technique (4 weeks)

   A. Techniques of using general purpose program

   B. Formulation of models for simulation

   C. Simulation

5. Application of Models (1 week)

   A. Measurement of Parameters

   B. Evaluation of Policies

6. Study of Additional Areas (5 weeks)

---

* Supplementary references, noted by an asterisk, indicate useful related or background material.

A. Consumer Behavior

(1) Durable goods demand--Stone "...Demand for Durable Goods," *Econometrica,* July 1957
Chow, *Demand for Automobiles in the U.S.*, 1957

(2) Effect of advertising--Vidale, "...Sales Response to Advertising," *Operations Research,* June 1957

B. Financial and Investment of the Firm

(1) Cash flows--* Autio, *Dynamic Cash Flows,* M.S. Thesis, SIM, 1958. * Greenlaw, *Criteria...in the Management of Cash,* M.S. Thesis, SIM, 1957

(2) Investment behavior

Koyck, *Distributed Lags and Investment Analysis,* 1954

Chenery, "Overcapacity and the Acceleration Principle," *Econometrica,* January 1952

Fujiyama--*Capital Expansion in the Electric Power Industry,* M.S. Thesis, SIM, 1958

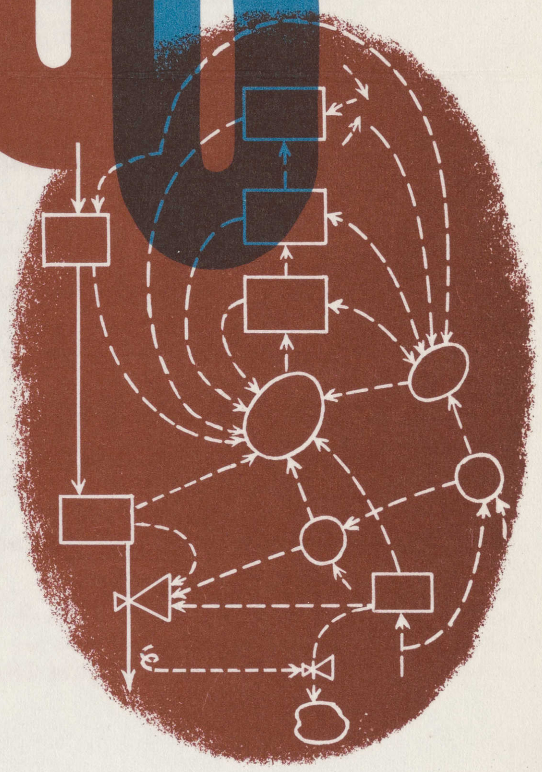
(3) Dividend Behavior

Lintner, "Distribution of Incomes of Corporations..", dittoed

C. Other topics

1958 Summer Session Material

INDUSTRIAL DYNAMICS



Massachusetts Institute of Technology
August 18 — August 29
Summer Session 1958

Until today, management education and management practice have been compartmentalized into such classic subdivisions as finance, marketing, production, and accounting. However, the major problems of the industrial manager arise not so much from the separate components of the company as from how these interact to produce instability in prices, production, employment, and profits. Building on the development of electronic computers, on military advances in understanding decision-making and in using simulation to solve complex systems, and on twenty years of research in feedback control systems, we may now develop new ways to analyze the industrial enterprise. Management is on the verge of a major breakthrough in understanding how industrial company success depends on the interaction between flows of information (including policies and decisions), materials, money, manpower, and capital equipment. To help provide a basis from which to understand the forces causing industrial fluctuation, growth, stability, and decline, the School of Industrial Management will offer a two-week Special Summer Program on **INDUSTRIAL DYNAMICS** *from Monday, August 18, through Friday, August 29, 1958.*

Outline of Program

In this Special Summer Program, models of the industrial company will be formulated and simulated on the I.B.M. 704 computer in the M.I.T. Computation Center. Emphasis will be on how organizational structure, policies, decision criteria, information distortion, and the time lags in information and material flow affect growth and can create fluctuation in sales, accentuation of seasonal tendencies, and amplification of external economic conditions.

The Program will be based on two years of industrial dynamics research, on a course for graduate students taught in 1957-58, and on a course for Sloan Fellows given in the spring of 1958. Principal attention will be given to how the significant aspects of the company can be represented, drawing on the prior experience of the participants.

This will be an intensive session devoted to method, rather than a descriptive survey course. It should be taken only by those who expect actively to work in industrial dynamics during the next several years, for the field has not yet reached a point where the participant can expect to make immediate profitable application of what he learns.

Qualifications of Applicants

The Program will be limited to about 35 participants. Applicants must have a wide interest in and experience with the problems of the company as a whole and the interaction of the separate industrial functions of research, engineering, production, marketing, and finance. Thereafter, preference will be given to those who have had mathematics (through differential equations), an introduction to digital computer programming, and practical or educational contact with feedback control systems. Applications should show industrial experience and the extent of training in mathematics, digital computation, and feedback control.

Tuition for the two-week Program will be $325, due and payable upon notification of admission. Academic credit will not be offered. Exercises will be held from 9 to 12 and 2 to 5, Mondays through Fridays.

Staff

This Program is under the direction of Dr. Jay. W. Forrester, Professor of Industrial Management. The staff will include Assistant Professors Joseph V. Yance and John L. Enos, and other staff members of the industrial dynamics group of the M.I.T. School of Industrial Management.

Special Summer Programs—1958

	BEGINNING TUESDAY, JUNE 17
2.269	Metal Cutting
2.379	Behavior of Metals and Design Requirements for Elevated Temperatures
3.149	Casting of Light Metals*
6.569	Switching Circuits
	BEGINNING MONDAY, JUNE 23
2.219	Random Vibration
2.459	Thermodynamics for Engineers and Chemists
15.909	Organization for Research and Development
16.919	Aeroelasticity
17.409	The Dwelling House—An Emerging Technology
	BEGINNING TUESDAY, JULY 8
1.619	Cavitation Mechanics in Engineering Practice
2.138	Strain Gages: Fundamentals*
2.529	Modern Developments in Heat Transfer
2.789	Fluid Power Control
6.529	Electrical Energy Conversion and Control*
7.899	Modern Research Methods in Biology and Medicine
13.039	Stability and Control of Ship Motion
M379	Introduction to Probability Theory and its Applications to Operations Research
	BEGINNING MONDAY, JULY 14
2.139	Strain Gages: Applications*
5.168	Physics of Infrared Radiation
N209	Principles of Radioisotope Utilization
	BEGINNING MONDAY, JULY 21
4.049	Color in Art and Science
4.689	City and Regional Planning
6.609	Advanced Feedback Control Theory
15.719	Mathematical Analysis and Simulation in Industrial Operations
	BEGINNING MONDAY, AUGUST 4
5.163	Infrared Spectroscopy: Technique*
6.628	Microwave Ferrites
14.379	Design and Analysis of Scientific Experiments
15.899	Planning Marketing Strategy and Tactics
	BEGINNING MONDAY, AUGUST 11
5.164	Infrared Spectroscopy: Applications*
19.329	Air Pollution
	BEGINNING MONDAY, AUGUST 18
2.939	Techniques in Textile Research*
3.539	Metallurgical Applications of X-ray Diffraction
15.589	Industrial Dynamics
	BEGINNING MONDAY, AUGUST 25
2.929	The Mechanics of Textile Structures*
	BEGINNING TUESDAY, SEPTEMBER 2
1.369	The Design and Construction of Earth Embankments
2.739	Design and Dynamics of Mechanisms
6.627	Problems of High Powered Radar Design
E359	Editing Technical Reports

Programs marked with an asterisk (*) are of one week's duration. All other Programs will extend through two weeks.

GENERAL INFORMATION

Admission

In order to maintain highest standards, the enrollment in each Special Summer Program is limited according to the facilities and staff which are available. If a Program is not oversubscribed, applications for it will be considered up to two weeks before it begins. However, early application is strongly advised. Application forms are available from the Director of the Summer Session, M.I.T.

The Institute reserves the right to select those applicants whose qualifications and experience suggest that they will receive the most benefit from the Program for which they are applying. Neither admission nor dormitory reservations are transferable except by specific Summer Session Office authorization, and then only when evidence of the qualifications of the proposed substitute have been filed in advance.

A limited number of Special Summer Program Scholarships are available to defray in part the Program tuition of members of teaching staffs (rank of instructor or higher) of other educational institutions. Requests for such scholarships should accompany applications for admission.

Payments

Applicants whose admission has been approved will be notified as early as possible and will receive Bursar's Remittance and Registration Cards. Both cards, together with full payment for tuition and housing in check, draft, or money order made payable to W. A. Hokanson, Bursar, should be returned immediately to the Office of the Summer Session, Room 7-103, Massachusetts Institute of Technology, Cambridge 39, Massachusetts; they must be received one week before the opening of the Program. The amounts of tuition and room rent will be stated in the notification of admission; both may be included in one remittance, and both must be paid in full before the opening session of a Program. No bills are sent.

Those whose tuition is to be covered by contract with a government agency or with industry should apply for admission to the Office of the Summer Session in the usual way, as promptly as possible, noting on their applications that a contract or agency approval is being requested. At the same time, they should arrange to have the *agency* send a purchase order or other evidence of its intention to authorize tuition payments to the Office of the Summer Session. This purchase order must reach the Office of the Summer Session at least one week before the Program begins. It should include the full name of the prospective registrant(s), as well as the name of the Program. If it is not sent, and if the Program is a limited one for which others have applied, the Institute reserves the right to cancel admission.

Since housing charges (if any) are not covered by government contracts, these payments must be received independently from prospective registrants at least one week before the beginning of a Program.

Please note that a contract from industry or a government agency does not reserve a place in a Special Summer Program; it must be preceded or accompanied by a completed application for admission.

Refunds

Registrants who notify the Summer Session Office of cancellation of their plans before they arrive in Cambridge will be refunded their payments. However, no refunds of either tuition or rent will be made to those who leave before completing a Program in which they have been registered, and refunds of room rentals cannot be made for Saturdays, Sundays, or holidays during a Program.

Registration

Each person admitted to a Program should report at the Summer Session Office, Room 7-103, at 8:30 a.m. on the day that his Program begins. At this time he completes formal registration, receives a receipt for payments, and is given cards of admission to the Program. Detailed information about special M.I.T. facilities and services will be available, as will printed guides to Metropolitan Boston and other New England regions.

Mail and Messages

Registrants for Special Summer Programs may have mail and messages addressed to them at the Office of the Summer Session. Please have the name of the Program included in the address.

Dormitory Accommodations

The M.I.T. dormitory system will be available to all registrants in Special Summer Programs. Rooms for single men, as well as a limited number of accommodations for married couples and single women, will be reserved if specifically requested in the application for admission. Applicants who wish other accommodations should correspond directly with Boston or Cambridge hotels.

For single men or women, the housing rate in an M.I.T. dormitory is $3.50 per person for each 24-hour unit, beginning at 6 p.m. The rate for married couples is $2.50 per person. In general, reservations will be available from 6 p.m. of the Sunday night preceding the opening of a Program until 6 p.m. of the Saturday following its conclusion. Dormitory reservations will be confirmed in the letter of notification of admission; this letter should be presented at the Institute House upon arrival. Please include payments for dormitory accommodations with tuition payments, made at least one week in advance. Registrants in Special Summer Programs should make no rent payments at any dormitory office.

Each House is open 24 hours every day. Children under twelve years of age may not be housed in any dormitory during the Summer Session, nor are pets allowed. No dormitory employee is authorized to modify these regulations.

College dining facilities on the campus will be available to Special Summer Program registrants and their guests. In addition, there are restaurants within easy walking distance of the Institute, and many famous eating places and hotels of Metropolitan Boston are situated just across the Charles River Basin.

Ample free outdoor parking space near the dormitories is available during the summer for those attending Special Summer Programs.

Recreational Facilities

Summer guests are welcome to use the Institute's libraries and recreational facilities, including tennis courts, softball fields, squash courts, the Alumni Swimming Pool, and a fleet of small sailboats operating from the M.I.T. Sailing Pavilion.

Within easy access of the Institute campus are some of the world's finest museums, libraries, and art galleries. The famous Hatch Memorial Shell, outdoor summer concert hall for the Boston Pops Orchestra, is just across the Charles River Basin. Major League baseball games are scheduled throughout the summer at Fenway Park, only a short ride from the campus. Several summer theaters and the historical areas of Salem, Concord, Lexington, and Plymouth are within easy driving distance, while Cape Cod, the White Mountains of New Hampshire, the Green Mountains of Vermont, and many parts of the Maine seacoast may easily be visited on week-end trips.

Design by the M.I.T. Office of Publications (GA); art work by the M.I.T. Illustration Service. Photon typography by the Machine Composition Company; production by the M.I.T. Photographic Service.

List of Participants
Industrial Dynamics Summer Course
Massachusetts Institute of Technology
August 18 - 19, 1958

Name	Company	Title
Carlson, Bruce R.	Sprague Electric, North Adams, Mass.	Statistical Assistant to President
Dzielinski, Bernard P.	I.B.M., Yorktown Heights	Associate Engineer
Galloway, R. Lynn	Eastman Kodak, Rochester, New York	Assistant Comptroller
Greene, George M.	Hughes Aircraft, Culver City, California	Assistant to the Vice President, Engineering
Greenlaw, David S.	Eastman Kodak, Rochester, New York	Assistant Technical Advisor to the General Manager
Henderson, George P.	Imperial Oil, Toronto, Canada	Researcher (Math., Stat., Computation)
Learson, Vincent	I.B.M. New York City	Vice President
McDonald, William Q.	Sprague Electric, North Adams, Mass.	Systems and Procedures Analyst
Middleswart, Frank F.	du Pont, Wilmington, Delaware	Director, Systems Engineering Division
Muije, Cornelius S.	Proctor and Gamble, Cincinnati, Ohio	Systems Analyst
Newchurch, Edwin J.	Esso Engineering Company Linden, New Jersey	Assistant Director Petroleum Development Division
Steffens, Lester	Socony Mobil Oil Co. New York City	Economist

Name	Company	Title
Swanson, Arthur K.	U. S. Air Force Tinker Air Force Base Oklahoma	Colonel
Ward, Edward J.	Wright Air Development Center, Dayton, Ohio	Allocation of funds and manpower in R & D.
Wells, Howard A.	Wright Air Development Center, Dayton, Ohio	General Engineer, Program Planning
Zito, Arthur J.	General Electric, Syracuse, New York	Operations Research Specialist

Schedule of Industrial Dynamics 1958 M.I.T. Summer Session

Course 15.589

August 18 through August 29

Monday, August 18

9:00 - 10:00	Registration, coffee, class members, meet staff, class members fill out questionnaire.
10:00	General objectives of course --Forrester
10:15	Industrial Dynamics I -- Forrester
11:45	Explain lunch arrangements.
	Industrial Dynamics II
2:00	Discuss examples in HARVARD BUSINESS REVIEW article--Forrester.
3:30	Recess
	Class Simulation I -- Forrester and others
3:45	Information feedback experiments
4:45	Discuss evening assignment. Disagreements and questions regarding HBR paper. Discussion of the most serious and important management problem in the class -- member's company or industry.
5:00 - 6:00	Faculty Club

Tuesday, August 19

9:00	Discussion of homework and answer questions.
9:30	Class Simulation II -- Pugh Simplified Production-Distribution System
10:30	Coffee
10:45	Class Simulation III - Pugh Continue previous exercise.
12:00	Lunch
1:30	Formulating System Description I -- Forrester Equation notation to be used Review of Memo D-16 Discussion of how to set up calculation sequence

3:00 Recess

3:15 Formulating System Description II -- Forrester
Equations 1, 2, 3, 4, 5
Overt and implicit decisions

4:45 Evening assignment
Read Memo D-16.
Simulation of ordering rule

Wednesday, August 20

9:00 Questions -- Discussion

9:15 Formulating System Description III -- Forrester
Smoothing methods
Moving average
Exponential
Non-linear
Equation 6

10:30 Coffee

10:45 Formulating System Description IV -- Pugh
Representing Delays

12:00 Lunch

1:30 Formulating System Description V -- Pugh
Delays (continued)

2:15 Formulating System Description VI -- Forrester
Equation 7

3:00 Recess

3:15 Formulating System Description VII -- Forrester
Equations 8, 9, 10, 11, 12

4:30 Discuss evening home problem -- Yance
Individual paper simulation of simple production-distribution system.

Thursday, August 21

9:00 Discussion of home problem

9:30 Formulating System Description VIII -- Forrester
Equations 13, 14, 15, 16
Mention of Equations 17 through 32 as similar to through 16.

10:00 Digital Computer I -- Pugh
General characteristics of digital computer and simulation.

10:30 Coffee

10:45 Digital Computer II -- Pugh
Elementary example to be simulated on IBM704.
Equations
Steps required

12:00 Lunch

1:30 Formulating System Description IX -- Forrester
Equations 33 through 46

3:00 Recess

3:15 Digital Computer III -- Pugh and others
Go to computation center.
Observe steps in getting results from the example.
Homeword -- reading

Friday, August 22

9:00 Formulating System Description X -- Forrester
Computer results from production distribution part of system

10:15 Coffee

10:30 Class Simulation IV -- Fey and Forrester
Factory scheduling, personnel, money flow, backlog, delivery delays

12:00 Lunch

1:30 Class Simulation V
Continue morning session.

3:30 Recess

3:45 Formulating System Description XI -- Forrester
Equations 47 through 53

4:45 Weekend assignment
Reading
Describe in detail, to show the dynamic interplay, a situation or problem from your own past experience which you feel best illustrates the information feedback characteristics being discussed in this course.

Monday, August 25

9:00 Class Simulation VI -- Fey
Discussion and interpretation of results of class simulation sessions IV and V.

9:45 Industrial Dynamics III -- Enos
Dynamics of consumer market

10:30 Coffee

10:45 Industrial Dynamics IV -- Enos or Amstutz
Consumer market continued

12:00 Lunch

1:30 Formulating System Description XII -- Forrester
Equations 54 through 57

2:15 Digital Computer IV
Class sub-groups study variation on Harvard Business Review curves by getting computer simulation results.

Tuesday, August 26

9:00 Industrial Dynamics V -- Yance
Adding price behavior to production-distribution model. General description, types of behavior

10:30 Coffee

10:45 Class Simulation VI -- Forrester
Copper mining

12:00 Lunch

1:30 Digital Computer V
Sub-groups continue
Computer simulation

Wednesday, August 27

9:00 Industrial Dynamics VI -- Enos
Models of the national economy.
History and literature

10:15 Coffee

10:30 Industrial Dynamics VII -- Roberts
Information and decision flow in research and development management.

12:00	Lunch
1:30	Industrial Dynamics VIII -- Forrester Research and Development Management
3:00	Recess
3:15	Digital Computer VI Sub-groups work on problems.

Thursday, August 28

9:00	Industrial Dynamics IX -- Enos New approaches to dynamic behavior of national economy.
10:15	Coffee
10:30	Industrial Dynamics X -- Fey A case study in information, scheduling, and order characteristics.
12:00	Lunch
1:30	Industrial Dynamics XI -- Yance A study of ordering, price, and supply relationships in the hide-leather-shoe industry.
3:00	Recess
3:15	Digital Computer VII Sub-groups continue work.
6:00	Group dinner -- Faculty Club

Friday, August 29

9:00	Industrial Dynamics XII -- Enos National Economy continued.
10:30	Coffee
10:45-5:00	Open for previous topics not completed, sessions arranged to meet special requests of the class, discussion, future expectations in management practice and education.

List of Theses

Memorandum D-16

Memorandum D-23

List of Theses

Memorandum D-16

Memorandum D-23

Massachusetts Institute of Technology
School of Industrial Management
Cambridge 39, Massachusetts

Industrial Dynamics Research

MEMO: D-16

SUBJECT: Formulating Quantitative Models of Dynamic Behavior of Industrial and Economic Systems, Part I.
(Part II to appear as a separate memo)

WRITTEN BY: Jay W. Forrester

DATE: April 5, 1958

ABSTRACT

This memo discusses an approach to analyzing the stability and fluctuation of industrial and economic systems that recognizes their closed-loop, information-feedback characteristics and their decision-making procedures. Five interacting sub-systems are used--information flow (including overt and implicit decisions), material flow, money flow, capital equipment generation and usage, and manpower employment and mobility.

The situations to be studied can usually be realistically represented only by very non-linear mathematical systems and will be so formulated. The system formulation can be unstable if the world it represents is unstable. It can incorporate hundreds or even thousands of variables to achieve sufficient reality to be useful. The computation needed for solution is only proportional to the number of variables and lies well within the capability of today's computers. The immediate goal is to find improved but not "optimum" system design; we have no meaningful definition of an "optimum" in highly non-linear systems nor any method of proving that any existing system behavior is the best achievable.

The mathematical formulation and notation is continuously evolving. Section D discusses the present grouping of equations into three classes of variables--levels, auxiliary variables, and rates. Time notation, flow diagram symbols, and length of solution interval are discussed.

Part II, as a separate memo, will show in detail how the generalities discussed in this memo are applied to the illustrative example outlined in section B of this Part I. It will contain sections on (1) detailed mathematical description of the example; (2) kinds of results obtainable; and (3) areas of future extension of the industrial dynamics research.

Contents

FORMULATING QUANTITATIVE MODELS OF DYNAMIC BEHAVIOR OF INDUSTRIAL AND ECONOMIC SYSTEMS

Part I. General Procedure

A. Introduction

Management science now stands at a frontier leading to a new understanding of industrial growth and stability. This new insight into the underlying factors that control industrial and economic behavior will come from an ability to analyze the time-varying interactions between the major facets of industrial activity.

This study of the industrial enterprise as an integrated system, relating the flows of information, materials, money, manpower, and capital investment, is here referred to as "industrial dynamics". Understanding the way these five flow systems interlock to amplify and control one another will take us far in explaining the effects of decisions, policies, organizational forms and investment plans.

Industrial dynamics will help us understand how underlying, unchanging characteristics of an enterprise like its organizational form and its management policies can cause production fluctuations, over-expansion of factory capacity, failure to keep pace with technological change, and the other dramatic manifestations of industrial difficulty.*

---

* For a more general discussion of the background of industrial dynamics and its importance to the future of management, see:

1. Jay W. Forrester, "Systems Technology and Industrial Dynamics", Technology Review, Vol. 59, No. 8 (June 1957), Massachusetts Institute of Technology.

2. Jay W. Forrester, "Industrial Dynamics--Understanding the Forces Causing Industrial Fluctuation, Growth, Stability and Decline", a manuscript dated February 9, 1958, privately circulated by the MIT School of Industrial Management, and submitted for the July-August 1958 issue of the Harvard Business Review.

The analysis of industrial systems herein discussed grows out of four developments which had their start in the country's military research effort. The first is the concept of servomechanisms, or information feedback systems, as developed during World War II.[3,4] Second, are the digital computer and electronic data processing products of post-war military projects. Third, is the better understanding of the decision-making process which became necessary in automatizing high-speed military tactical operations. Fourth, is the use of simulation to give empirical answers to problems that are far too complex for explicit mathematical solution.

It now appears possible to formulate industrial behavior with sufficient reality to cope with the interactions of production, distribution, advertising, research, investment, and the consumer market. Within such a formulation, both physical and psychological factors must be included. The ultimate goal is to take our literature and knowledge of "descriptive management" and formalize what we know about the separate parts. It then will become possible to determine the implications of how these parts interact in the complete system.

As we will see later, one is unlikely to succeed in deducing the internal characteristics of a complex social system by working backward from its external manifestations. We propose, therefore, <u>not</u> to follow the approach common in the field of econometrics where an attempt is made to derive the laws of the system from statistical analysis of over-all behavior. Time-series correlation and regression analysis methods are not valid when applied to non-linear, potentially unstable, noisy, closed-cycle information feedback systems. Yet, the important characteristics that interest us in economic and industrial behavior probably arise primarily from the non-linearities, random disturbances, and interconnected flow loops.

---

[3] Brown, Gordon S. and Campbell, Donald P., <u>Principles of Servomechanisms</u>, John Wiley & Sons, 1948.

[4] Tustin, Arnold. <u>The Mechanism of Economic Systems</u>, Harvard University Press, 1953.

Instead, therefore, the analytical approach followed here builds upward from the level of everyday human activity. We will formally describe those actions and situations which we observe in normal economic operations. These individual aspects of the system are then combined to determine how the separate parts interact to create the over-all system behavior. We shall see that apparently volatile, erratic, external manifestations can actually result directly and unavoidably from stable unchanging organizational forms, from the times required to make decisions and transmit goods and information, from enduring operating policies, and from the choice of information sources used for decision making.

In the sections that follow and in Part II, I will use an example of a very simplified production-distribution-advertising-marketing system as a vehicle around which to discuss the general analytical principles. In the remainder of Part I is a verbal description of the example, a general discussion of the formulation of a quantitative model, and an explanation of the types of equations to be used. In Part II, the generalities will be applied to the example by developing a full set of equations for information and material flow and by illustrating the results which can be obtained.

B. Description of a Simplified Example

Analysis methods in the later sections will be illustrated by application to the situation of Figure 1. Here are a factory, a factory warehouse, distributors, retailers, prospective purchasers, and advertising. To begin study of this system, we would need to know three things--organizational form, delays in information and material flow, and the purchasing and inventory policies being followed.

ADVERTISING AND CONSUMER MARKET

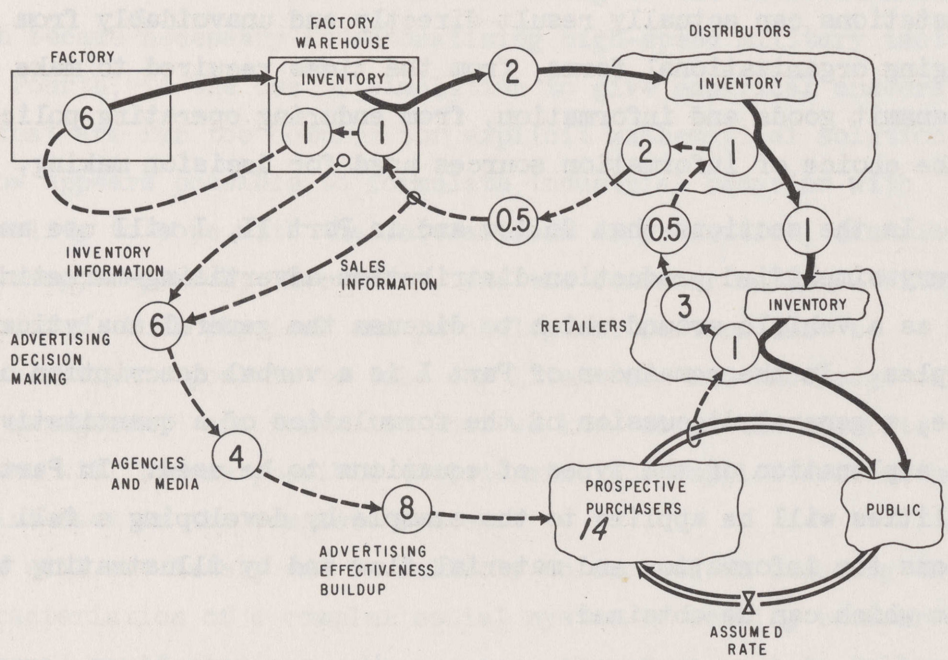


FIG. 1

1. Organization

We must know the organizational form of the information and material flow channels. These are illustrated in Figure 1. Dotted lines show information flow, and the solid lines the flow of materials. Information, in the form of orders, flows upward in the distribution system. Also, information, in the form of advertising authorizations and actual advertising presentations to the public, flows downward in the advertising loop.

Materials flow from production line to factory warehouse inventory, and thence to distributors, retailers, and the consumers. Information generates decisions that control the flow of goods.

2. Delays

To generate the proper time relationships between the flows of information and materials, we must know the delays in all actions and decisions. Typical time delays are shown on the diagram in weeks. Orders placed with the retailers are filled in one week. Retailers analyze their sales information and place purchase orders after a time averaging three weeks. A half-week is required for mailing of orders, one week for the filling of orders at the distributor, and one week for shipping of goods from distributor to retailer. Similar delays exist between the distributor and the factory warehouse. Six weeks' factory lead time is required to change factory output rate. In the advertising loop, decisions to change advertising level require six weeks for data analysis and decision making. Agencies and media are assumed to average four weeks for their work. Eight weeks are assumed for building up full advertising effectiveness in the consumer's mind after the appearance of a new level of advertising activity. The system sensitivity to these numerical assumptions will be discussed later. They are presented as intuitively plausible values for consumer-durable goods. As we will see later, the sensitivity to the particular chosen values can be determined. If the system is insensitive to any plausible range of values, we need not be concerned further. If, some particular assumptions are critical, we then know that a more careful determination is required.

3. Policies and Decisions

To complete our simplified system, we must estimate the policies that determine purchases, advertising, and consumer buying.

As a start we will consider only three major motivations in the placing of orders by the retailers, distributors and factory schedulers. First, the purchases will reflect the actual sales made so that goods sold can be replaced. Second, as the average level of business activity changes, there is a corresponding desire to change inventory levels. Third, as business activity changes, it is necessary that a corresponding change occur in the in-process orders and shipments in the supply pipelines of the system. These will be taken up more fully in later sections. We are presently excluding orders motivated by speculation and anticipated price changes, forecasts of future sales, general economic conditions, product model changes, etc.

Advertising expenditure can here be based on either the level of business activity (sales) or distress (excess inventories). These are two common bases for advertising decisions and will be explored before incorporating other factors.

The consumer-level representation is even more over-simplified than the distribution system. We will provide for two modes of operation. One is to specify retail sales as an independently given time-function. In that case we exclude all dynamic interactions with the consumer level (thereby eliminating the effect of advertising). Alternatively, we can specify the rate at which "prospective purchasers" come into existence. The behavior illustrated here presumes a pool of prospective purchasers who have some awareness of their need for the product and who are susceptible to advertising and price discounts. The prospective purchaser is assumed to exist for an average period of 14 weeks before purchasing. The average time he remains as a prospective purchaser is influenced by advertising. Advertising influences the rate of purchases by "prospective purchasers" but does not affect the source of supply of prospective purchasers. It will be noted that Figure 1 describes a system in which advertising has no long-range effect on total sales but can affect short-term sales within the range of fluctuation possible by varying the size of the pool of prospective purchasers. This does not imply absence of advertising effects in the real world but only that they will be added in future extensions of company and market model studies.

Before continuing (in Part II) with the detailed equations to describe this system, the following sections discuss the general principles to be employed.

C. Principles for Formulating a Dynamic System Model

Some general principles can be stated to guide a mathematical formulation of company, industry and economic activity. They all aim at a high degree of reality in representing economic behavior. As a result, we will develop a system of mathematical equations containing a large number of variables. An adequate description of a modern corporation will require several hundred inter-acting variables. A representation of the national economy could easily lead to a system of several thousand variables. This need not be frightening. The inclusion of additional reality often relieves us of demonstrating the validity of simplifying assumptions. Greater reality takes us away from simultaneous equations into a formulation where solution time and difficulty is only proportional to the number of variables and does not increase at a higher rate as in many types of models. The greater reality yields a model that is entirely parallel to actual functions of the organization and is easier to understand and easier to verify. It leads to easier communication between company executives and those doing the model formulation. The modern digital computer gives us a low-cost tool that makes practical the manipulation of complex, realistic models.

1. Five Interacting Flow Systems

We no longer think only in terms of the classical industrial functions such as production, research, advertising, etc., but in terms of the basic factors which make up industrial activity. These seem to be:

a. <u>Information Flow</u>. Information must not be confused with the quantities that it represents. Information about the state of the industrial system is available at different places with different time lags and accuracies. Very often we will see that the "true" state of the system is being generated by the model and that, simultaneously, differing information about this "true" behavior is being produced by the way in which the system functions.

All decision-making is included in the information flow system because decisions are generated, not from physical quantities but from information about the physical behavior. Prices are, by nature, information and are generated in the information flow system.

b. <u>Material Flow</u>. Here we include all movement of physical goods, whether raw materials, in-process inventories, or finished product.

c. <u>Money Flow</u>. Money flow will include credit, the banking system, cash position of the company, accounts receivable, etc. At points of sale, goods (material flow) multiplied by price (information) lead later to payment (money flow).

d. <u>Manpower Mobility</u>. Many of the transient behavior patterns in economic systems arise from the availability of labor supply and the time necessary to change employment levels. Here we must clearly distinguish between men, length of work week, man-hours, and labor productivity changes.

e. <u>Capital Equipment</u>. By the <u>flow</u> of capital equipment we mean the way factories and machines come into existence, the stock of existing capital equipment, what part (to measure the level of productivity) of our capital equipment stock is in use at any instant, and the motivations that lead to the discard of capital equipment.

2. Information-Feedback Systems

An information-feedback system exists when the environment leads to a decision and action that alters the original environment. This is a broad definition covering most human, social, and technical activities. Our economic fluctuations are a manifestation of the time-varying interactions that occur in information-feedback loops. At the company level, rising sales deplete inventories, leading to new production decisions to fill demand and rebuild inventory. A new technological product, in rising demand, permits high profit margins that attract competitors. This leads to increased production capacity which equals or exceeds demand, thence to falling profits that seek equilibrium with other investment opportunities. The need for product improvement leads to research expenditure and to technological progress that encourages still more research. These and all other management decisions are made in the framework of an information-feedback system.

The concept of information-feedback is important because the system as a whole exhibits behavior which is not attributable to the parts separately. The pattern of system interconnection, the delays, and the information distortion combine to determine stability and growth. As we will see later, the interconnection of entirely ordinary corporate actions can lead to production fluctuations, unemployment, and excess plant capacity. As one action feeds into another and eventually back to the first, we find instability which is the counterpart of "hunting" in mechanical servomechanisms. Careful attention must be given to time delays, information distortion, and the basis for decisions.

a. Phase and Time Relationships

The behavior of information feed-back systems is intimately associated with the time-sequence relationships between the parts of the system. These lags can arise from reservoirs in the system and from delays in decisions and actions.

Reservoirs and variable accumulations of all types must be carefully noted and represented. Their very nature produces time delays and their purpose is often to permit or introduce changes between input and output rates of flow. We must include those reservoirs in information channels, such as unfilled order backlogs, in-transit orders in the mail, and decisions that have been made but not executed. In material flow, the reservoirs include in-process and static inventories and goods in transit. In money flow, the reservoirs include bank balances, accounts receivable, and loans. In man-power we include number of employees, various categories of unemployed, and the classes of consumers and investors. In capital equipment the reservoirs include plant and machines, segregated as necessary by age, type, and productivity index.

Time delays must be carefully observed and introduced in their proper places in the system formulation. Time delays should be explicitly introduced into the equations and not combined with the solution interval of a set of difference equations. While one might make some proper use of the solution interval for introducing time delays, this is a dangerous procedure. It has, in my opinion, caused disastrous confusion in many formulations of economic models. This has been especially true when the completely irrelavent time intervals at which data had been collected in the past has dictated the structure of time delays in system models and the solution interval to be used in solving the resulting difference equations.

b. <u>Information Amplification and Distortion</u>

Information amplification occurs many places. In placing orders, inventory accumulation and the filling of supply pipe lines in proportion to the level of business activity cause orders to exceed sales on rises of business activity and vice versa. Amplification occurs in many forecasting and predicting procedures. It often occurs when growth rates are extrapolated ineptly, leading to over-investment in factory capacity. Amplification arises from the tendency to order ahead when the deliveries of goods slow down. Ordering ahead during price rises and delayed ordering during price declines produces amplification. All these are of the greatest importance in representing the factores that control industrial growth and stability.

Likewise, many sources of information distortion occur. Information is modified by averaging procedures, by noise and random disturbances, by delays in information transmission, and by summarizing individual business transactions into composite data for executive decision.

Since information is the raw material of decisions, information amplification and distortion must be included if we are to properly generate realistic system behavior. Particular attention must be given to determining what information is actually available and used at each point in the system -- much information is not available, much of what is available is not actually used, what is used is not necessarily the most useful of that available.

c. <u>Decision Making</u>

Decision making in the information-feedback system couples information and action channels. To properly represent the dynamic system behavior we must have formal expressions to indicate how decisions are made. The flow of information is continuously converted into decisions and action. No plea about the inadequacy of our understanding of the decision-making process can be allowed to excuse us from estimating the decision-making criteria. To omit the basis for decision making is to deny its presence -- a mistake of far greater magnitude than any errors in our best estimate of the process. Two kinds of "decisions" will be recognized in the equations in later sections:

Overt Decisions consist of the conscious decisions by people as part of the management and economic process.

Implicit Decisions are the decisions that determine the behavior of the system as an unavoidable result of the state of the system. Included here are (1) ability to deliver depending on the status of inventories, (2) the output rate (goods delivered) from a transportation system resulting from the input rates, the goods in transit, and the transport delays, or (3) factory production output as a result of the status of employment, capital equipment and available materials. Factory output is an implicit decision since output as such is not actually controlled by overt management decision. The overt decisions relate to hiring rates, ordering of raw materials and machine tools, and setting the length of the work week.

3. Continuous Flows

Most persons have a natural tendency to be unduly concerned by the discreteness of events in industrial activity. Even such identifiable actions as the granting of a bank loan, the construction of a factory, or the signing of a contract are but points in a long sequence of plans, preparations and preliminary decisions. The international incident or the presidential heart attack that appears to affect the stock market or precipitates an economic rise or decline is often a mere trigger or excuse for a change which is already imminent.

Discreteness of events is entirely compatible with the concept of information-feedback systems. However, we must be on guard against unnecessarily cluttering our formulation with the detail of discrete events which will but obscure the real momentum and continuity that our industrial system exhibits.

The study of the individual event is of great importance to us, not because we want to handle individual events in most of our models, but because tracing the single item divulges the paths and delays existing in the channels of our system.

4. Broad View of Total System

The economic system, made up of its component industries, contains technical, managerial, psychological, organizational, monetary, and historical factors. All these must find their proper place in our formulation of how the system components interact. No model will serve all purposes. Its usefulness will depend on our judgment of what to include. The decision on model scope must not, however, be limited to some preconceived notion of what is "economics" or "management".

In formulating a quantitative model, we must have the courage to include all the facets which we would consider pertinent in a verbal description of the subject. A strange cleavage exists between those who verbally describe the real world and those who construct formal models. Qualitative plausibility is important in a model even when quantitative accuracy is not possible. We often see the model maker inclined to omit a function for which he has no measurements, rather than put in an estimate. This is equivalent to setting the value to zero, which is often far from the most probable value that best fits the available intuitive judgments.

5. Attention to Basic Assumptions

Too often we find, in the literature of quantitative economics and management science, articles that dispose of the basic assumptions about the system in the first paragraph or page -- assumptions that are not explained or defended, and sometimes not even believed, by the author. There then follow pages of elegant and fruitless mathematic derivation of results from this tenuous foundation.

Very often we find mathematical accuracy (to three decimal places) obscuring the possibility that the function being measured is entirely non-existent and meaningless. To avoid these errors we must build our system representation from a much deeper level in the social structure than has been customary.

Rather than attempting to derive an abstract model from external manifestations (like sales, manufacturing rates, inventory levels, gross national product, freight car loadings, and bank balances), we should build from the level of observable events. We must represent typical actions in the system, and the levels at which decisions are made. The assumptions then become a reflection of the factors that one discusses in descriptive economics or descriptive management. The assumptions, as long as they must remain assumptions, can then be evaluated against every day observation and experience. Only thus can we expect to bridge the chasm between the practicing manager and the analytical approach. From any analysis, the results can be no more valid than the assumptions, the answers are no more pertinent than the questions asked. The analysis must not obscure the assumptions to the point where their validity can no longer be evaluated by the responsible manager.

6. Linearity and Stability

Many past models have been formulated on the assumption that the system being represented was inherently stable and that a linear representation was a reasonable approximation. I believe that neither of these presumptions should be allowed to dictate the model form because both may be grossly wrong.

The systems which we are here working with are highly non-linear and most of the time are operating against limitations of over-employment, money shortage, political pressure to overcome inflation or recession, or capital equipment inadequacy. Furthermore, the justification that we are working only with small deviations in a linear range is untenable because the events of greatest interest are apt to be those associated with the changing pattern of the non-linear restraints on the system.

In formulating a model, we should not presume that the system is necessarily stable in the mathematical and control system sense. It is entirely possible, and I believe likely, that many of our industrial and economic systems are inherently unstable except when non-linear restraints are controlling the behavior. Formulating a model through methods which impose or require that the system be stable may exclude from our consideration some of the most interesting and important areas of study. It is probable that the behavior of our economic system is characterized by the behavior of an unstable, non-linear system continually shifting from one set of restraints to another.

7. Limits of Functional Relationships

In line with the preceding comments on linearity and stability, it is essential that the functional relationships formulated in the equations be plausibly correct in all of the extreme conditions of possible model operation. Rather than making our task more difficult, this will often simplify the problem of getting proper model behavior. Intercepts at axes, directions of curvature, and ultimate asymptotic limits of our various functions should correctly reflect what we know about the system. Then the model, like the real world, can move along these curves until a balance point is found that will satisfy the total requirements of the complete feedback system. As a result, a particular variable in our model may take on a slightly different value from its counterpart in the real world. Nevertheless it can still have the right qualitative performance with proper timing and direction of change relative to other variables in the system. Furthermore, we often find that thresholds, limits, and directions of curvature can be specified with much greater certainty than can the exact quantitative values in the mid-ranges of operation. Once the limiting conditions have been identified, they often restrict the freedom of choice and the possibility of error when we complete the function through the intermediate regions.

8. Dimensional Units of Equations

Just as in the physical sciences, we must give meticulous attention to the correctness of physical units in our equations. Much confusion has been caused in economic models by carelessness on this point. In the literature we often find equation systems built in dollar-equivalent terms, as if refrigerators were made of dollar bills instead of from materials, by labor, using only existing capital equipment. Looseness in handling units of measure vastly increases the likelihood of omitting the all-important time relationships between the flows of the five basic systems suggested above. For example, demand (information) leads to shortages (in material, goods) that lead to a decision (information) to build more productive capacity that leads to construction (flow of materials and labor) that increases productive capacity (capital equipment) that requires payment (money flow) for the capital equipment. A time-sequential relationship exists between these factors which must be represented by our analysis methods if the true information-feedback characteristics of the system are to be preserved. The preoccupation with money flow and with the money equivalent of labor and materials (and the complete omission of information as a separate quantity) has obscured the character of the industrial system in many analytical attempts. Somewhat facetiously, one might say that information controls what will happen while money is only an indication of what has happened. (Of course, information on what has happened determines decisions on what will be done next.) As we will see in the simplified example later, if one is to omit part of the complete system, money flow is more dispensable than information flow.

9. Goals

In model building of the type here discussed, we must be realistic about the goals of our work. We will be contending with highly non-linear, interconnected, potentially unstable, information feedback control systems. This is the province of the intuitive manager and decision-maker in industry. Rather than starting with the classical approach in the physical sciences by working with simplified, linear, mathematical relationships in which analytic expressions for optimum or maximized solutions can be obtained, we should here begin by accepting the terms of the task.

Our approach will be much the same as that of the practicing industrial manager who attempts, by trial and error and by making changes in his methods, to discover improved business practices. Our tremendous advantage over experimentation with the real-life industrial organization is our ability to perform, at low cost and in a short time, what amount to controlled experiments to determine system behavior under diverse operating conditions.

In the realistic non-linear systems with which we propose to work, there is no reason to hope that we can immediately develop methods which will produce "optimum" or "best" answers. We can choose between any set of alternative systems and decide which we like best. By understanding their characteristics better, we will find ourselves able to design better and better systems that are more and more satisfactory. The methods should not be expected, however, to lead directly to any meaningful form of "maximized" solution. Indeed, merely defining an optimum solution may present a problem of the first magnitude.

We are dealing here, not with the simple concept of the servomechanism which is often treated as having a single "error function" (difference between actual and desired results) and a single control mechanism. Instead, our economic systems have distributed "error functions" represented by the individual goals of many participating persons. The control function is likewise dispersed so that it exists, in part, at each decision point in the system.

Our realistic goal is, therefore, to learn more about the system characteristics in order to accomplish improvements, without yet setting ourselves the task of looking for an "optimum".

10. <u>Magnitude of Job</u>

As our final guiding principle, we should be realistic about the size of the task which we are undertaking. We are proposing to analyze systems which are far more complex than have yet been undertaken in the physical sciences. The history of human endeavor is one of tackling new frontiers of exponentially increasing difficulty. Each new task is many times the scope of its predecessors but is, nevertheless, accomplished in approximately the same length of time, since society can build on part accomplishments. For example, about equal periods have been needed for perfecting the subsonic internal engine aircraft, the supersonic jet aircraft, the earth satellite, and presumably, in the future, man's flight to the moon. In this same sense, we are now ready to begin analysis of industrial behavior. This in turn will lead directly to progress in the field of economics since economic conditions are but the reflection of the composite of industrial, market and governmental actions.

In the present day, we would not expect a few men in a few months to develop fully a major new field in the physical sciences, nor would we attempt to jump directly to the final result without laying a solid foundation of experiment and theory. By contrast, there is a tendency in the social sciences to attach major problems with insignificant resources and to attempt to use these resources, not for building foundations, but for trying to jump directly to final, useful results. This is spectacular when successful, but usually leads to a superficial and useless encounter with an overwhelming task.

We must recognize the size of our problem for what it is and gear our efforts accordingly. This means that our approach should be to develop a foundation and a structure as we go, and initially to measure progress by the rate of advancement rather than by the usefulness of final results. A properly integrated optimum program of research and experiment might easily grow for a period of five years before beginning to yield noteworthy results, and might well take one or two decades before our knowledge of industrial and economic behavior begins to take on an appearance of rigor and reliability.

D. Mathematical Equation System

The following comments on the equations used herein indicate only a present status which is changing daily. We must not be narrowly restricted to any one method of formulating the systems under study. Too often in the management sciences, we see a solution looking for a problem. Instead, we should first identify the most urgent corporate and economic ailments; then develop the methods that can cope with cause and correction. The equation forms here presented do not provide for all circumstances that will arise. It will be necessary to expand them from time to time. For example, in what follows, we do not show any way to incorporate knowledge of discrete past events. Such would be needed in bringing forward annual sales curves from past years or in segregating existing capital equipment by age and productivity so that the profile of existing plant capacity is available in making certain decisions in an economic model.

1. General

The following sections describe a set of difference equations which can be evaluated one at a time without requiring simultaneous solution.[5] They are to be solved at time intervals that are short compared to the shortest delay intervals which are part of the system description. The equations relate primarily to two classes of variables -- "levels" and "rates". Levels are calculated at the end of time intervals and are generated by rates of flow which are considered constant over the time interval. Decisions are calculated at the beginning of each time interval and are used to determine the rates which will exist over the succeeding interval.

It is generally trus, but is not necessary, that information and decision making will be represented as a continuous process in the sense that new information, and new decisions, are calculated in each time period. Information, including decisions, is therefore thought of as a continuously flowing quantity. This does not exclude discontinuous decision such as an authorization of a new factory, since this authorization occurs after a sufficient build-up of in-flowing information to justify the decision.

2. Classes of Equations

We will use here four classes of equations which will segregate types of variables and determine the sequence in the computing process in which the equations must be evaluated. In future work it is probable that other classes of equations will be included to cover situations not herein provided.

---

5. Some of the steps in evolving the present system of equations are documented in the memos of the SIM Industrial Dynamics Group, particularly D -2 of Sept. 25, 1957 by Jay W. Forrester on "Ordering Decision Criteria to Use in Production-Inventory Model 4" and also in memo D-9 of December 31, 1957 by Richard K. Bennett on "Model 4 Equations for the IBM 704 Computer Program.

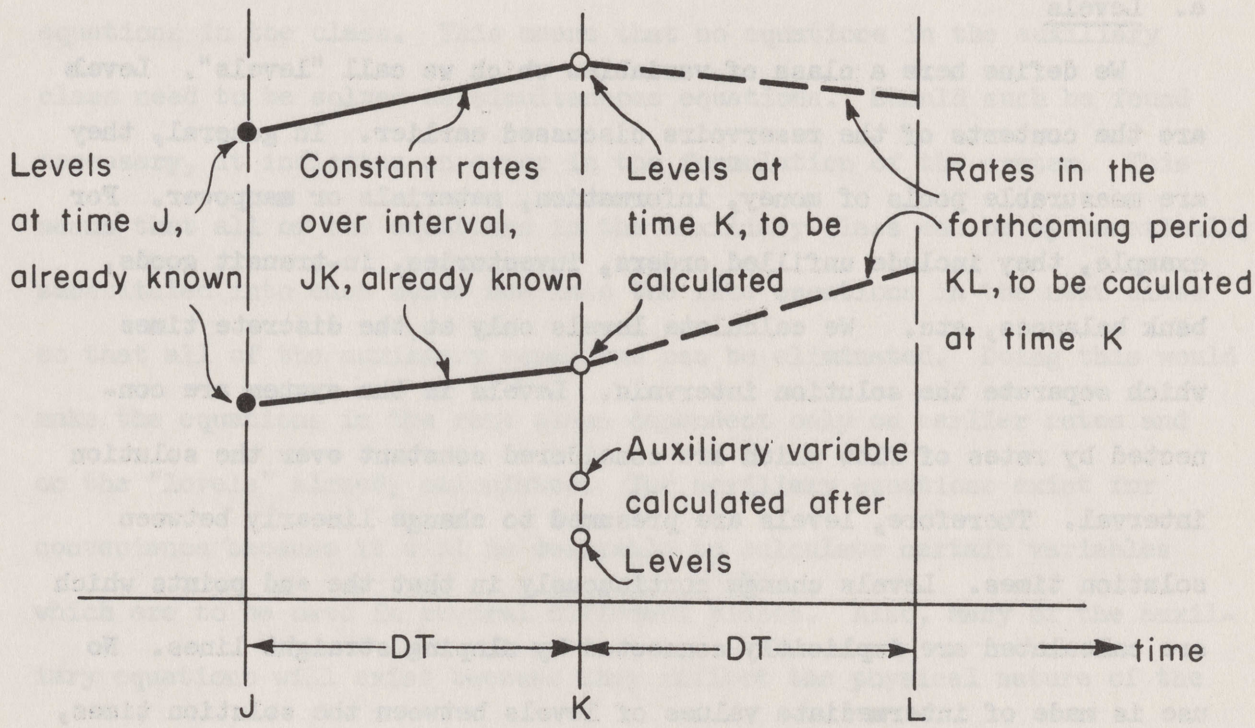


FIG. 2 CALCULATIONS AT TIME K

The continuous flow of time is broken up into discrete intervals, each of length, DT. See Figure 2. The discrete points in time separating these intervals are given the designation J, K, L. The equations are solved at these discrete times and are written in terms of the generalized time step, K. In other words, we assume that the progress of the solution has just reached time, K, but that the equations have not yet been solved for levels at time, K, nor for rates over the interval, KL. The equations therefore show how to obtain levels at time, K, and rates for the interval, KL, based on levels at time, J, and rates over the interval, JK.

a. Levels

We define here a class of variables which we call "levels". Levels are the contents of the reservoirs discussed earlier. In general, they are measurable pools of money, information, materials or manpower. For example, they include unfilled orders, inventories, in-transit goods, bank balances, etc. We calculate levels only at the discrete times which separate the solution intervals. Levels in the system are connected by rates of flow which are considered constant over the solution interval. Therefore, levels are presumed to change linearly between solution times. Levels change continuously in that the end points which are calculated are implicitly connected by sloping straight lines. No use is made of intermediate values of levels between the solution times, J, K, L. As one arrives at the time, K, the first task is to calculate the new values of levels at time, K. These equations for levels are independent of each other, and the order of solution is immaterial. This is true because theydepend only on values of levels at time, J, and rates of flow over the interval, JK, all of which are available.

b. Auxiliary Equations

After evaluating all "levels" at time, K, we will then come to a group of equations which we will call "auxiliary equations". These are equations which may depend on rates in the interval, JK, and the values of levels at time, K. Here, in contrast to the equations for levels, the evaluation of some variables in the auxiliary equation group may depend on other variables in that same group. This imposes a restriction on the order in which the auxiliary equations can be solved. By definition of this class of equation, there must always be an order in which the equations can be solved so that the necessary variables are available from preceding equations in the class. This means that no equations in the auxiliary class need to be solved as simultaneous equations. Should such be found necessary, it indicates an error in the formulation of the system. This means that all of the equations in the auxiliary class can be systematically substituted into each other and into the rate equations in the next class so that all of the auxiliary equations can be eliminated. Doing this would make the equations in the rate class dependent only on earlier rates and on the "levels" already calculated. The auxiliary equations exist for convenience because it will be desirable to calculate certain variables which are to be used in several different places. Also, many of the auxiliary equations will exist because they reflect the physical nature of the system and are built around variables which we wish to define and calculate and whose meaning would be lost or submerged if these equations were consolidated by substitution. The auxiliary equations must therefore be evaluated in a certain order after the equations for levels have been evaluated and before the equations for rates are evaluated.

c. <u>Rates and Decisions</u>

We now come to the "rates of flow" into and out of the "levels" discussed above. These rates are controlled by the decisions in the system. These decisions may be either the overt or the implicit decisions which were discussed earlier. These equations, being decision equations, are thought of as controlling what is to happen next in the system. Therefore, we think of them as being calculated at time, K, and determining the rates of flow in the next interval of time, KL. Each of these decisions controlling a rate of forthcoming action is based on certain information about the past and present which is available at the decision point. It is philosophically impossible for these decisions to be dependent on each other. One decision made in the economic system cannot be based on any other decision made at the same instant of time at some other place. These equations determining rates are therefore entirely independent of each other and can be evaluated in any order. These equations will, in general, involve levels and auxiliary variables evaluated at time, K, and rates which existed over the preceding interval, JK. The rates which are calculated are considered to be constant over the succeeding interval, KL.

d. Parasitic Equations

We have left, a group of equations which are not directly involved in the dynamic behavior of the system. We will call them "parasitic equations". They may be summations of other variables or calculations for output and recording purposes. They are not part of the generation of rates and levels within the dynamic system itself. These equations can be evaluated independently of each other and presumably at any time consistent with their purpose.

3. Time Progression

After all variables have been evaluated, the computing procedure is to prepare for the next time step by indexing all time indicators so that the levels and auxiliary variables at time, K, become the ones designated with time notation, J. Likewise, the rates, KL, just calculated, become the rates JK when the next cycle of computation begins.

4. Notation for Equations

The symbols used for these equations will be selected to have as much mnemonic significance as possible, and also to allow direct printing on conventional computing machinery. Thus, the symbols will not need translation in going to and from computing machine punches and printers.

We are therefore restricted to the use of groups of capital letters, English alphabet, and no subscripts or superscripts. The necessary size of a letter group used to represent a term in an equation will be determined by the complexity of the system being explored. We will use,in this article, three capital letters to represent a constant or a variable. The first two letters will represent the nature of the term and the last letter the sector of the system in which it applies.

In larger systems than illustrated here a code group of 4,5 or even 6 letters may be found desirable for representing terms in equations. In systems of hundreds of variables, a dictionary will clearly be necessary as a reference guide to the meaning of the code groups. This is probably an unavoidable inconvenience.

a. Variables

Variables will be represented by three-letter groups. Any combinations can be used for variables other than variable time delays, except those beginning with A, D, and F. For example, RRR represents Requisitions (orders) Received at the Retail sector.

b. Constants

Three letter groups beginning with A will be used for parameters which are constants until reassigned different numerical values. For example, AIR is the Inventory constant at Retail.

c. Delays

Time "constants" of delays, whether constant or variable will be designated by letter groups starting with D. If the delay is a time variable, it will carry a time notation as do other variables. For example, DCR is the delay in Clerical data processing at Retail. DUD.J is the variable delay in processing Unfilled orders at the Distributor, evaluated at time, J.

d. Functions

The letter, F, used as the initial letter in a group, indicates a designated function of the quantities that follow in parentheses. Figures and letters grouped with the F indicate the nature of the functional relationship. For example, FDI.KL (DCR, PCR.K) indicates the output rate during interval KL from a first-order (1) Delay Function involving the constant, DCR, and the variable level, PCR, at time, K.

e. Time Notation

A letter group representing a variable will be followed by a period to separate the designation of the variable from the time notation. The time notation will indicate whether the letter group is a rate variable, a level or auxiliary variable, or a constant.

As an example of our notation for a rate of flow, RRR.JK represents the rate at which Requisitions were Received at Retail during the time interval,JK. The double time notation indicates the time interval over which the rate of flow exists.

Levels and Auxiliary Variables will carry a single time notation letter indicating the instant of time for which they have been evaluated. For example, UOR.J indicates Unfilled Orders at Retail at the instant of time, J. Likewise, Inventory Desired at Retail at time K, would be written, IDR.K.

Constants are evident by the absence of a time notation.

The time increment between evaluation of the difference equations, normally thought of as "delta t", will be represented by the double symbol,DT.

5. Examples of Equations

Some examples of the previously described equations and notations may be helpful. The following equation tells us how to calculate the level of

Unfilled Orders at Retail at time K :

$$UOR.K = UOR.J + DT\ (RRR.JK - SSR.JK) \qquad (1L)$$

This equation tells us that the unfilled orders at retail at time, K, are equal to the unfilled orders at retail at the previous time, J. plus the inflow minus the outflow. The incoming Requisitions Received at Retail in the time interval, JK, is shown by the notation, RRR.JK, multiplied by the solution time interval, DT. The orders filled are given by the rate of Sales Shipped from Retail during the time period, JK, multiplied by DT. These rates are in terms of quantity per unit of time (for example, units ordered pwe week). In general, rates will be measured in some customary unit of time. This may be in the units of flow per day, week, month, quarter, or year. Flow will not be measured in terms of the solution time interval, DT, because we may wish to change DT without re-evaluating all the constants in the system. By putting the solution interval, DT, in explicitly, it can be changed from time to time without affecting the validity of the equations. The equation number on the righthand margin shows that this is equation number 1 and the suffix, L, shows that it belongs to the class of equations defining "levels".

Equations determining levels will involve terms in which rates are multiplied by the solution interval, DT. They are equations which would contain integrals if the formulation were in differential rather than difference equations. The dimensions of the variable cannot be taken to distinguish "levels" from rates. For example, RSR.K is the Requisitions Smoothed at Retail and gives the averaged level of retail requisitions. It is measured in "orders per week" but is a "level" not a "rate". IT is the average level of business activity and is not a rate which flows into or out of any reservoir in the system. It is given by the following equation involving the solution interval, DT :

$$RSR.K = RSR.J + (ASR)(DT)(RRR.JK - RSR.J) \qquad (2L)$$

An example of an auxiliary equation follows:

$$IDR.K = (AIR)(RSR.K) \qquad (3A)$$

Equation 3A is seen to be an auxiliary equation because it defines Inventory Desired at Retail at time K, in terms of variables also defined at time, K. This means that equation 3A can only be evaluated after the equation which evaluates RSR.K. "Desired" inventory, IDR, is measured in units of inventory, just as an actual inventory which would be defined as a "level". Inventory desired at retail is, however, only a concept or piece of information used in reaching a decision. It meets the test of systematic substitution because equation 3A could be substituted into the decision equation (4R), this eliminating equation 3A. The auxiliary equation is kept separate only for convenience in maintaining simple equation forms, or because the variables they express have some special meaning to us, or because the variable which is evaluated is used in several places in other equations and we can avoid evaluating it more than once in the computation.

By taking care to classify as such all equations which define auxillary variables, we avoid unintentional propagation delays through the system created by the particular equation structure we choose. Subdividing equations for convenience does not then introduce unintended delays. All intended delays in the transmission of information, materials, money and manpower should be represented by explicit terms in the equations.

The following equation is one defining a rate in the forthcoming time period, KL, in terms of quantities which are known at time, K. It is equation number 4 in our system and belongs to the class of equations defining rates as designated by "R".

$$SSR.KL = \frac{UOR.K}{DUR.K}\sqrt{\frac{IAR.K}{IDR.K}} \qquad (4R)$$

The rate during period, KL, is here defined in terms of information existing at time, K.

6. Flow Diagram Symbols

I find that a flow diagram is invaluable in describing the system for which equations are being written. It seems best to develop the diagram and the equations simultaneously. Doing so helps prevent errors and aids in visualizing the relationships that are being expressed mathematically. A flow diagram requires a set of symbols to display the various functions being performed. I have chosen not to represent the detailed mathematical relationships but rather to show the organizationsl structure of the system and to show the sources of all information used at the decision points. To do this we need different kinds of lines to separate the flows of information and materials. In more advanced diagrams one would also need to distinguish the flows of money, manpower, and capital equipment.

Figure 3 illustrates the flow diagram symbols used in this article. At Fig. 3a is shown the solid line used for material flow and the dotted line used for information flow. Beside each line is shown the symbol representing the quantity whose path is illustrated.

At Fig. 3b is shown the rectangle used for s reservoir or accumulation. It here represents unfilled orders with requisitions flowing in at one rate and filled orders being removed at a different rate. The symbol, UOR, representing the quantity stored in the box is indicated as well as the equation number used for its evaluation.

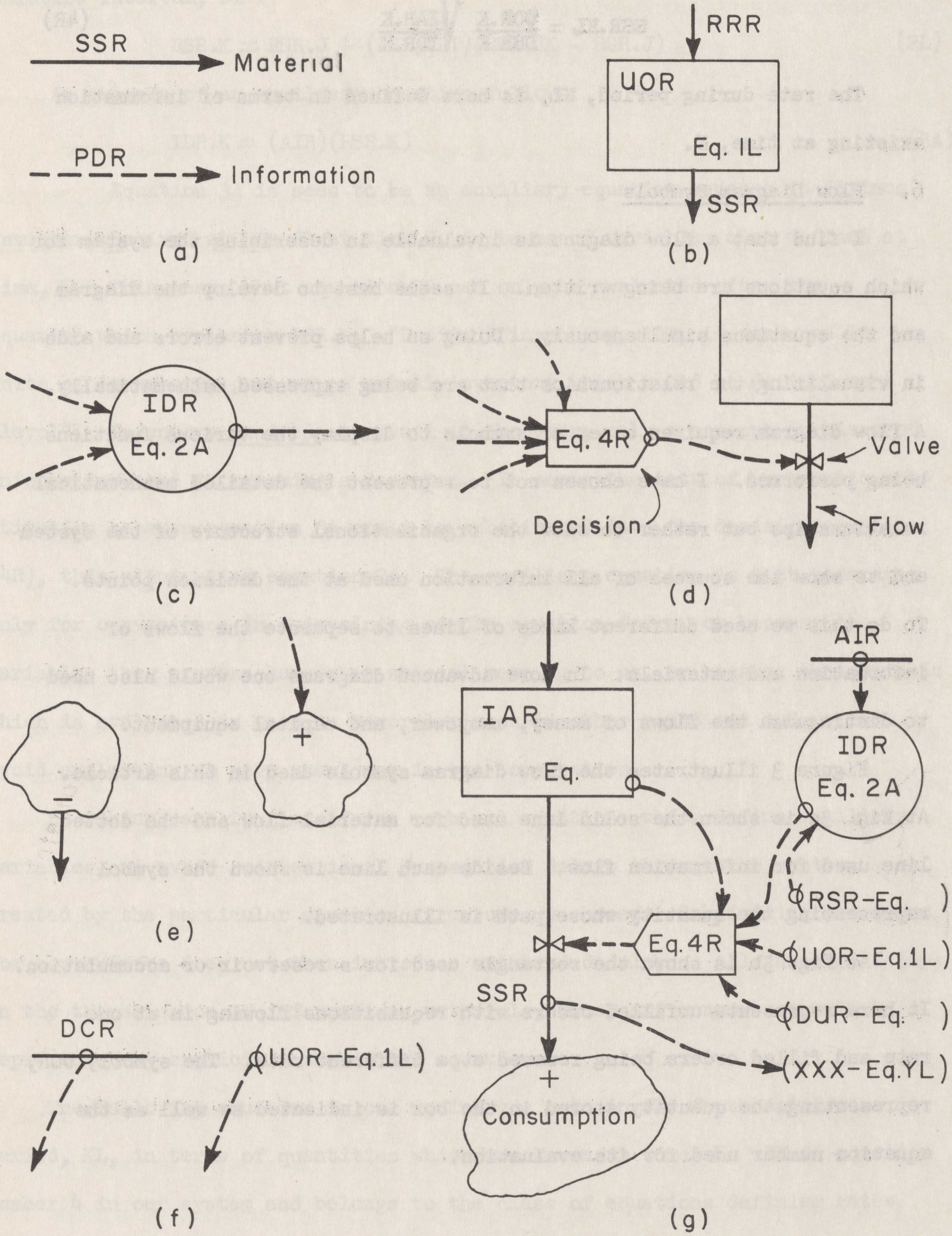


FIG. 3 FLOW DIAGRAM SYMBOLS

At Fig. 3c is shown a circle used to represent the position of an auxiliary variable and auxiliary equation in the system. The variable, IDR, is here calculated according to the relationship shown in equation 3a. The small circle on the periphery of the large circle with the dotted line leading away shows an information take-off. The outflowing arrow shows the destination of information about IDR. Unlike an outflowing rate carrying a minus sign and no small circle, as shown in Fig. 3b, the outflowing information has no effect on the quantity in the circle. Information about a quantity in a rectangular box or about a rate of flow can be indicated by a similar information take-off symbol.

At Fig. 3d we see the indication of a decision controlling a rate of flow. The small double triangle indicates a valve which determines the flow rate based on the incoming decision. The decision is made at the arrow box where information channels are combined according to the equation indicated.

At times we will have need to indicate a source or destination outside the system being represented. The irregular outline in Fig. 3e is used to indicate a source of materials whose supply characteristics are not being considered or a destination of goods sold outside of the system. It is also convenient as indicating the source and discard of pieces of information suchas purchase orders, since this will often help us to maintain the correct units of measure in the system.

At Fig. 3f we have a symbol showing the input of the constant, DCR, which is underlined with an information take-off. Also shown is a value of a variable which is not generated within the flow diagram on this sheet. It is enclosed in parenthesis along with an indication of the equation number which evaluates it. Presumably the flow diagram for this equation appears on a different sheet depicting another part of the system.

At Fig. 3g is a typical combination of the symbols already used, showing the information inputs which are used for controlling the outflow rate of goods, SSR, from the inventory, IAR. We also see an information take-off from the rate, SSR, which goes to variable XXX in Equation YL in another diagram.

Abbreviated symbols for a delay function are given later in Section C.8.c.

7. Redundancy of Equation Type and Time Notation

It will be noted that, in the preceding equations, the time notation and the equation type notation are largely redundant. Equations for levels carry the type notation, L, and in the left hand term carry the time notation, K, and, on the right, the time notation, K, on each side. Rate equations of type, R, contain time symbols, KL, on the left and time notations, K and JK, on the right.

Experience has shown that confusion is readily possible in defining equation types and handling the time notation. Therefore, in this paper, both will be used. It is probable that in actually solving these equations on a computer, one may choose to drop one or the other of the notations.

8. Representation of Delays

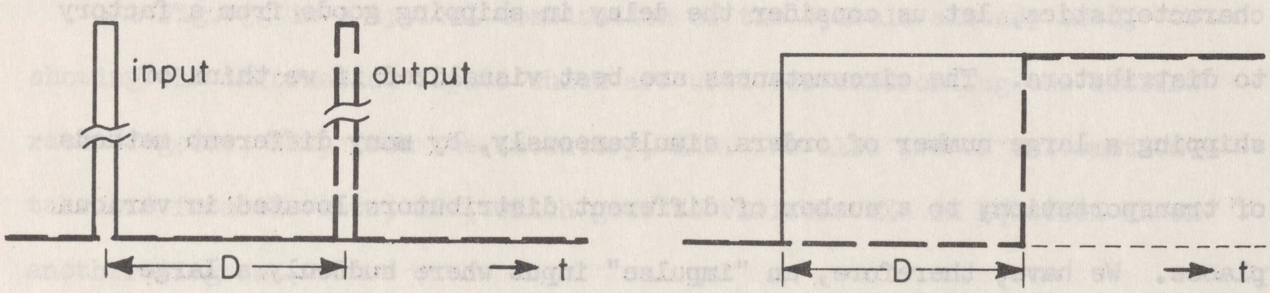
Preceding sections have already stressed the importance of properly representing the times required to accomplish various real-world actions when we formulate a model of system dynamic behavior. We will interpose here this section on the handling of delays so that the methods can be used later without further discussion.

In formulating an equation to express a time-delay relationship, we are usually concerned with an aggregate of a large number of individual actions which form the input to the delay function and where we are interested in the shape of the output response. As a situation for discussing delay characteristics, let us consider the delay in shipping goods from a factory to distributors. The circumstances are best visualized if we think of shipping a large number of orders simultaneously, by many different methods of transportation, to a number of different distributors located in various places. We have, therefore, an "impulse" input where suddenly a large number of items are inserted into the system at one moment and we wish to consider their rate of arrival at their destinations.

Two characteristics of a delay are of immediate interest to us. One is the length of time expressing the average delay, D. The other is the functional relationship giving the time shape of the output response. For our example, we clearly could observe average delay fairly readily and would expect it to be a number of days depending on the types of transportation and distance involved. Determining the proper shape of the output rate-of-arrival curve will require a more thoughtful examination.

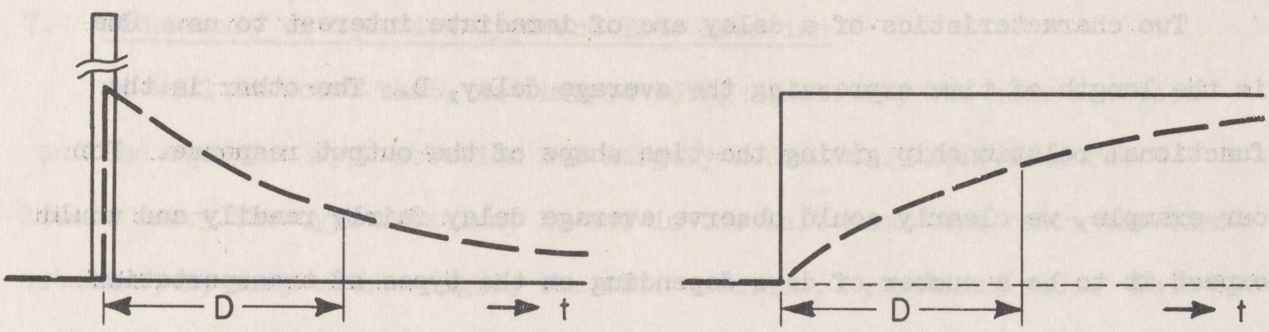
a. Qualitative Shapes of Delay Function

We must take care in selecting the functional form (time shape of the output) of the delay to be used. The functional form of the delay can have a major effect on the qualitative behavior of our model as we will see. Having the wrong form of delay may affect the model results more than errors as large as a factor of two or more in the length of the delay interval which is assumed.



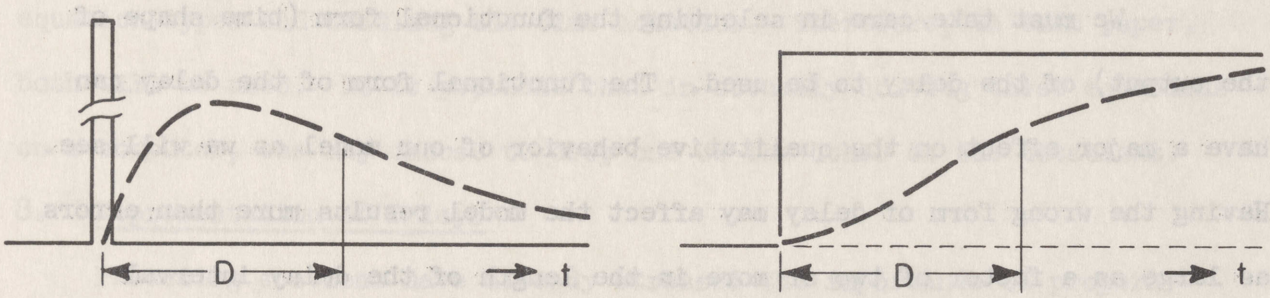
(a) (b)

FIG. 4–DISCREET OR PIPELINE DELAY



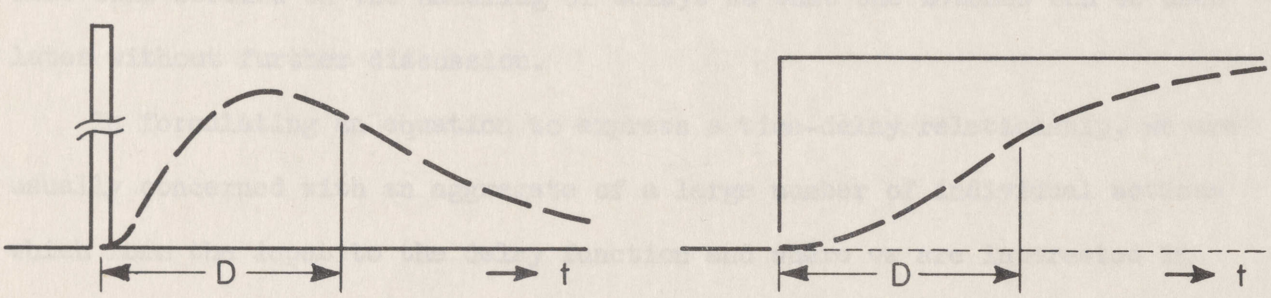
(a) (b)

FIG. 5–1st ORDER EXPONENTIAL DELAY



(a) (b)

FIG. 6–2nd ORDER EXPONENTIAL DELAY



(a) (b)

FIG. 7–3rd ORDER EXPONENTIAL DELAY

Figures 4, 5, 6, and 7 show several of many possible forms of delay functions. Figure 4 shows the discrete or "pipeline" delay in which nothing happens at the output until after the elapsed delay, D, when we find an exact reproduction of the input. Figure 4a shows an impulse input to the delay with the resulting impulse output appearing at time, D. Figure 4b shows the response to a step input. The input rises suddenly from zero to its final value and the output does likewise at D days later. This is clearly not a realistic representation of our shipping delay since it implies that all shipments are delivered at all points in the country at exactly the same number of days after they are sent. This type of delay has often been used in economic models but is probably not representative of any economic function.

Figure 5 shows a first order exponential delay. In Figure 5a we see that the maximum rate of output occurs immediately after an impulse input, and the output rate declines exponentially thereafter. In response to a step change in input rate as in Figure 5b, the output rises exponentially toward the new input rate. This, likewise, is not representative of our shipping delay since Figure 5a shows that the maximum rate of arrival at the destination occurs at the instant the multiplicity of shipments is made. Clearly the maximum rate of arrival does not occur at the moment of shipment.

Figure 6 is the output response of a second order exponential delay. The second-order delay is the equivalent of two first-order delays cascaded one after the other so that the output of the first is the input to the second. We see in Figure 6a that the initial output rate in response to an impulse input is zero, and that the output curve has its maximum slope at the origin. This is also qualitatively unacceptable as a shipping delay because we would not expect the rate of shipment arrivals to start rising rapidly at the moment that shipments leave the factory.

In Figure 7 is a third-order, exponential delay. This shape of output response is the first of the sequence that satisfies the more obvious characteristics of the actual shipping process. In Figure 7a the output response to an impulse input is initially zero. Also, the

initial slope of the output curve is zero. The curve begins to rise slowly, reaches a maximum slope, and then a peak value, and falls off. In Figure 7b is the way the output rises following a step change in the input rate.

If one wished to further refine the response shape of the delay time-function, he would now be forced to make a careful examination of the behavior of the system being represented and to measure actual characteristics.

Once we have a functional form that exhibits a qualitative shape that matches our empirical knowledge of the facts, we will usually find neither the necessity nor adequate data to refine the function further. This is a specific example of the general comments in section C.7 above on "Limits of Functional Relationships". So long as a functional relationship violates knowledge that is available to us, we should strive for greater realism. Once the known asymptotes, intercepts, and directions of curvature are satisfied for the transient as well as the steady-state behavior, it is probably time to proceed with other parts of the model until such time as we can make an actual quantitative determination of how sensitive the model is to the various assumptions.

A different physical circumstance, for example the consumer buying rate response to a single advertisement placed in a daily newspaper by a department store, might lead one to select a different shape of delay response. Here, one might feel that the first-order exponential delay was more representative of the output than one of the other forms shown, on the basis that the buying response is greatest immediately and falls off with time. Likewise, one may occasionally, as in representing the lead time delay in establishing a revised factory manufacturing rate, want to have a more pronounced initial delay than is given by the third-order exponential. Cascading exponential delays increases the initial delay and steepens the slope of the rising part of the curve. The limiting case of an infinite order exponential delay is the discrete or pipeline delay shown in Figure 4.

There is no necessity to restrict ourselves to the exponential functions for representing delays. For the present, the ones shown are convenient and give adequate scope to satisfy our limited knowledge of the behaviors being described. It is entirely likely that more

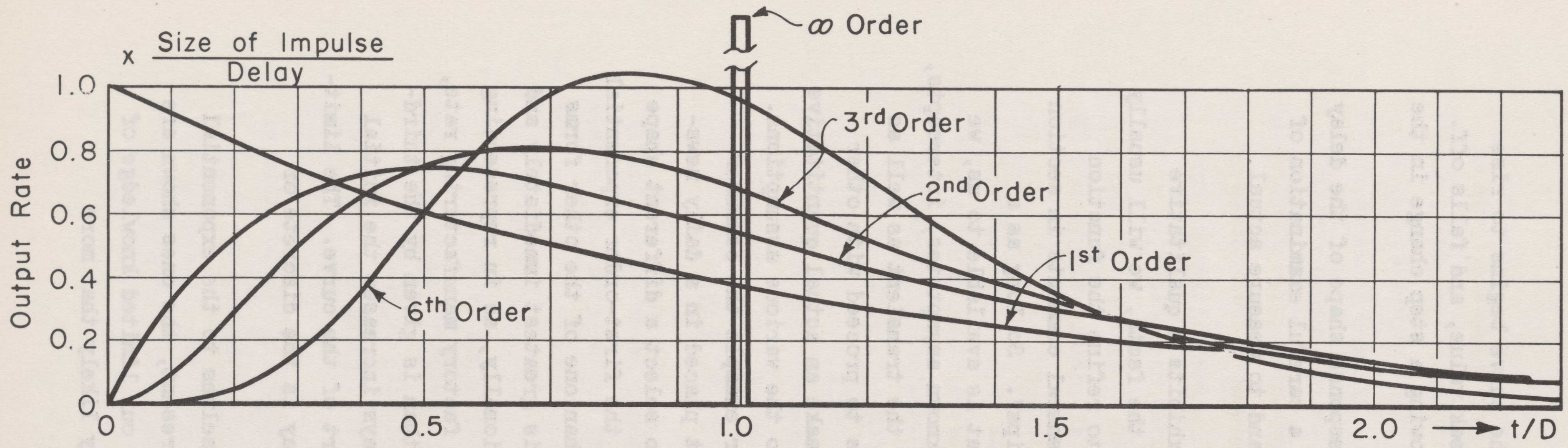


FIG. 8 VARIOUS ORDER EXPONENTIAL RESPONSES TO A UNIT IMPULSE

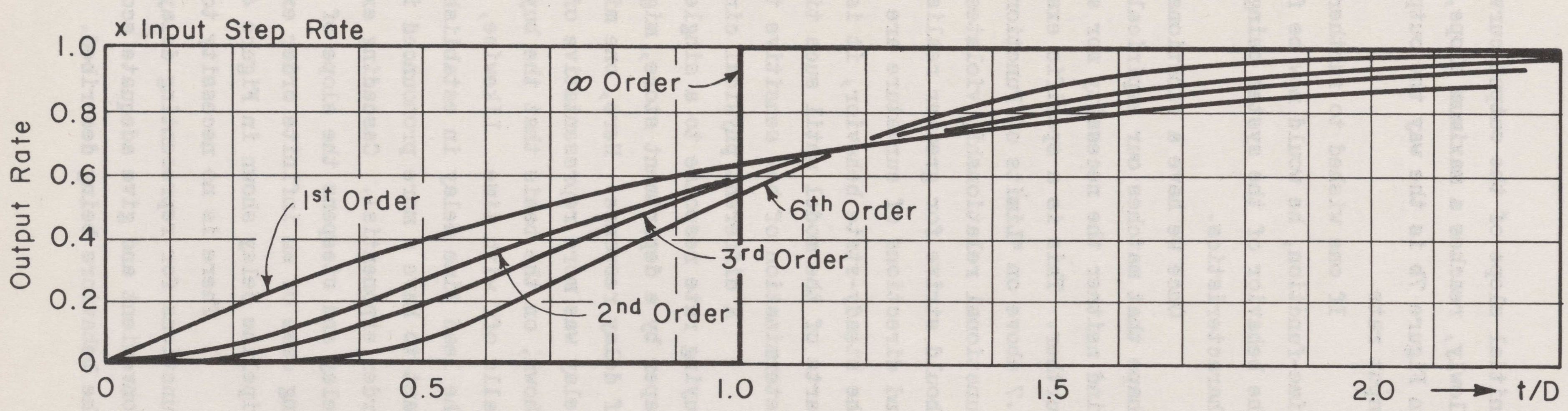


FIG. 9 VARIOUS ORDER EXPONENTIAL RESPONSES TO A UNIT STEP

investigation of this subject will divulge simpler formulations for equally satisfactory results, or formulations which will permit larger steps in the solution interval.

b. Quantative Delay Functions

Figure 8 shows the accurate output response of first, second, third, sixth, and infinite order exponential delays when an "impulse" is the input (that is, a quantity is put in at zero time, nothing in thereafter, and we observe the output).[6] Along the time axis, time is indicated in terms of the total average delay, D. The delay is defined in such a way that in a steady state flow rate multiplied by the delay. In other words, all of the curves are normalized in such a way that for a delay of, D, and a uniform flow through the delay of R items per unit of time, the number of items in-transit in the delay is equal to (R)(D).

Mathematically speaking, an nth-order delay is equivalent to n cascaded single order delays, with each single-order delay having a time constant of D/n. Each first-order delay has the same rate and consequently the same number of units stored in it. Thus the total number of units is n times the number stored in any one of the delays.

The vertical scale shows the output rate in terms of units of I/D where I is the impulse quantity inserted initially and D is the delay. I/D has the proper dimensions of units/time.

Figure 9 shows the output from first, second, third, sixth and infinite order delays when a step function of input rate occurs. A sudden input flow rate is impressed at zero time and the resulting output rates are plotted.

The vertical scale shows the ratio, at any instant, of the output rate to the suddenly applied input rate.

---

[6] As presented earlier by Jack Pugh, "Nth Order Delays", Dec. 2, 1957, SIM Industrial Dynamics Memo D-7.

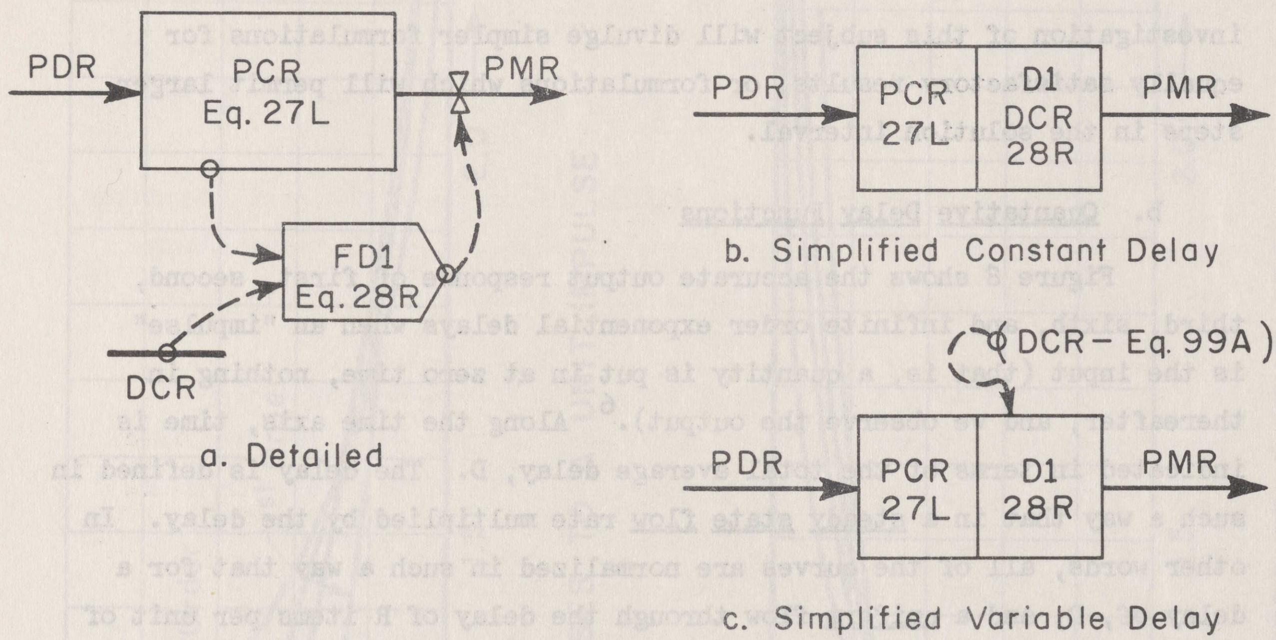


FIG. 10 DELAY FLOW DIAGRAM AND SYMBOLS

c. Delay Equations and Flow Diagram Symbols.

Figure 10 shows flow diagrams representing a delay. In Figure 10a the incoming flow rate is shown as PDR. The box is the reservoir holding the quantity (level), PCR, in-transit in the delay. The outgoing rate, PMR, is controlled by the value which responds to the implicit decision function, FD1, representing a first-order exponential delay. The average delay constant is DCR.

Because the delay functions will be so common, it is worth while to use an abbreviated notation in flow diagrams. Figure 10b shows such for a constant delay, DCR, which is written in the second section of the box along with D1, indicating the kind of delay, and 28R which is the equation number defining the outgoing rate. In the first section is the symbol, PCR, for the level of items in the reservoir and the equation number, 27L, defining PCR. Figure 10c is similar except that the variable delay time is supplied from the outside.

The equations which would represent the first-order delay are:

$$PCR.K = PCR.J + DT(PDR.JK - PMR.JK) \qquad (5L)$$

$$PMR.KL = FD1(PCR.K, DCR) = \frac{PCR.K}{DCR} \qquad (6R)$$

Equation 5L states that the quantity now, at time K, in the delay equals the quantity at time, J, plus what was inserted during the interval, DT, minus what was removed. Equation 6R states that the outflow rate, PMR, in the forthcoming time period, KL, is a first-order delay function, FD1, of the quantity in the delay, PCR, and the nominal delay, DCR. The time delay, DCR, is here shown as a constant but could be a variable and, if so, it would carry a time notation. Equation 6R also gives the explicit form of the function, FD1. Since the explicit form, PCR.K/DCR, for the first-order delay is shorter to write than the functional notation, we will often write it directly, not using the FD1 form.

In a similar way, a second-order would be represented as in Figure 10 with the kind of delay shown as FD2. The equations for the second-order delay can be written as follows:

$$PCR.K = PCR.J + DT(PDR.JK - PMR.JK) \qquad (7L)$$

$$PMR.KL = PMR.JK + \frac{4(DT)}{DCR}\left[\frac{PCR.K}{DCR} - PMR.JK\right] \qquad (8R)$$

$$= FD2(PCR.K, DCR)$$

The equations for two cascaded single-order delays have been combined in such a way that PCR still represents the total in-transit quantity stored in the delay; it is not split between two quantities representing the two delays. The second stored quantity which is needed for the second order delay is supplied through the use of the previous output rate, PMR.JK. In general, an nth-order delay will require storing n quantities from one time step to the next.

The functional notation for a third-order delay is FD3. Evaluating the third-order delay is still more complicated and involves an intermediate new variable for each third-order delay as follows:

$$PCR.K = PCR.J + DT(PDR.JK - PMR.JK) \qquad (9L)$$

$$PMR.KL = FD3(PCR.K, DCR) = PMR.JK + \frac{9(DT)}{DCR}\left[\frac{3(xxx.K)}{DCR} - PMR.JK\right] \qquad (10R)$$

$$xxx.K = xxx.J + DT\left[\frac{PCR.J}{DCR} - PMR.JK\right] \qquad (11L)$$

(There are many ways in which the equations for the third-order delay might be written, especially with regard to the extra variable which is required. The form chosen here makes the extra equation of a form that fits the pattern already established for other equations in the model).

9. <u>Solution Time Interval</u>

To properly generate the time-phase relationships described by the equations, it is important that the solution interval, DT, be chosen sufficiently short. On the other hand, it should be as long as permissible, to conserve computation time.

Figure 11 shows the output of a first-order delay to illustrate how its generation depends on the solution interval, DT. For small values of DT we see that the exponential curves are closely traced. When DT is equal to the value of the delay, D, we see that the representation acts much like a pipeline delay; the impulse input is emptied completely during the interval DT, and the step input is delayed and reproduced exactly at the end of time, D, which equals the solution interval. For values of solution interval greater than the delay, the output becomes oscillatory because, at the end of the first interval, more is transferred out of the delay than exists in it, leaving a negative value which calls for a return flow in the second solution interval. This, of course, is physically impossible for the kinds of systems we propose to represent and must be avoided. Beyond a solution interval equal to twice the delay, the output oscillation increases with time, because in each time interval the sign of the quantity in the delay reverses and its absolute magnitude increases exponentially.

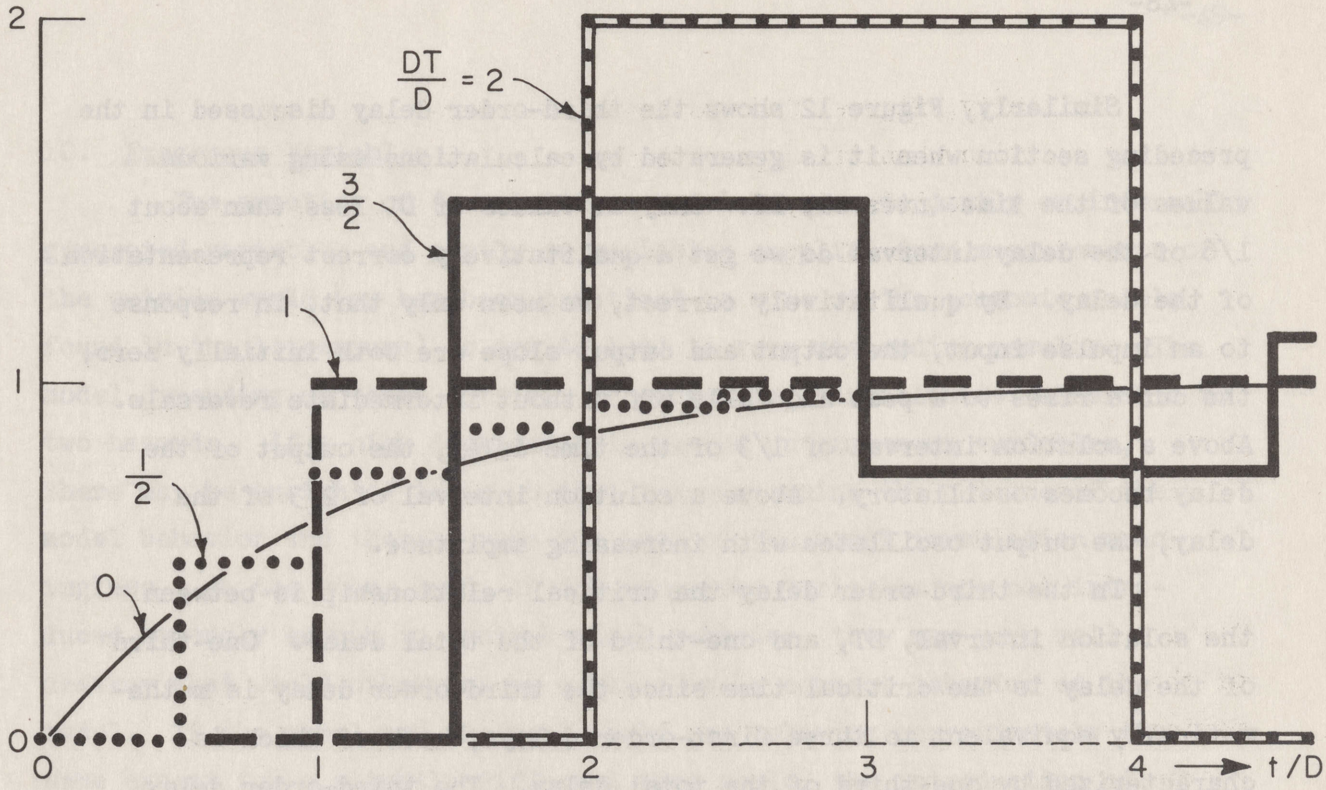


FIG. 11 STEP RESPONSE OF 1st ORDER DELAY WITH VARIOUS RATIOS OF DT TO D

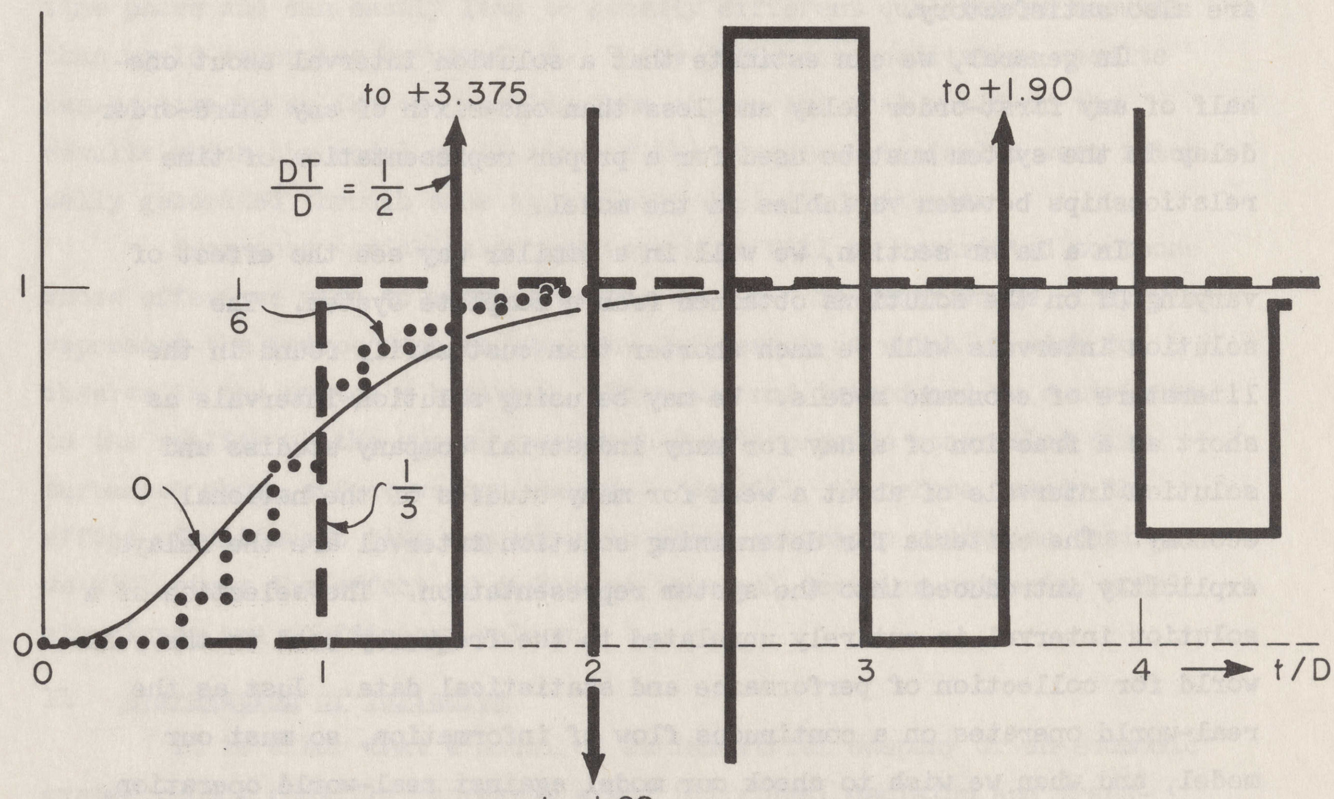


FIG. 12 STEP RESPONSE OF 3rd ORDER DELAY WITH VARIOUS RATIOS OF DT TO D

Similarly, Figure 12 shows the third-order delay discussed in the preceding section when it is generated by calculations using various values of the time interval, DT. Only at values of DT less than about 1/6 of the delay interval do we get a qualitatively correct representation of the delay. By qualitatively correct, we mean only that, in response to an impulse input, the output and output slope are both initially zero, the curve rises to a peak and falls off without intermediate reversals. Above a solution interval of 1/3 of the time delay, the output of the delay becomes oscillatory. Above a solution interval of 2/3 of the delay, the output oscillates with increasing amplitude.

In the third-order delay the critical relationship is between the solution interval, DT, and one-third of the total delay. One-third of the delay is the critical time since the third-order delay is mathematically equivalent to three first-order delays, each of which is characterized by one-third of the total delay. The third-order delay gives a satisfactory representation only if the cascaded first-order delays are also satisfactory.

In general, we can estimate that a solution interval about one-half of any first-order delay and less than one-sixth of any third-order delay in the system must be used for a proper representation of time relationships between variables in the model.

In a later section, we will in a similar way see the effect of varying DT on the solutions obtained from a complete system. The solution intervals will be much shorter than customarily found in the literature of economic models. We may be using solution intervals as short as a fraction of a day for many industrial company studies and solution intervals of about a week for many studies of the national economy. The criteria for determining solution interval are the delays explicitly introduced into the system representation. The selection of a solution interval is entirely unrelated to the frequency used in the real-world for collection of performance and statistical data. Just as the real-world operates on a continuous flow of information, so must our model, and when we wish to check our model against real-world operation, we can do so at those intervals when data become available.

10. Exogenous Variables

The practice of formulating a model to operate partly on internally generated variables and partly on variables supplied from measurements of the outside world (as has been practiced in some of the economic models found in the literature) is apt to lead to very misleading results. The model depending on these outside, or exogenous, variables is vulnerable to two hazards. If a high dependence is placed on exogenous variables, there may be enough of these so that they determine the outcome of the model behavior and thereby make the rest of the model formulation meaningless. On the other hand, if the oxogenous variables are not introduced strongly enough to control model behavior, they may very well destroy what would otherwise be qualitatively correct behavior of the model. If the model and the real world are behaving approximately the same except for a slightly different time scale, the introduction of variables from the outside world will appear in the model in the wrong time phase and can easily lead to grossly different qualitative results than would otherwise be obtained. These different qualitative results can be totally unlike the results from either the real world or the results which the model would give if the exogenous variables were internally generated through some approximate decision criteria.

Exogenous variables in our work here will be restricted to those whose effect we wish to measure on the system. In other words, they will represent the independent inputs, the influences of which we wish to observe in the system's behavior. They are not introduced to contribute to the reality of the system operation but to provide controlled disturbances whose effect we can observe. We will, therefore, study the effect of different time sequences in these exogenous variables just as we will study the effect of different internal decision criteria, model structures and coefficient values.

11. Aggregation of Variables

It is clear that a model of an industrial company or an economic system cannot possibly represent every individual decision and transaction taking place in the system. We are therefore forced to aggregate many of these transactions into common categories. How this is to be done is of the greatest importance but must still based on intuitive judgments since we have not yet succeeded in formulating explicit principles which will determine the level of detail which one must deal with in a dynamic model. At the present time, we can only make some general comments which may need to be changed or elaborated in the future.

a. Preserve Actual Decision Criteria.

The actual decision-making level of the system seems to be the key for guiding our model formulation. It appears that we can usually aggregate together all similar kinds of decisions but should not aggregate up to the point where it becomes impossible to discuss an aggregate flow in terms of its relationship to individual decisions. In other words, we can aggregate into a single stream all purchases of a given class but in this stream we should represent the sequence of actions which will be encountered by any individual order as it traverses the system. At each point in the system where an important decision is made, we should have available those variables on which we presume the decision is actually based. Therefore, we are speaking of a kind of model which takes explicit recognition of the psychological and social as well as economic forces which we believe determine decisions. This means that a model of the national economy and a model of industrial company behavior have much in common. We can aggregate companies into industries, and perhaps industries together on the basis of similar dynamic characteristics, but, in the composite aggregate, will still retain the mechanisms for making production, investment, sales promotion, dividend, and inventory decisions.

b. Preserve Non-Linear Limitations

If the actual decision criteria are properly represented, we will be preserving the essential non-linear characteristics of the system. For example, we will not produce goods in the model in excess of our plant productive capacity; we will not employ more labor than exists; we will not sell goods which we do not have. I believe it is not possible to preserve in a model the important behavior patterns unless aggregation is stopped before we eliminate the non-linear decision-making relationships. It is my feeling therefore that most aggregate models of economic behavior which have been explored in the past can have no real significance or meaning. By their very nature they exclude the characteristics of the system which determine its behavior.

c. <u>Aggregate Similar Actions</u>

Very often we can aggregate together a large class of similar decisions and actions, depending on the purpose of our study. Each model is presumably formulated for a particular purpose; otherwise, we will have no basis for making decisions on what to include. For many purposes in the study of an industrial company we can aggregate together all items in a particular product line, or, perhaps very often, all sales even in diverse product lines. Here, for the time being, we must be guided by experience and by judgment as to what is pertinent to the answers which we seek.

d. <u>Aggregate by Similar Time Constants</u>

Very often we can aggregate apparently dissimilar items because they are controlled by similar decision criteria and have similar behavior time constants. For example, in a model of the economic system we might expect for many purposes to aggregate together all items of food; and into another category soft goods, such as clothing; into another, hard goods including appliances and automobiles; and into another, housing. These major categories would, for most purposes, need to be kept separate from each other because they represent major dynamic differences as to the motivation for purchase, the deferrability of purchase, and the life of the inventory of the item held by the consumer. Likewise, we might aggregate together machine tools and industrial factory construction on the basis that changes in these occur together with approximately the same timing and on the basis of similar decisions.

e. <u>Using Discrete Events to Formulate Aggregate Flow.</u>

Even though we speak of aggregating large groups of individual physical items and decisions together, the aggregate must, nevertheless, traverse a channel which is descriptive of the channel traversed by the individual item. As an analogy, if we were developing a model of a city water works, we would include individual reservoirs, pumping stations, major distribution lines, and groups of water consumers segregated by type and by geographical location. We would derive a diagram which properly represents the circumstances encountered by an individual drop of water as it passes from wells, to reservoir, through piping, through pumps, and through various impediments to water flow until it is used. We would make no effort to identify the particular drop or molecule of water but we would have a flow diagram which properly represents the circumstances that a particular drop of water would encounter.

Even though we intend to aggregate many similar types of decisions, we must nevertheless formulate our system representation by examining the circumstances leading to a particular decision and its execution.

f. <u>Effect of Aggregation</u>

We can probably expect that the effect of aggregating more and more items together will be to smooth out and average a variety of slightly different reaction times. For example, one company might conceivably take exactly two weeks for processing a certain kind of information and reaching a decision. Another company might take, likewise, a fixed amount of time but an amount which is different from the first. The averaging of these effects will tend to smooth out the response curve and convert discrete "pipeline" delays into something more like the third-order exponential delay discussed earlier. For the time being, this qualitative effect should be kept in mind. In the future, it can be explored more fully in its own right by a simulation study that combines discrete events for the purpose of getting curves of typical aggregate behavior.

12. <u>Comments</u>

a. <u>Difference versus Differential Equations</u>

It would be possible to construct the equations as a system of differential equations rather than difference equations. It is my present feeling that this would lead to a system less understandable by the average management and economics student who will need to work with them. Furthermore, since the equations will be solved on a digital computer, which requires that differential equations be approximated by difference equations, we hereby avoid the necessity for translation and the necessity for having two systems of equations, one written by the formulator of the original situation and another as the interpretation of these by the digital computer programmer.

b. First versus Higher Order Integration

It will be noted that we have a system of difference equations employing first-order integration for coupling the equations for rates to those for levels. First-order integration instead of a higher order integration of the equations might be questioned. The relative merits are as yet unresolved. It is not clear that higher order integration will give more useful results and it will complicate the formulation of the equations. The relationship between the shorter of the time delays characterizing the system and the solution interval will often be such that only a few solutions will be obtained during the interval of these delays. This raises the possibility that the usual considerations regarding highly precise solution of difference equations may not apply. Our objective is to obtain the proper qualitative behavior of the system and approximately correct quantitative results. Errors of a few per cent in getting solutions to the particular equations are not important because the equations themselves will seldom be that close to representing the real world we are studying. Unlike certain mathematical procedures involving simultaneous equations, we would not expect problems of mathematical instability (as distinguished from real instability which may be a characteristic of the system under study) to arise, so long as the precautions, discussed earlier are observed in the selection of the solution interval.

c. Transparency of the Market.

In the system of equations described, decisions are made based only on that information that is presumed to be actually available at the decision-making point. This means, for example, that the ordinary "accounting equations" found in many economic models will not appear in this kind of equation system. It is clear that a consumer deciding to

buy a product does not first determine whether gross national product will continue to equal consumption plus savings.

Instead his decision will here be based on availability of the product, price, his own financial position, and other information which is, in fact, available to him. We therefore derive a set of equations which do not presume "market transparency", or the existence at decision-making points of information which would not normally be available.

Part II

Part II will appear as a separate memo to show the application of these general principles to the example outlined above in Section B.

List of Theses

Staff Biogr

Memorandum D-23

Massachusetts Institute of Technology
School of Industrial Management
Cambridge 39, Massachusetts

Industrial Dynamics Research

MEMO: D-23

SUBJECT: Production-Distribution System

FROM: Jay W. Forrester

DATE: October 22, 1958

TO: Industrial Dynamics Group

ABSTRACT

Equations and flow diagram for the production-distribution-advertising model used in the industrial summer session, August 1958.

15.589

INDUSTRIAL DYNAMICS

Production-Distribution System Equations

Model 9
Retail Stage

1L $$UOR.K = UOR.J + (DT)(RRR.JK - SSR.JK)$$

2L $$IAR.K = IAR.J + (DT)(SRR.JK - SSR.JK)$$

3R $$SSR.KL = \frac{UOR.K}{DFR.K}$$

4A $$DFR.K = (DUR)\sqrt{\frac{IDR.K}{IAR.K}}$$

5A $$IDR.K = (AIR)(RSR.K)$$

6L $$RSR.K = RSR.J + (DT)\left[\frac{1}{DRR}\right](RRR.JK - RSR.J)$$

7R $$PDR.KL = RRR.JK + \left[\frac{1}{DIR}\right]\left[(IDR.K - IAR.K) + (LDR.K - LAR.K) + (UOR.K - UNR.K)\right]$$

8A $$LDR.K = (RSR.K)\left[(DCR) + (DMR) + (DFD.K) + (DTR)\right]$$

9A LAR.K = (CPR.K) + (PMR.K) + (UOD.K) + (MTR.K)

10A UNR.K = (DUR)(RSR.K)

11L CPR.K = CPR.J + (DT)(PDR.JK - PSR.JK)

12R PSR.KL = FD3 (CPR.K, DCR)

13L PMR.K = PMR.J + (DT)(PSR.JK - RRD.JK)

14R RRD.KL = FD3(PMR.K, DMR)

15L MTR.K = MTR.J + (DT)(SSD.JK - SRR.JK)

16R SRR.KL = FD3 (MTR.K, DTR)

Distributor Stage

17L UOD.K = UOD.J + (DT)(RRD.JK - SSD.JK)

18L IAD.K = IAD.J + (DT)(SRD.JK - SSD.JK)

19R $SSD.KL = \frac{UOD.K}{DFD.K}$

20A $DFD.K = (DUD)\sqrt{\frac{IDD.K}{IAD.K}}$

21A $$\mathrm{IDD.K = (AID)(RSD.K)}$$

22L $$\mathrm{RSD.K = RSD.J + (DT)\left[\frac{1}{DRD}\right](RRD.JK - RSD.J)}$$

23R $$\mathrm{PDD.KL = RRD.JK + \left[\frac{1}{DID}\right]\Big[(IDD.K - IAD.K) + (LDD.K - LAD.K) + (UOD.K - UND.K)\Big]}$$

24A $$\mathrm{LDD.K = (RSD.K)\left[(DCD) + (DMD) + (DFF.K) + (DTD)\right]}$$

25A $$\mathrm{LAD.K = (CPD.K) + (PMD.K) + (UOF.K) + (MTD.K)}$$

26A $$\mathrm{UND.K = (DUD)(RSD.K)}$$

27L $$\mathrm{CPD.K = CPD.J + (DT)(PDD.JK - PSD.JK)}$$

28R $$\mathrm{PSD.KL = FD3\ (CPD.K,\ DCD)}$$

29L $$\mathrm{PMD.K = PMD.J + (DT)(PSD.JK - RRF.JK)}$$

30R $$\mathrm{RRF.KL = FD3\ (PMD.K,\ DMD)}$$

31L $$\mathrm{MTD.K = MTD.J + (DT)(SSF.JK - SRD.JK)}$$

32R $$\mathrm{SRD.KL = FD3\ (MTD.K,\ DTD)}$$

List of Theses
Staff Biographies

Factory Stage

33L $$UOF.K = UOF.J + (DT)(RRF.JK - SSF.JK)$$

34L $$IAF.K = IAF.J + (DT)(SRF.JK - SSF.JK)$$

35R $$SSF.KL = \frac{UOF.K}{DFF.K}$$

36A $$DFF.K = (DUF)\sqrt{\frac{IDF.K}{IAF.K}}$$

37A $$IDF.K = (AIF)(RSF.K)$$

38L $$RSF.K = RSF.J + (DT)\left[\frac{1}{DRF}\right](RRF.JK - RSF.J)$$

39R $$MDF.KL = RRF.JK + \left[\frac{1}{DIF}\right]\Big[(IDF.K - IAF.K) + (LDF.K - LAF.K) + (UOF.K - UNF.K)\Big]$$

40A $$LDF.K = (RSF.K)\Big[(DCF) + (DPF)\Big]$$

41A $$LAF.K = (CPF.K + OPF.K)$$

42A $$UNF.K = (DUF)(RSF.K)$$

43L $$CPF.K = CPF.J + (DT)(MDF.JK - MOF.JK)$$

44R $$MOF.KL = FD3\ (CPF.K,\ DCF)$$

45L OPF.K = OPF.J + (DT)(MOF.JK - SRF.JK)

46R SRF.KL = FD3 (OPF.K, DPF)

Advertising and Consumer Behavior

47R VIF.KL = (AVI)(IAF.K) + (AVS)(MOF.JK)

48L VPF.K = VPF.J + (DT)(VIF.JK - VCF.JK)

49R VCF.KL = FD3 (VPF.K, DVF)

50L VPA.K = VPA.J + (DT)(VCF.JK - VMC.JK)

51R VMC.KL = FD3 (VPA.K, DVA)

52L $$\text{VEC.K} = \text{VEC.J} + (\text{DT}) \left[\frac{1}{\text{DVC}}\right] (\text{VMC.JK} - \text{VEC.J})$$

53L PPC.K = PPC.J + (DT)(BGC.JK - BBC.JK)

54R BGC.JK = Given, independent input rate

55R $$\text{BBC.KL} = (\text{PPC.K}) \left[\frac{1}{\text{DPP.K}}\right]$$

56A $$\text{DPP.K} = \frac{1}{\left(\frac{1}{\text{ANC}}\right) + (\text{ACV})(\text{VEC.K})}$$

57R RRR.KL = (APC)(BBC.KL)

3rd Order Delay Equations

$$\mathrm{OUT.KL} = \mathrm{FD3}\ (\mathrm{LEV.K},\ \mathrm{DEL})$$

$$\mathrm{OUT.KL} = \mathrm{OUT.JK} + (\mathrm{DT}) \left[\frac{9}{\mathrm{DEL}} \right] \left[\left(\frac{3}{\mathrm{DEL}} \right) (\mathrm{EXV.K}) - \mathrm{OUT.JK} \right]$$

$$\mathrm{EXV.K} = \mathrm{EXV.J} + (\mathrm{DT}) \left[\frac{\mathrm{LEV.J}}{\mathrm{DEL}} - \mathrm{OUT.JK} \right]$$

$$\mathrm{LEV.K} = \mathrm{LEV.J} + (\mathrm{DT})(\mathrm{IND.JK} - \mathrm{OUT.JK})$$

List of Theses
Staff Biographies

Variables and Constants

1. ACV = constAnt relating Consumer buying and adVertising (reciprocal advertising units)

2. AID = proportionality constAnt for Inventory at Distributor (weeks)

3. AIF = proportionality constAnt for Inventory at Factory (weeks)

4. AIR = proportionality constAnt for Inventory at Retail (weeks)
(Ratio of desired inventory to smoothed sales)

5. ANC = constAnt, Normal (average) Customer purchasing delay time between entering consumer pool and actual purchase (weeks)

6. APC = constAnt, units Purchased per Customer (units/customer)

7. AVI = constAnt, adVertising, Inventory (reciprocal weeks)
(fraction of value of factory inventory spent on advertising per week)

8. AVS = constAnt, adVertising, Sales (dimensionless)
(fraction of factory sales rate devoted to advertising)

9. BBC = Buyers Buying at Consumer sector (customers/week)

10. BGC = prospective Buyer Generation in Consumer sector (customers/week)

11. CPD = Clerical Processing, Distributor (units)

12. CPF = Clerical Processing, Factory (units)

13. CPR = Clerical Processing, Retail (units)

14. DCD = Delay, Clerical at Distributor (weeks)

15. DCF = Delay, Clerical at Factory (weeks)

16. DCR = Delay, Clerical at Retail (weeks)

17. DFD = Delay in Filling orders at Distributor (weeks)

18. DFF = Delay in Filling orders at Factory (weeks)

19. DFR = Delay in Filling orders at Retail (weeks)

20. DID = Delay in Inventory (and pipeline) adjustment at Distributor (weeks)

21. DIF = Delay in Inventory (and pipeline) adjustment at Factory (weeks)

22. DIR = Delay in Inventory (and pipeline) adjustment at Retail (weeks)

23. DMD = Delay in order Mailing at Distributor (weeks)

24. DMR = Delay in order Mailing at Retail (weeks)

25. DPF = Delay in Production at Factory (weeks)

26. DPP = Delay between formation of Prospective purchaser and actual Purchase (weeks)

27. DRD = Delay in smoothing Requisitions at Distributor (weeks)

28. DRF = Delay in smoothing Requisitions at Factory (weeks)

29. DRR = Delay in smoothing Requisitions at Retail (weeks)

30. DT = solution time interval (weeks)

31. DTD = Delay in Transit to Distributor (weeks)

32. DTR = Delay in Transit to Retail (weeks)

33. DUD = "normal Delay," Unfilled orders at Distributor (weeks)

34. DUF = "normal Delay," Unfilled orders at Factory (weeks)

35. DUR = "normal Delay," Unfilled orders at Retail (weeks)

36. DVA = Delay in processing adVertising in Agencies and media (weeks)

37. DVC = Delay in adVertising becoming effective in Consumer actions (weeks)

38. DVF = Delay in adVertising decision at Factory (weeks)

39. IAD = Inventory Actually at Distributor (units)

40. IAF = Inventory Actually at Factory (units)

41. IAR = Inventory Actually at Retail (units)

42. IDD = Inventory Desired at Distributor (units)

43. IDF = Inventory Desired at Factory (units)

44. IDR = Inventory Desired at Retail (units)

45. LAD = pipeLine orders Actually in transit to Distributor (units)

46. LAF = pipeLine orders Actually in transit to Factory (units)

47. LAR = pipeLine orders Actually in transit to Retail (units)

48. LDD = pipeLine orders Desired in transit to Distributor (units)

49. LDF = pipeLine orders Desired in transit to Factory (units)

50. LDR = pipeLine orders Desired in transit to Retail (units)

51. MDF = Manufacturing Decision impending at Factory before clerical and decision delays (units/week)

52. MOF = Manufacturing Orders into Factory (units/week)

53. MTD = Material in Transit to Distributor (units)

54. MTR = Material in Transit to Retail (units)

55. OPF = Orders in Process at Factory (units)

56. PDD = Purchasing rate Decision at Distributor (units/week)

57. PDR = Purchasing rate Decision at Retail (units/week)

58. PMD = Purchase orders in Mail from Distributor (units)

59. PMR = Purchase orders in Mail from Retail (units)

60. PPC = Prospective Purchasers in Consumer sector (customers)

61. PSD = Purchase orders Sent from Distributor (units/week)

62. PSR = Purchase orders Sent from Retail (units/week)

63. RRD = Requisitions Received at Distributor (units/week)

64. RRF = Requisitions Received at Factory (units/week)

65. RRR = Requisitions Received at Retail (units/week)

66. RSD = Requisitions Smoothed at Distributor (units /week)

67. RSF = Requisitions Smoothed at Factory (units/week)

68. RSR = Requisitions Smoothed at Retail (units/week)

69. SRD = Shipments Received at Distributor inventory (units/week)

70. SRF = Shipments Received at Factory inventory (units/week)

71. SRR = Shipments Received at Retail inventory (units/week)

72. SSD = Shipments Sent from Distributor inventory (units/week)

73. SSF = Shipments Sent from Factory (units/week)

74. SSR = Shipments Sent from Retail inventory (units/week)

75. UND = Unfilled orders, Normal, at Distributor (units)

76. UNF = Unfilled orders, Normal, at Factory (units)

77. UNR = Unfilled orders, Normal, at Retail (units)

78. UOD = Unfilled Orders at Distributor (units)

79. UOF = Unfilled Orders at the Factory (units)

80. UOR = Unfilled Orders at Retail (units)

81. VCF = adVertising rate Committed from Factory (sales units/week)

82. VEC = adVertising level Effective in influencing Consumer (advertising units/week)

83. VIF = adVertising Impending decision from Factory rate of advertising (sales units/week)

84. VMC = adVertising presented by Media to Consumer (sales units/week)

85. VPA = adVertising in Process in Agencies (sales units)

86. VPF = adVertising decisions in Process at Factory (sales units)

3rd Order Delay Variables and Constants

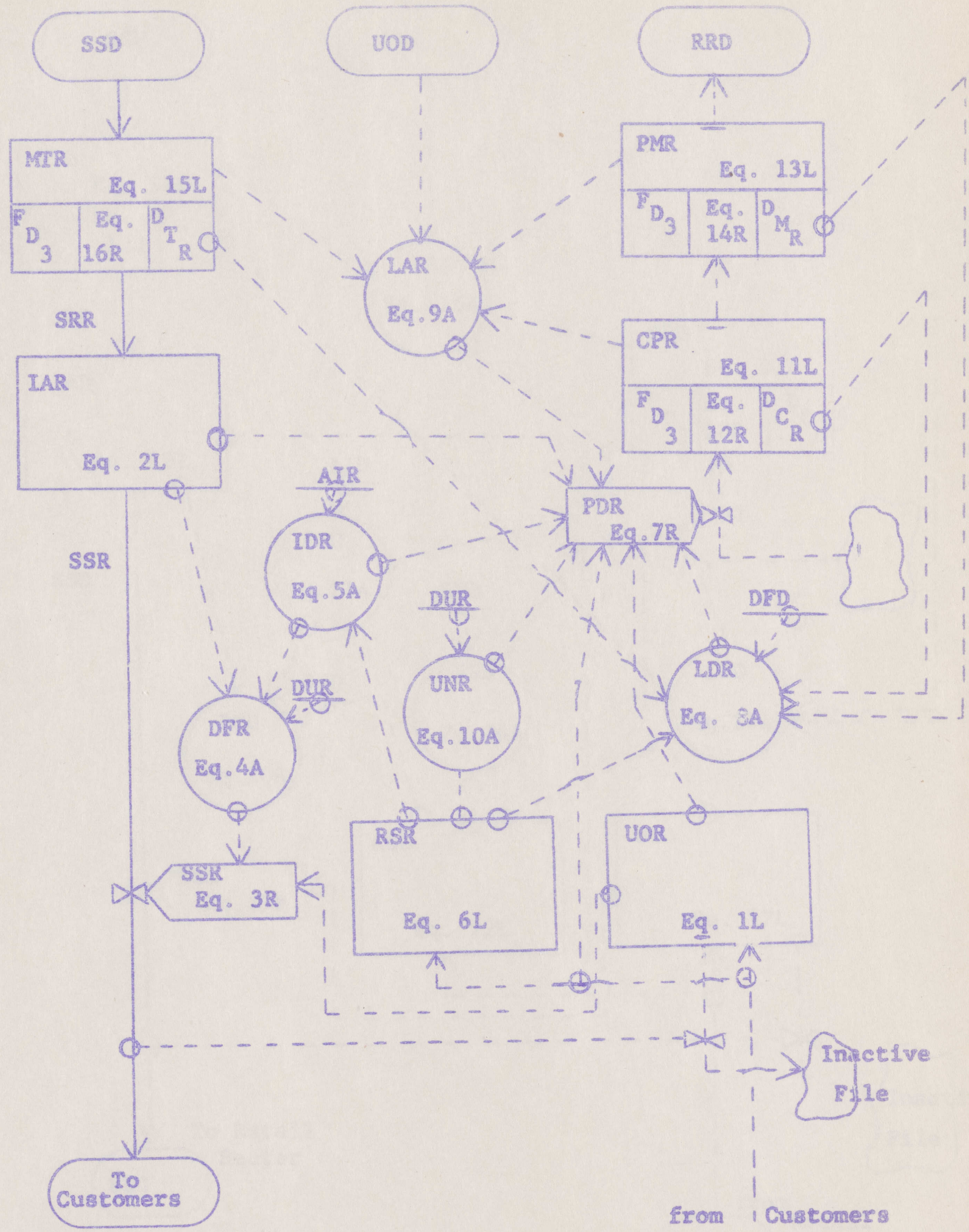
1. EXV = EXtra Variable needed for mathematics (units)

2. DEL = average DEL ay in 3rd order delay (weeks)

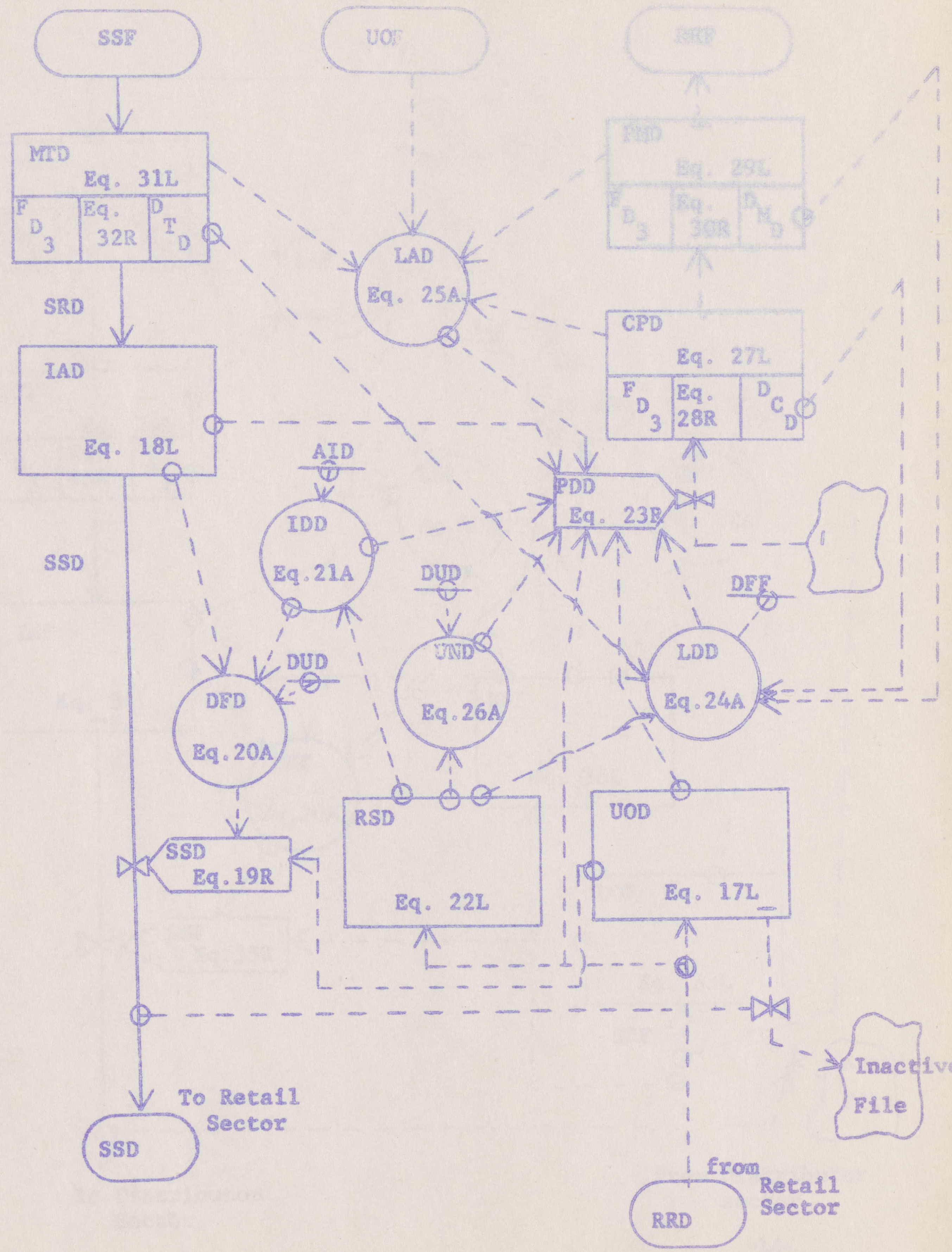
3. IND = INput to 3rd order Delay (units/week)

4. LEV = LEVel of orders in 3rd order delay (units)

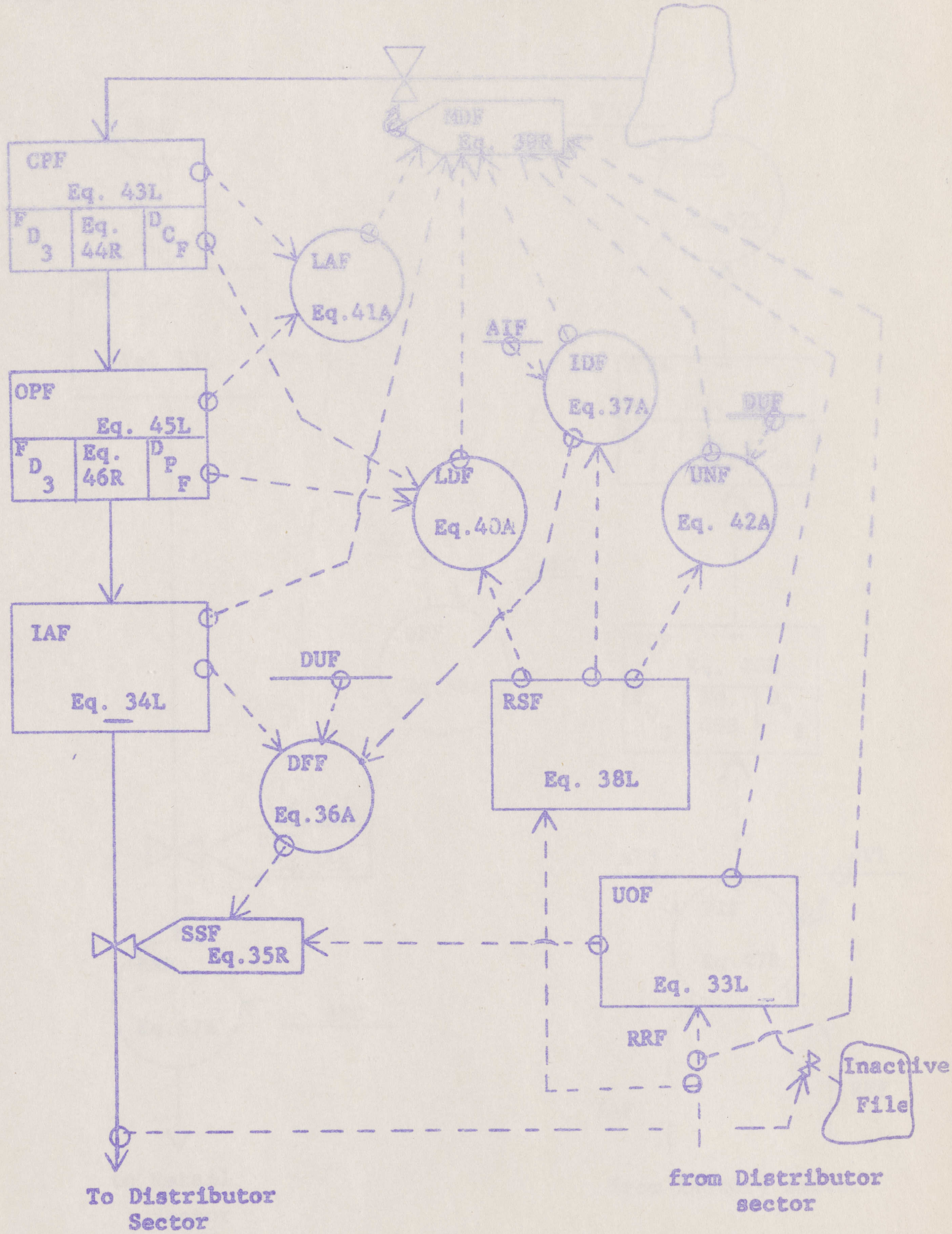
5. OUT = OUTput of 3rd order delay (units/week)


SSD
UOD
RRD
MTR Eq. 15L
F D 3
Eq. 16R
D T R
PMR Eq. 13L
F D 3
Eq. 14R
D M R
LAR Eq. 9A
SRR
CPR Eq. 11L
F D 3
Eq. 12R
D C R
IAR Eq. 2L
AIR
PDR Eq. 7R
IDR Eq. 5A
SSR
DUR
DFD
UNR Eq. 10A
LDR Eq. 8A
DUR
DFR Eq. 4A
RSR Eq. 6L
UOR Eq. 1L
SSR Eq. 3R
Inactive File
To Customers
from Customers

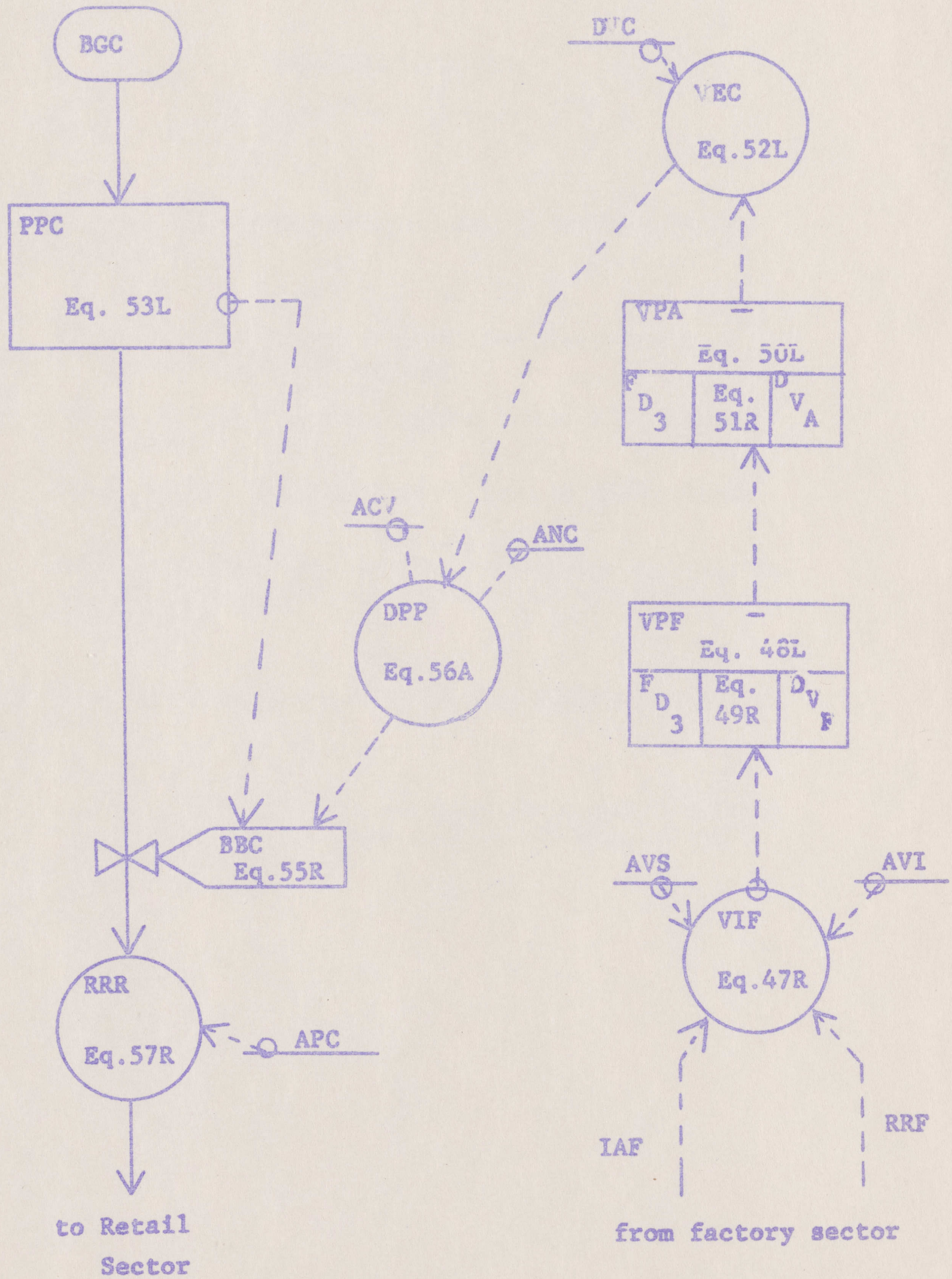
Flow Diagram for Distribution Sector of Production-Distribution, Model 9



Flow Diagram for Factory Sector of Production-Distribution, Model 9



Flow Diagram of Advertising Sector of Production-Distribution, Model 9



List of Theses

Staff Biographies

Partial List of Theses

Industrial Dynamics Staff

The following were under the supervision of Professor Jay W. Forrester:

Blitz, Melvin, "Digital Computers and Simulators of Dynamic Industrial Problems"

Hadley, George, "Towards a Dynamic Model of the National Economy"

Katz, Abraham, "An Operations Analysis of an Electronic Systems Firm"

Krasnow, Howard, "The Dynamics of Production and Distribution in the Rayon and Acetate Sector of the Textile Industry"

The following was under the supervision of Professor Joseph V. Yance:

Fujiyama, Kakuichiro, "Capacity Expansion in the Electric Power Industry: A Study in Fluctuation in Capital Investment Process"

Staff Biographies

John L. Enos

Assistant Professor
of Marketing

Born: July 13, 1924

S.B., (Economics and Chemical Engineering)
M.I.T., 1949
M.A., (Economics) Western Reserve, 1956
Ph.D., (Economics) M.I.T., (in process)

First appointed: 1955

Entered M.I.T. in 1942; served with the U. S. Air Force and O.S.S. from 1943-45; returned to M.I.T. In 1949, went to Standard Oil of Ohio for three years after which he spent some time with the Federal Reserve Bank of Cleveland. Received a Fulbright award to the Netherlands School of Economics in 1952 and worked in the Central Planning Bureau on the problem of stimulating private investment in Holland. Returned to M.I.T. in 1953 on a Goodyear Fellowship in Industrial Economics. Secretary, Graduate Economic Seminar, 1954-55. Member of the American Economic Association and Econometric Society. Consultant in Industrial Management. Publications include: "A Note on the Current Status of National Economic Planning in Great Britain"; "The High Price of Gold"; and "A Measure of the Rate of Technological Progress".

Revised 9/1/57

Jay W. Forrester

Professor of
Industrial Management

Born: July 14, 1918

B.Sc., (Elec. Eng.) Univ. of Nebraska, 1939
M.Sc., (Elec. Eng.) M.I.T., 1945
D.Eng., Univ. of Nebraska, 1954

First appointed: 1956

Co-founder of servo-mechanisms laboratory at M.I.T. in 1940, developing electric and hydraulic servo-mechanisms for gun mounts and radar; Supervisor of Whirlwind I digital computer development from 1944-1951 when he founded the Digital Computer Laboratory at M.I.T. and acted as its Director until 1956. In 1951 became a Division Head at Lincoln Laboratory in charge of developing and engineering the SAGE (Semi-Automatic Ground Environment) air defense system for the continental United States now being installed for the Air Defense Command. He left the Lincoln Laboratory to join the School in July, 1956. Consultant for the Department of Defense, he holds patents in servo-mechanisms, digital information storage, etc. and has published numerous papers on digital computers and their applications. Member of the Scientific Advisory Board of the National Security Agency, and of the editorial board of the Institute of Radio Engineers.

Revised 9/1/57

Phyllis Fox
(Mrs. George Sternlieb)

Research Associate

Born: March 13, 1923

B.A., Wellesley, 1944
B.S. (E.E.) Univ. of Colorado, 1948
M.S. (E.E.) M.I.T., 1949
Ph.D. (Mathematics) M.I.T., 1954

First appointed: 1958

After receiving her degree at Wellesley, Dr. Fox operated a Differential Analyzer for the General Electric Company for two years; while working toward M.I.T. degrees served as Research Assistant in Digital Computer Laboratory later as Fellow in Professor Morse's Committee on Machine Computation. Went with AEC Computing Center, Institute of Mathematical Sciences, N.Y. University, using the Univac and 704 computers in the solution of problems in the fields of general numerical analysis, fluid mechanics and shock waves, reactor calculations, etc. Publications include: Servomechanisms Laboratory, M.I.T. Report R-169, "The Solution of Power-Network Problems on Large-Scale Digital Computers". June 1949; "Propagation of Shock Waves in the Generalized Roche Model" (with P.A. Carrus, Felix Haas and Zdenek Kopal) The Astrophysical Journal Vol. 113, No. 3, May, 1951, pp. 496-518; "Perturbation Theory of Wave Propagation Based on the Method of Characteristics", Journal of Mathematics and Physics, Vol. 34, No. 3, October, 1955, pp. 133-151; "On the Numerical Solution of the Equations for Spherical Waves" (with Anthony Ralston), Journal of Mathematics and Physics, Vol 36, No. 4, January, 1958, pp. 313-328. Dr. Fox will be associated with Dr. Forrester in Industrial Dynamics research.

Martin Greenberger

Assistant Professor of
Industrial Management

Born: November 30, 1931

A.B., Harvard (Summa Cum Laude), 1953
A.M., (Applied Mathematics) Harvard, 1956
Ph.D., (Applied Mathematics) Harvard, 1958

First appointed: 1958

Joined the M.I.T. Faculty after forming and managing IBM Applied Science Cambridge, a research group associated with the M.I.T. Computation Center. Formerly at Harvard, was a teaching fellow in Mathematics and a resident adviser. Earlier held a National Science Foundation Fellowship and was on the staff of the Harvard Computation Laboratory engaged in a data processing study of accounting systems in the Public Utility Industry. With the Seeligson Engineering Committee of San Antonio, taught statistical analysis and conducted a computer project to predict oil recovery factors for sandstone reservoirs. While serving with the Air Force, designed iterative procedures for solving multiple regression equations by computer and organized (with others) a Southwest Section to the Association for Computing Machinery. Several technical papers: one on equipment replacement was awarded first place in a national contest sponsored by the Illinois Institute of Technology. Doctoral thesis dealt with a simulation of the U.S. Economy.

George F. Hadley

Assistant Professor of
Industrial Management

Born: November 12, 1930

B.S. (C.E.) Carnegie Tech., 1952
A.M. (Physics) Harvard, 1954
Ph.D. (Physics) Harvard, 1955

First appointed: 1957

Mr. Hadley was a Teaching Fellow in Chemistry and Physics at Harvard from 1952-1954; lectured in group theory occasionally. With the Standard Oil Company of California for over a year (as a Research Physicist). He worked on the application of the IBM 701 and 704 computers to solve the partial differential equations of petroleum reservoir dynamics. He has also had industrial experience in the chemical and aircraft industries. Mr. Hadley has written technical papers in theoretical physics, mechanical engineering, reservoir engineering, and linear programming. Member of Tau Beta Pi, Sigma Xi, and Phi Kappa Phi, he was Texas Company Fellow in Physics at Harvard, where he also received a Harvard Foundation Scholarship as well as Carnegie and Wilson Scholarships at Carnegie Tech. Mr. Hadley will join our faculty in the fall of 1957.

Alexander L. Pugh III
Research Associate

Born: September 24, 1930
B.S. in E.E. University of Pennsylvania
S.M. in E.E. M.I.T.

After completing the Masters degree, Mr. Pugh worked as a research assistant in Operations Research coding waiting line problems for the C.P.C. computer, and as a research assistant at Lincoln Laboratory developing basic circuits for the transistorized computer, TX-0. From 1955-1957 he served in the Air Force as a lieutenant where he developed analogue circuitry for a radar system. In 1957 he joined the Industrial Dynamics group as a research assistant and assisted in the early work on the national economy model. Presently he is a research associate in that group and is coding the input portion of DYNAMO, a computer routine to test the models that the industrial dynamics formulates.

Joseph V. Yance

Assistant Professor
of Industrial Management

Born: July 10, 1926

B.S., M.I.T., 1948
Ph.D., Harvard, 1955

First appointed: 1957

While at Harvard, Dr. Yance was a Research Assistant on the Harvard Economic Research Project concerned with investment models. After receiving his degree at Harvard, he served with the U. S. Army where he worked with a civilian operations research group studying the determination of the optimum organization and weapon assignment of rifle squads. Coming to the School in the Spring of 1957 as a Research Associate, he was appointed Assistant Professor for the coming academic year 1957 - 1958.

Biographies of Full-time Research Assistants

Fey, Willard R.: B.S.(E.E.) and in Business and Engineering Administration MIT 1957; now studying in the Electrical Engineering Graduate School for a M.S. in E.E. 1959. He joined the Industrial Dynamics Group after graduation to work on the Sprague Electric Project. At present, he is concerned with the derivation of a mathematical model of the operations of one Sprague product line.

Howard, David J.: B.S.(E.E.) McGill University 1957. At present he is engaged in programming research for the School of Industrial Management, MIT.

Parker, Theodore: B.S.(E.E.) Duke University 1957; now working toward S.M.(E.E.) MIT 1959. Previous experience includes patent work, design and construction of transistorized test equipment for a magnetic core buffer memory, and studies into the multihole magnetic cores for performing logical operations and possible analogue applications. Present work is in developing the industrial sector of a national economy model.

Reis, Wendyl: B.S. Bus. Eng. Admin. MIT 1956; graduate work at Columbia University Graduate School of Business; now a candidate for M.S. Industrial Management, MIT. He is presently working on a case study of the production-distribution operations of the Sprague Electric Company.

Roberts, Edward: B.S.(E.E.), MIT 1958; M.S.(E.E.), MIT 1958; M.S. (Industrial Mgt.) MIT 1959 (in process). He has been a member of the Cooperative Course in Electrical Engineering associated with the General Electric Company for three years where he worked on business systems applications of digital computers. In the Industrial Dynamics Group Mr. Roberts has prepared an automatic multi-variable graph plotting program using the IBM 704 and is now working to formulate a dynamic model of military research and development activities.

Spiro, Michael: B.S. Carnegie Inst. of Technology 1956; M.A. Yale University 1958; Ph.D. Economics MIT--in process. He has worked in the cost accounting and production department of the Star-Drill Keystone Company of Beaver Falls, Penn., and for the past two summers he has been with the Datamatic Division of Minneapolis Honeywell as a junior-systems engineer. Presently he is engaged in the formulation of the consumer-household sector of a macro-economic model.

Computer Programmer

Duren, Grace A.: B.A. Radcliffe 1957. Mrs. Duren worked for IBM as technical librarian and 704 programmer from Sep. 1957 - June 1958, and she started to work for the Industrial Dynamics Group in October 1958 as a programmer assisting Dr. Martin Greenberger and Dr. Phyllis Fox.